

SOLVED PROBLEMS IN SOIL MECHANICS

SUMMARY OF (SOME) VERY IMPORTANT IDEAS AND/ EQUATIONS

General expression for the bulk unit weight (sometimes referred to as **moist** unit weight)

$$\gamma_{bulk} = \gamma = \frac{W}{V} = \frac{(1 + m)G_s\gamma_w V_s}{V}$$

$$\text{When } V = 1; V_s = \frac{1}{1 + e} = 1 - n$$

$$\Rightarrow \gamma = \frac{(1 + m)G_s\gamma_w V_s}{V} = \frac{(1 + m)G_s\gamma_w}{1 + e}$$

$$\gamma = \frac{(1 + m)G_s\gamma_w}{1 + e} \Rightarrow \rho = \frac{(1 + m)G_s\rho_w}{1 + e}$$

$$\gamma = (1 + m)(1 - n)G_s\gamma_w \Rightarrow \rho = (1 + m)(1 - n)G_s\rho_w$$

Also, recall from the basic fundamental relationships that:

$$\gamma = \frac{W}{V} = \frac{W_w + W_s}{V}$$

$$\gamma_d = \frac{W_s}{V} \Rightarrow V = \frac{W_s}{\gamma_d}$$

$$m = \frac{W_w}{W_s} \Rightarrow W_w = mW_s$$

$$\Rightarrow \gamma = \frac{W_w + W_s}{V} = \frac{mW_s + W_s}{\frac{W_s}{\gamma_d}} = \gamma_d(1 + m)$$

$$\Rightarrow \gamma = (1 + m)\gamma_d$$

Likewise

$$\gamma = \frac{(1 + m)G_s\gamma_w}{1 + e} = (1 + m) \frac{G_s\gamma_w}{1 + e}; \frac{G_s\gamma_w}{1 + e} = \gamma_d$$

$$\Rightarrow \gamma = (1 + m)\gamma_d$$

$$\Rightarrow \gamma_d = \frac{\gamma}{1+m}; \quad \rho_d = \frac{\rho}{1+m}$$

$$\Rightarrow W_s = \frac{W}{1+m}; \quad M_s = \frac{M}{1+m}$$

Also

$$S_r = \frac{V_w}{V_v}$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{mG_s \gamma_w V_s}{\gamma_w} \Rightarrow V_w = mG_s V_s$$

$$\Rightarrow S_r = \frac{V_w}{V_v} = \frac{mG_s V_s}{V_v}$$

Now, with

$$V_s = 1; \quad V_v = e$$

$$\Rightarrow S_r = \frac{mG_s V_s}{V_v} = \frac{mG_s}{e}$$

$$\Rightarrow S_r e = mG_s \text{ or } m = \frac{S_r e}{G_s}$$

OR

With

$$V = 1; \quad V_s = \frac{1}{1+e}; \quad V_v = n = \frac{e}{1+e}$$

$$\Rightarrow S_r = \frac{mG_s V_s}{V_v} = mG_s \left(\frac{V_s}{V_v} \right); \quad \frac{V_s}{V_v} = \frac{1}{e}; \text{ since } \frac{V_v}{V_s} = e$$

$$\Rightarrow S_r = \frac{mG_s}{e} \text{ or } m = \frac{S_r e}{G_s}$$

$$\Rightarrow \gamma = \frac{(1+m)G_s \gamma_w}{1+e} = \frac{(1 + \frac{eS_r}{G_s})G_s \gamma_w}{1+e} = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

Hence

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1 + e} \Rightarrow \rho = \frac{(G_s + eS_r)\rho_w}{1 + e}$$

Therefore

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1 + e} \Rightarrow \rho = \frac{(G_s + eS_r)\rho_w}{1 + e}; \quad 0 < S_r < 1$$

Also;

$$S_r = 0; \Rightarrow \gamma = \frac{(G_s + 0)\gamma_w}{1 + e} = \frac{G_s\gamma_w}{1 + e} = \gamma_d$$

Therefore

$$\gamma_d = \frac{G_s\gamma_w}{1 + e} \Rightarrow \rho_d = \frac{G_s\rho_w}{1 + e}; \quad S_r = 0$$

$$S_r = 1; \gamma = \frac{(G_s + e)\gamma_w}{1 + e} = \gamma_{sat}$$

Therefore

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e} \Rightarrow \rho_{sat} = \frac{(G_s + e)\rho_w}{1 + e}; \quad S_r = 1$$

γ_{sat} , γ and γ_d EXPRESSED IN TERMS OF THE POROSITY

The last expression for γ_{sat} can also be re-written as

$$\gamma_{sat} = \frac{G_s\gamma_w}{1 + e} + \left(\frac{e}{1 + e}\right)\gamma_w = \gamma_d + n\gamma_w \Rightarrow \rho_{sat} = \rho_d + n\rho_w$$

since

$$\frac{G_s\gamma_w}{1 + e} = \gamma_d \text{ and } \frac{e}{1 + e} = n$$

$$\Rightarrow \gamma_{sat} = \gamma_d + n\gamma_w$$

$$\Rightarrow \rho_{sat} = \rho_d + n\rho_w$$

Also;

$$\gamma_d = \frac{G_s\gamma_w}{1 + e} = G_s\gamma_w \left(\frac{1}{1 + e}\right) = G_s\gamma_w(1 - n)$$

$$\Rightarrow \gamma_d = G_s \gamma_w (1 - n)$$

Again;

$$\gamma_{sat} = \gamma_d + n\gamma_w = G_s \gamma_w (1 - n) + n\gamma_w = [(1 - n)G_s + n]\gamma_w$$

$$\Rightarrow \gamma_{sat} = [(1 - n)G_s + n]\gamma_w$$

$$\Rightarrow \rho_{sat} = [(1 - n)G_s + n]\rho_w$$

Lastly

$$\gamma = \frac{(1 + m)G_s \gamma_w V_s}{V}$$

$$V = 1; V_s = 1 - n$$

$$\Rightarrow \gamma = \frac{(1 + m)G_s \gamma_w V_s}{V} = (1 - n)(1 + m)G_s \gamma_w$$

$$\Rightarrow \gamma = (1 - n)(1 + m)G_s \gamma_w$$

$$\Rightarrow \rho = (1 - n)(1 + m)G_s \rho_w$$

Again, with

$$m = \frac{S_r e}{G_s} = \left(\frac{n}{1 - n} \right) \frac{S_r}{G_s}$$

$$\gamma = (1 - n) \left(1 + \left(\frac{n}{1 - n} \right) \frac{S_r}{G_s} \right) G_s \gamma_w = (1 - n) \left(G_s + \left(\frac{n}{1 - n} \right) S_r \right) \gamma_w$$

$$= [(1 - n)G_s + nS_r]\gamma_w$$

$$\Rightarrow \gamma = [(1 - n)G_s + nS_r]\gamma_w$$

$$\Rightarrow \gamma = [(1 - n)G_s + nS_r]\gamma_w; \mathbf{0 < S_r < 1}$$

$$\Rightarrow \rho = [(1 - n)G_s + nS_r]\rho_w; \mathbf{0 < S_r < 1}$$

VOLUME OF AIR V_a , AIR CONTENT N_a , AND AIR VOIDS RATIO (OR PERCENTAGE AIR VOIDS), A_v

Air content:

$$N_a = \frac{V_a}{V}$$

$$V_a = V_v - V_w$$

Now, with

$$V_v = n; \quad V_w = nS_r; \quad V = 1$$

$$\Rightarrow N_a = n(1 - S_r)$$

Air voids ratio:

$$A_v = \frac{V_a}{V_v} = \frac{n(1 - S_r)}{n}$$

$$\Rightarrow A_v = 1 - S_r$$

OR

$$A_v + S_r = 1$$

This last equations tells us an important relationship that exists between these two quantities that if let's say, the air voids ratio is 0.2, then the degree of saturation is 0.8 or, when the air voids ratio is 0.6, the degree of saturation is 0.4 so that the two ratios must necessarily sum to a value of 1

ILLUSTRATIVE EXAMPLES/ QUESTIONS:

ILLUSTRATIVE EXAMPLE 1:

The void ratio e of a given sample of a soil was found to be 0.60. Using this only information:

- (a) Determine the volume of solids V_s and volume of voids V_v of the sample if its total volume V is

- (i) $1m^3$
- (ii) $10m^3$
- (iii) $15m^3$
- (b) Assuming the soil above has moisture content of 20% and specific gravity 2.70, determine:
 - (i) The density of solids, ρ_s of the soil. What is the empirical relationship between this quantity and the dry density ρ_d ?
 - (ii) its bulk density, ρ and its bulk unit weight γ
 - (iii) its dry density, ρ_d and its bulk unit weight γ_d
 - (iv) its saturated density, ρ_{sat}
 - (v) its total weight W ; and its dry weight W_s for each case
 - (vi) its total mass M and its dry mass M_s for each case
 - (vii) its current degree of saturation, S_r
 - (viii) its volume that is occupied by water V_w , for each case
 - (ix) the volume occupied by air V_a , air content, N_a and the air voids ratio A_v for each case
- (c) What is the value of moisture content at complete saturation m_{sat} , and the associated SATURATED WEIGHTS W_{sat} for each case?
- (d) Using the information obtained in parts (b) (iv)-(vii) and (c) above, determine the amount of water for each particular volume case, that would be required to be added to the soil in order
 - (i) To raise its dry density moisture content to the current value of $m = 0.20$
 - (ii) Convert its dry density ($m = 0$) to saturation state
 - (iii) Convert its bulk density ($m = 0.20$) to saturation state

- (e) What volume of water was added in each of the cases in (d) above
- (f) What are the boundary values of m for this type of soil?

Given:

$$e = 0.60$$

$$V = V_1 = 1m^3; V = V_2 = 10m^3; V = V_3 = 15m^3$$

$$G_s = 2.70$$

Required:

$$V_s = ?$$

$$V_v = ?$$

$$\rho_s = ?$$

$$\rho = ?$$

$$\rho_d = ?$$

$$\rho_{sat} = ?$$

$$W = ?$$

$$W_s = ?$$

$$M = ?$$

$$M_s = ?$$

$$S_r = ?$$

$$V_w = ?$$

$$m = ?$$

$$m_{sat} = ?$$

$$W_{sat} = ?$$

SOLUTION:

(a) Volumes of solids and voids

$$e = 0.60$$

From the relationship between e and V_s we have

$$V_s = \frac{1}{1+e} = \frac{1}{1+0.6} = \frac{1}{1.6} = \frac{10}{16} = 0.625$$

Now, we know that

$$V_s = \frac{1}{1+e}; \text{ when } V = 1$$

Therefore the volume of solids is;

$$(i) \quad V_s = \frac{1}{1+0.60} = \frac{1}{1.6} = 0.625$$

$$V_s = 0.625m^3 \text{ for } V = 1m^3$$

and the volume of voids corresponding to the above volume of solids is therefore

$$V_v = V - V_s$$

$$V_v = 1.0 - 0.625$$

$$V_v = 0.375m^3 \text{ for } V = 1m^3$$

ALTERNATIVELY:

$$V = 1m^3; V_v = n$$

$$\Rightarrow V_v = \frac{e}{1+e} = \frac{0.6}{1.6} = 0.375m^3$$

Likewise;

$$(ii) \quad V_s = 10 \times 0.625 = 6.25m^3; V_v = 10 \times 0.375m^3 = 3.75m^3 \text{ for } V = 10m^3$$

$$(iii) \quad V_s = 15 \times 0.625 = 9.375m^3; V_v = 15 \times 0.375m^3 = 5.625m^3 \text{ for } V = 15m^3$$

(b) Densities

Since we are also given the values $G_s = 2.70$; $m = 0.20$, so with $e = 0.60$ being also known, we can simply use the relationship

$$\frac{\rho_s}{\rho_w} = G_s$$

(i) Density of solids, ρ_s

From the relationship

$$\frac{\rho_s}{\rho_w} = G_s$$

$$\Rightarrow \rho_s = G_s \rho_w = 2.70 \times (1000 \text{ kg m}^{-3}) = 2700 \text{ kg m}^{-3}$$

$$\rho_s = 2700 \text{ kg m}^{-3}$$

Relationship between density of solids and dry density:

$$\rho_s = G_s \rho_w$$

$$\rho_d = \frac{G_s \rho_w}{1 + e}$$

Upon combining the two equations, we arrive at a relationship

$$\rho_d = \frac{\rho_s}{1 + e}$$

or

$$\rho_s = \rho_d (1 + e)$$

(ii) Bulk density, ρ and bulk unit weight γ

Bulk density:

$$\rho = \frac{(1+m)G_s \rho_w}{1 + e}; m = 0.20, e = 0.60$$

$$\rho = \frac{(1 + 0.20) \times 2700 \text{ kg m}^{-3}}{1 + 0.60} = \frac{3240 \text{ kg m}^{-3}}{1.60} = 2025 \text{ kg m}^{-3}$$

$$\rho = 2025 \text{kgm}^{-3}$$

ALTERNATIVELY:

$$\rho = \frac{(G_s + eS_r)\rho_w}{1 + e}; S_r = ?$$

$$S_r e = mG_s \Rightarrow S_r = \frac{mG_s}{e}$$

$$S_r = \frac{0.20 \times 2.70}{0.60} = 0.9 = 90\%$$

$$\begin{aligned} \Rightarrow \rho &= \frac{(G_s + eS_r)\rho_w}{1 + e} = \frac{(2.70 + 0.60(0.9))1000 \text{kgm}^{-3}}{1 + 0.60} \\ &= \frac{(2.70 + 0.54)1000 \text{kgm}^{-3}}{1.60} = \frac{3240 \text{kgm}^{-3}}{1.60} = 2025 \text{kgm}^{-3} \end{aligned}$$

Bulk unit weight:

$$\gamma = \rho g$$

$$\gamma = 2025 \text{kgm}^{-3} \times 10 \text{ms}^{-2} = 20250 \text{Nm}^{-3}$$

$$\gamma = 20.25 \text{kNm}^{-3}$$

(iii) Dry density, ρ_d and dry unit weight γ_d

Dry density:

$$\rho_d = \frac{\rho}{1 + m} = \frac{2025 \text{kgm}^{-3}}{1.20} = 1687.5 \text{kgm}^{-3}$$

$$\rho_d = 1687.5 \text{kgm}^{-3}$$

ALTERNATIVELY:

$$\rho_d = \frac{G_s \rho_w}{1 + e}$$

$$\rho_d = \frac{2.70(1000)kgm^{-3}}{1.60} = \mathbf{1687.50kgm^{-3}}$$

OR

$$\rho_d = G_s \rho_w (1 - n)$$

$$n = \frac{e}{1 + e} = \frac{0.60}{1 + 0.60} = 0.375$$

$$\rho_d = 2.70 \times 1000 kgm^{-3} (1 - 0.375) = 2700 kgm^{-3} \times 0.625 = \mathbf{1687.50 kgm^{-3}}$$

OR

$$\rho_d = \frac{M_s}{V} = \frac{G_s \rho_w V_s}{V}$$

$$V_s = 0.625m^3 \text{ when } V = 1m^3$$

$$\Rightarrow \rho_d = \frac{G_s \rho_w V_s}{V} = \frac{2.70 \times 1000 kgm^{-3} \times 0.625m^3}{1m^3} = 2700 kgm^{-3} \times 0.625m^3$$

$$= \mathbf{1687.50 kgm^{-3}}$$

Dry unit weight:

$$\gamma_d = \rho_d g$$

$$\gamma_d = \mathbf{16.875 kNm^{-3}}$$

(iv) Saturated density, ρ_{sat} and SATURATED UNIT WEIGHT; γ_{sat}

Saturated density:

$$\rho_{sat} = \rho_d + n \rho_w$$

$$\begin{aligned}
\Rightarrow \rho_{sat} &= 1687.50 \text{kgm}^{-3} + 0.375((1000) \text{kgm}^{-3}) \\
&= 1687.50 \text{kgm}^{-3} + 375 \text{kgm}^{-3} \\
\rho_{sat} &= \mathbf{2062.50 \text{kgm}^{-3}}
\end{aligned}$$

ALTERNATIVELY:

$$\begin{aligned}
\rho_{sat} &= \frac{(G_s + eS_r)\rho_w}{1 + e} \\
\Rightarrow \rho_{sat} &= \frac{(G_s + e)\rho_w}{1 + e}; S_r = 1
\end{aligned}$$

Therefore

$$\rho_{sat} = \frac{(2.70 + 0.60)(1000) \text{kgm}^{-3}}{1 + 0.60} = \frac{3300 \text{kgm}^{-3}}{1.60} = \mathbf{2062.50 \text{kgm}^{-3}}$$

OR

$$\begin{aligned}
\rho_{sat} &= [(1 - n)G_s + n]\rho_w \\
n &= 0.375 \\
\rho_{sat} &= [(1 - 0.375)2.70 + 0.375]1000 \text{kgm}^{-3} \\
\rho_{sat} &= [(0.625 \times 2.70) + 0.375]1000 \text{kgm}^{-3} = [1.6875 + 0.375]1000 \text{kgm}^{-3} \\
&= 2.0625 \times 1000 \text{kgm}^{-3} = \mathbf{2062.50 \text{kgm}^{-3}}
\end{aligned}$$

OR

$$\begin{aligned}
\rho_{sat} &= \frac{(1 + m_{sat})G_s\rho_w}{1 + e}; m_{sat} = ? \\
S_r e &= m_{sat} G_s; S_r = 1 \\
\Rightarrow m_{sat} &= \frac{e}{G_s} = \frac{0.6}{2.7} = \frac{2}{9} = 0.2222 = 22.22\%
\end{aligned}$$

$$\begin{aligned}\Rightarrow \rho_{sat} &= \frac{(1 + m_{sat})G_s\rho_w}{1 + e} = \frac{\left(1 + \frac{2}{9}\right)2700kgm^{-3}}{1.6} \\ &= \frac{2700kgm^{-3} + 600kgm^{-3}}{1.6} = \frac{3300kgm^{-3}}{1.6} = \mathbf{2062.50kgm^{-3}}\end{aligned}$$

Saturated UNIT WEIGHT:

$$\begin{aligned}\gamma_{sat} &= \rho_{sat}g \\ \gamma_{sat} &= \mathbf{20.625kNm^{-3}}\end{aligned}$$

(v) Total weight ***W***; and dry weight ***W_s*** for each case

Total weights

$$V = \mathbf{1m^3}; \gamma = 20.25kNm^{-3}$$

$$\frac{W}{V} = \gamma \Rightarrow W = \gamma V$$

$$W = 20.25kNm^{-3} \times 1m^3 = 20.25kN$$

$$\mathbf{W = 20.25kN}$$

Likewise; for

$$V = \mathbf{10m^3}; 10 \times 20.25kN = \mathbf{202.5kN}$$

$$V = \mathbf{15m^3}; 15 \times 20.25kN = \mathbf{303.75kN}$$

Dry weights:

$$V = \mathbf{1m^3}; \gamma_d = 16.875kNm^{-3}$$

$$W_s = \gamma_d V$$

$$W_s = 16.875kNm^{-3} \times 1m^3 = 16.875kN$$

$$\mathbf{W_s = 16.875kN}$$

Likewise; for:

$$V = 10m^3; W_s = 10 \times 16.875kN = 168.75kN$$

$$V = 15m^3; W_s = 15 \times 16.875kN = 253.125kN$$

ALTERNATIVELY:

All the values of W_s corresponding to the three above values of V can also be found by using the relationship

$$W_s = \frac{W}{1 + m}$$

where m is the moisture content

(vi) total mass M and dry mass M_s for each case

Total masses:

$$V = 1m^3; W = 20.25kN$$

$$W = Mg \Rightarrow M = \frac{W}{g}$$

$$M = \frac{20.25kN}{10ms^{-2}} = \frac{20250N}{10ms^{-2}} = 2025kg$$

$$M = 2025kg$$

Likewise; for:

$$V = 10m^3; M = 10 \times 2025kg = 20250kg$$

$$V = 15m^3; M = 15 \times 2025kg = 30375kg$$

Dry masses

$$V = 1m^3; W_s = 16.875kN$$

$$M_s g = W_s \Rightarrow M_s = \frac{W_s}{g}$$

$$M_s = \frac{16.875kN}{10ms^{-2}} = \frac{16875N}{10ms^{-2}} = 1687.5kg$$

$$\mathbf{M_s = 1687.5kg}$$

Likewise; for:

$$\mathbf{V = 10m^3; M_s = 10 \times 1687.5kg = 16875kg}$$

$$\mathbf{V = 15m^3; M_s = 15 \times 1687.5kg = 25312.50kg}$$

Alternatively; If the values of M_s corresponding to the three above values of V could also have been found by using the relationship

$$\mathbf{M_s = \frac{M}{1 + m}}$$

where $\mathbf{m}$ is the moisture content

(vii) Degree of Saturation, S_r

From the relationship $S_r e = m G_s$ we have

$$S_r = \frac{m G_s}{e}$$

$$\Rightarrow S_r = \frac{0.20(2.70)}{0.60} = 0.9 = 90\%$$

$$\mathbf{S_r = 90\%}$$

(viii) Volumes of water in each of the three (total volume) cases

We know that:

$$S_r = \frac{V_w}{V_v}$$

$$\Rightarrow V_w = V_v S_r$$

$$V = 1m^3; V_v = 0.375m^3; S_r = 0.90$$

$$\Rightarrow V_w = 0.375m^3(0.90) = 0.3375m^3$$

Therefore

$$V_w = 0.3375m^3; V = 1m^3$$

Likewise;

$$V_w = 10 \times 0.3375m^3 = 3.375m^3; V = 10m^3$$

$$V_w = 15 \times 0.3375m^3 = 5.0625m^3; V = 15m^3$$

(ix) volume occupied by air V_a , air content, N_a and the air voids ratio A_v

From the relationship $V_v = V_a + V_w$ we have

$$V_a = V_v - V_w$$

However, we note from above that

$$V = 1m^3; V_v = 0.375m^3; V_w = 0.3375m^3$$

$$\Rightarrow V_a = 0.375m^3 - 0.3375m^3 = 0.0375m^3$$

$$V_a = 0.0375m^3$$

Then;

$$N_a = \frac{V_a}{V}$$

$$\Rightarrow N_a = \frac{0.0375m^3}{1m^3} = 0.0375 = 3.75\%$$

$$N_a = 3.75\%$$

And finally;

$$A_v = \frac{V_a}{V_v} = \frac{0.0375m^3}{0.375m^3} = 0.10 = 10\%$$

Likewise; for:

$$V = 10m^3; V_a = 10 \times 0.0375m^3 = 0.375m^3$$

$$V_a = 0.375m^3 \text{ and}$$

$$N_a = 3.75\%; A_v = 10\% \text{ as before}$$

$$V = 15m^3; V_a = 15 \times 0.0375m^3 = 0.5625m^3$$

$$V_a = 0.5625m^3 \text{ and}$$

$$N_a = 3.75\%; A_v = 10\% \text{ as before}$$

The ratios N_a and A_v remain the same in all cases because they represent the same soil, i.e.

$$N_a = \frac{0.0375m^3}{1m^3} = \frac{10 \times 0.0375m^3}{10m^3} = \frac{15 \times 0.0375m^3}{15m^3} = 0.0375 = 3.75\%$$

and

$$A_v = \frac{V_a}{V_v} = \frac{0.0375m^3}{0.375m^3} = \frac{10 \times 0.0375m^3}{10 \times 0.375m^3} = \frac{15 \times 0.0375m^3}{15 \times 0.375m^3} = 0.10 = 10\%$$

ALTERNATIVELY:

For $V = 10m^3$ and $V = 15m^3$, we could also have used the following procedure in obtaining the values of V_a though a somehow waste of time, unnecessary and tedious procedure, i.e.,

$$V = 10m^3; V_v = 3.75m^3; V_w = 10 \times 0.3375m^3$$

$$V_a = V_v - V_w$$

$$\Rightarrow V_a = 3.75m^3 - 3.375m^3 = 0.375m^3$$

$$V = 15m^3; V_v = 5.625m^3; V_w = 15 \times 0.3375m^3$$

$$V_a = V_v - V_w$$

$$\Rightarrow V_a = 5.625m^3 - 5.0625m^3 = 0.5625m^3$$

Here again, all the *three* volumes lead to the same ratios $N_a = 3.75\%$ and $A_v = 10\%$

Note also here that:

$$A_v = 10\%; S_r = 90\%$$

So that

$$A_v + S_r = 100\% \text{ or } A_v + S_r = 1$$

(c) Moisture content at complete saturation and saturated weights for each case

Moisture content at COMPLETE SATURATION:

$$\begin{aligned} S_r e &= m G_s; S_r = 1 \\ \Rightarrow e &= m_{sat} G_s \\ \Rightarrow m_{sat} &= \frac{e}{G_s} = \frac{0.6}{2.7} = 0.2222 = 22.22\% \\ m_{sat} &= \mathbf{22.22\%} \end{aligned}$$

Saturated weight:

$$\begin{aligned} W_{sat} &= \gamma_{sat} V \\ V &= \mathbf{1m^3}; \gamma_{sat} = 20.625kNm^{-3} \\ W_{sat} &= 20.625kNm^{-3} \times 1m^3 = 20.625kN \\ W_{sat} &= \mathbf{20.625kN} \end{aligned}$$

Likewise; for:

$$\begin{aligned} V &= \mathbf{10m^3}; W_{sat} = 10m^{-3} \times 20.625kNm^{-3} = \mathbf{206.25kN} \\ V &= \mathbf{15m^3}; W_{sat} = 15m^{-3} \times 20.625kNm^{-3} = \mathbf{309.375kN} \end{aligned}$$

(d) Amount of water (required to be) added

(i) From $m = 0$ to $m = 0.20$

$$\rho_d = 1687.5 \text{kgm}^{-3}; m = 0$$

$$\rho = 2025 \text{kgm}^{-3}; m = 0.20$$

$$\Rightarrow \rho_d + X_m(\text{water per cubic meter}) = \rho$$

where

$$X_m = \text{amount of water added per cubic meter}$$

$$\Rightarrow X_m = \rho - \rho_d$$

$$X_m = 2025 \text{kgm}^{-3} - 1687.5 \text{kgm}^{-3} = 337.5 \text{kgm}^{-3}$$

$$X_m = 337.5 \text{kgm}^{-3}$$

$$\Rightarrow W'_w = 337.5 \text{kg in } 1\text{m}^3$$

$$\Rightarrow 337.5 \text{kg of water should be added for every } 1\text{m}^3$$

This indicates that, $W'_w = 337.5 \text{kg}$ of water are needed for every 1m^3 in order to raise the moisture content of this type of soil from $m = 0$ to $m = 0.2$

Likewise;

$W'_w = 337.5 \times 10 = 3375 \text{kg}$ of water are needed for a 10m^3 volume of this type of soil in order to raise its moisture content from $m = 0$ to $m = 0.2$, and finally;

$W'_w = 337.5 \times 15 = 5062.50 \text{kg}$ of water are needed for a 15m^3 volume of soil for the same purpose

(ii) From $m = 0$ to $m_{sat} = 0.2222$

$$\rho_d = 1687.50 \text{kgm}^{-3}; m = 0$$

$$\rho_{sat} = 2062.50 \text{kgm}^{-3}; m_{sat} = 0.2222$$

Again as before, let

X_{sat} = amount of water added (per cubic meter) to saturation

$$\Rightarrow X_{sat} = \rho_{sat} - \rho_d$$

$$X_{sat} = 2062.50 \text{kgm}^{-3} - 1687.50 \text{kgm}^{-3} = 375 \text{kgm}^{-3}$$

$$X_{sat} = 375 \text{kgm}^{-3}$$

$$\Rightarrow W'_w = 375 \text{kg in } 1 \text{m}^3$$

This indicates that, $W'_w = 375 \text{kg}$ of water are needed for every 1m^3 in order to raise the moisture content of this type of soil from $m = 0$ to saturation, i.e.

$$m_{sat} = 0.2222$$

Likewise;

$$W'_w = 10 \times 375 \text{kg} = 3750 \text{kg for } 10 \text{m}^3 \text{ volume of soil}$$

$$W'_w = 15 \times 375 \text{kg} = 5625 \text{kg for } 15 \text{m}^3 \text{ volume of soil}$$

(iii) From $m = 0.20$ to $m_{sat} = 0.2222$

$$\rho = 2025 \text{kgm}^{-3}; m = 0.20$$

$$\rho_{sat} = 2062.50 \text{kgm}^{-3}; m_{sat} = 0.2222$$

Again as before, let

X_{sat} = amount of water added (per cubic meter) to saturation

$$\Rightarrow X_{sat} = \rho_{sat} - \rho$$

$$X_{sat} = 2062.50 \text{kgm}^{-3} - 2025 \text{kgm}^{-3} = 37.5 \text{kgm}^{-3}$$

$$X_{sat} = 37.50 \text{kgm}^{-3}$$

$$\Rightarrow W'_w = 37.50 \text{kg in } 1 \text{m}^3$$

Likewise;

$$W'_w = 10 \times 37.50 \text{kg} = 375 \text{kg for } 10 \text{m}^3 \text{ volume of soil}$$

$$W'_w = 15 \times 37.50 \text{kg} = 562.5 \text{kg for } 15 \text{m}^3 \text{ volume of soil}$$

(e) What volume of water was added in each of the cases in (d) above

Additional volume of water

(i) From $m = 0$ to $m = 0.20$

$$W'_w = 337.5kg \text{ in } 1m^3$$

$$\Rightarrow V_w = \frac{W'_w}{\gamma_w} = \frac{M'_w}{\rho_w}$$

$$V'_w = \frac{337.5kg}{1000kgm^{-3}} = 0.3375m^3$$

Therefore additional volume of water is

$$V'_w = 0.3375m^3 ; V = 1m^3$$

Hence

$$V'_w = 10 \times 0.3375m^3 = 3.375m^3 ; V = 10m^3$$

$$V'_w = 15 \times 0.3375m^3 = 5.0625m^3 ; V = 15m^3$$

(ii) From $m = 0$ to $m_{sat} = 0.2222$

$$W'_w = 375kg \text{ in } 1m^3$$

$$V'_w = \frac{375kg}{1000kgm^{-3}} = 0.375m^3$$

Therefore additional volume of water is

$$V'_w = 0.375m^3 ; V = 1m^3$$

Hence

$$V'_w = 10 \times 0.375m^3 = 3.75m^3 ; V = 10m^3$$

$$V'_w = 15 \times 0.375m^3 = 5.625m^3 ; V = 15m^3$$

(iii) From $m = 0.2$ to $m_{sat} = 0.2222$

$$W'_w = 37.50kg \text{ in } 1m^3$$

Just by looking at the figure $W'_w = 37.50$, it appears to be **10 times less** than the previous figure in (ii) above. Therefore, even without doing further calculations, we can easily deduce that

$$V'_w = 0.0375m^3 ; V = 1m^3$$

$$V'_w = 10 \times 0.0375m^3 = 0.375m^3 ; V = 10m^3$$

$$V'_w = 15 \times 0.0375m^3 = 0.5625m^3 ; V = 15m^3$$

(f) Boundary values of m

The minimum value of m is obtained when $\rho = \rho_d$ and the maximum when $\rho = \rho_{sat}$

Under this particular case therefore

$$\rho = \rho_d \Rightarrow m = 0$$

and

$$\rho = \rho_{sat} \Rightarrow m = m_{sat} = 0.2222$$

Hence, the boundary values of m are

$$0 \leq m \leq m_{sat} \Rightarrow 0 \leq m \leq 0.2222$$

ILLUSTRATIVE EXAMPLE 2:

Once again, calculate all the quantities stated in Example 1 above, but this time using a value for the void ratio as $e = 1.0$

SOLUTIONS GUIDE SUMMARY TABLE FOR ILLUSTRATIVE EXAMPLES 1 AND 2

	$m = 0.20; G_s = 2.70; e_1 = 0.60; e_2 = 1.0$		
	$V = 1m^3$	$V = 10m^3$	$V = 15m^3$
V_s	$\frac{1}{1+e}$	$10\left(\frac{1}{1+e}\right)$	$15\left(\frac{1}{1+e}\right)$
V_v	$\frac{e}{1+e}$	$10\left(\frac{e}{1+e}\right)$	$15\left(\frac{e}{1+e}\right)$
V	$V_v + V_s$	$V_v + V_s$	$V_v + V_s$
ρ_s	$\rho_s = G_s \rho_w$		
ρ_d	$\rho_d = \frac{G_s \rho_w}{1+e}$		
γ_d	$\gamma_d = \rho_d g$		
ρ	$\rho_d(1+m) = \frac{(1+m)G_s \rho_w}{1+e} = \frac{(G_s + eS_r)\rho_w}{1+e}$		
γ	$\gamma = \rho g$		
ρ_{sat}	$\rho_d(1+m_{sat}) = \frac{(1+m_{sat})G_s \rho_w}{1+e} = \frac{(G_s + e)\rho_w}{1+e}$		
γ_{sat}	$\gamma_{sat} = \rho_{sat} g$		
W_s	$\gamma_d V$	$10\gamma_d V$	$15\gamma_d V$
W	γV	$10\gamma V$	$15\gamma V$
W_{sat}	$\gamma_{sat} V$	$10\gamma_{sat} V$	$15\gamma_{sat} V$
M_s	$\frac{W_s}{g}$	$10\left(\frac{W_s}{g}\right)$	$15\left(\frac{W_s}{g}\right)$
M	$\frac{W}{g}$	$10\left(\frac{W}{g}\right)$	$15\left(\frac{W}{g}\right)$
S_r	$S_r = \frac{mG_s}{e}; S_r = 1 \Rightarrow e = m_{sat}G_s$		
V_w	$V_v S_r$	$10(V_v S_r)$	$15(V_v S_r)$

V_a	$V_v - V_w$	$10(V_v - V_w)$	$15(V_v - V_w)$
N_a	$\frac{V_a}{V}$	$\frac{10V_a}{10V}$	$\frac{15V_a}{15V}$
A_v	$\frac{V_a}{V_v}$	$\frac{10V_a}{10V_v}$	$\frac{15V_a}{15V_v}$
m_{sat}	$\frac{e}{G_s}$		
Amount of water added for each particular volume case			
$0 \leq m \leq 0.20$	$(\rho - \rho_d)x1m^3$	$(\rho - \rho_d)x10m^3$	$(\rho - \rho_d)x15m^3$
$0 \leq m \leq m_{sat}$	$(\rho_{sat} - \rho_d)x1m^3$	$(\rho_{sat} - \rho_d)x10m^3$	$(\rho_{sat} - \rho_d)x15m^3$
$0.2 \leq m \leq m_{sat}$	$(\rho_{sat} - \rho)x1m^3$	$(\rho_{sat} - \rho)x10m^3$	$(\rho_{sat} - \rho)x15m^3$
Boundary values of m; $0 \leq m \leq m_{sat}$			
$0 \leq m \leq m_{sat}$			

SOLUTION: ILLUSTRATIVE EXAMPLE 2

Given:

$$e = 1.0$$

$$V = V_1 = 1m^3; V = V_2 = 10m^3; V = V_3 = 15m^3$$

$$G_s = 2.70$$

Required:

$$V_s = ?$$

$$V_v = ?$$

$$\rho_s = ?$$

$$\rho = ?$$

$$\rho_d = ?$$

$$\rho_{sat} = ?$$

$$W = ?$$

$$W_s = ?$$

$$M = ?$$

$$M_s = ?$$

$$S_r = ?$$

$$V_w = ?$$

$$m = ?$$

$$m_{sat} = ?$$

$$W_{sat} = ?$$

SOLUTION:

(a) Volumes of solids and voids

$$e = 1.0$$

(i) Now, we know that

$$V_s = \frac{1}{1 + e}; \text{ when } V = 1$$

Volume of solids:

$$V_s = \frac{1}{1 + 1} = \frac{1}{2} = 0.50$$

$$V_s = 0.50m^3 \text{ for } V = 1m^3$$

Volume of voids:

$$V_v = V - V_s$$

$$V_v = 1.0 - 0.50$$

$$V_v = 0.50m^3 \text{ for } V = 1m^3$$

Note here that since $e = 1$, then $V_v = V_s = 0.50m^3$

Likewise;

$$(ii) \quad V_s = V_v = 10 \times 0.50m^3 = 5.0m^3 \text{ for } V = 10m^3$$

$$\Rightarrow V_s = V_v = 5.0m^3$$

$$(iii) \quad V_s = 15 \times 0.50 = 7.50m^3 \text{ for } V = 15m^3$$

$$\Rightarrow V_s = V_v = 7.50m^3$$

(b) Densities

Since we are also given the values $G_s = 2.70$; $m = 0.20$, so with $e = 0.60$ being also known, we can simply use the relationship

$$\frac{\rho_s}{\rho_w} = G_s$$

(i) Density of solids, ρ_s

From the relationship

$$\frac{\rho_s}{\rho_w} = G_s$$

$$\Rightarrow \rho_s = G_s \rho_w = 2.70 \times (1000kgm^{-3}) = 2700kgm^{-3}$$

$$\rho_s = 2700kgm^{-3}$$

Note that ρ_s still maintains its former in both cases, because its value does not depend on the void ratio e , unlike the rest of the densities, implying that the density of solids of a given volume of a soil does not get affected by the addition or removal of air and/water in it, unlike the dry, the bulk and the saturated densities (the latter are all dependent on the void ratio ratio e)

$$\rho_s = G_s \rho_w \text{ (not dependent on } e)$$

$$\rho_d = \frac{G_s \rho_w}{1 + e} \text{ (dependent on } e)$$

$$\rho_{bulk} = (1 + m) \frac{G_s \rho_w}{1 + e} = (1 + m) \rho_d (\text{dependent on } e)$$

$$\rho_{sat} = (1 + m_{sat}) \frac{G_s \rho_w}{1 + e} = (1 + m_{sat}) \rho_d (\text{dependent on } e)$$

Emperical relationship existing between density of solids and dry density:

$$\rho_s = G_s \rho_w$$

$$\rho_d = \frac{G_s \rho_w}{1 + e}$$

Upon combining the two equations, we arrive at a relationship

$$\rho_d = \frac{\rho_s}{1 + e}$$

or

$$\rho_s = \rho_d (1 + e)$$

(ii) Bulk density, ρ and bulk unit weight γ

Bulk density:

$$\rho = \frac{(1+m)G_s \rho_w}{1 + e}; m = 0.20, e = 1.0$$

$$\rho = \frac{(1 + 0.20) \times 2700 \text{ kgm}^{-3}}{1 + 1.0} = \frac{3240 \text{ kgm}^{-3}}{2} = 1620 \text{ gm}^{-3}$$

$$\rho = 1620 \text{ kgm}^{-3}$$

ALTERNATIVELY:

$$\rho = \frac{(G_s + e S_r) \rho_w}{1 + e}; S_r = ?$$

$$S_r e = m G_s \Rightarrow S_r = \frac{m G_s}{e}$$

$$S_r = \frac{0.20 \times 2.70}{1.0} = 0.54 = 54\%$$

$$\begin{aligned} \Rightarrow \rho &= \frac{(G_s + eS_r)\rho_w}{1 + e} = \frac{(2.70 + 1.0(0.54))1000 \text{ kgm}^{-3}}{1 + 1} \\ &= \frac{(2.70 + 0.54)1000 \text{ kgm}^{-3}}{2} = \frac{3240 \text{ kgm}^{-3}}{2} = \mathbf{1620 \text{ kgm}^{-3}} \end{aligned}$$

Bulk unit weight:

$$\gamma = \rho g$$

$$\gamma = \mathbf{1620 \text{ kgm}^{-3} \times 10 \text{ ms}^{-2} = 16200 \text{ Nm}^{-3}}$$

$$\gamma = \mathbf{16.20 \text{ kNm}^{-3}}$$

(iii) Dry density, ρ_d and dry unit weight γ_d

Dry density:

$$\rho_d = \frac{\rho}{1 + m} = \frac{\mathbf{1620 \text{ kgm}^{-3}}}{1.20} = 1350 \text{ kgm}^{-3}$$

$$\rho_d = \mathbf{1350 \text{ kgm}^{-3}}$$

ALTERNATIVELY:

$$\rho_d = \frac{G_s \rho_w}{1 + e}$$

$$\rho_d = \frac{2.70(1000) \text{ kgm}^{-3}}{1 + 1} = \frac{2700 \text{ kgm}^{-3}}{2} = 1350 \text{ kgm}^{-3}$$

$$\rho_d = \mathbf{1350 \text{ kgm}^{-3}}$$

OR

$$\rho_d = G_s \rho_w (1 - n)$$

$$n = \frac{e}{1+e} = \frac{1}{1+1} = 0.50$$

$$\rho_d = 2.70 \times 1000 \text{ kgm}^{-3} (1 - 0.50) = 2700 \text{ kgm}^{-3} \times 0.50 = \mathbf{1350 \text{ kgm}^{-3}}$$

OR

$$\rho_d = \frac{M_s}{V} = \frac{G_s \rho_w V_s}{V}$$

$$V_s = 0.50 \text{ m}^3; V = 1 \text{ m}^3$$

$$\begin{aligned} \Rightarrow \rho_d &= \frac{G_s \rho_w V_s}{V} = \frac{2.70 \times 1000 \text{ kgm}^{-3} \times 0.50 \text{ m}^3}{1 \text{ m}^3} = 2700 \text{ kgm}^{-3} \times 0.50 \text{ m}^3 \\ &= \mathbf{1350 \text{ kgm}^{-3}} \end{aligned}$$

Dry unit weight:

$$\gamma_d = \rho_d g$$

$$\gamma_d = \mathbf{13.50 \text{ kNm}^{-3}}$$

(iv) Saturated density, ρ_{sat} and SATURATED UNIT WEIGHT; γ_{sat}

Saturated density:

$$\rho_{sat} = \rho_d + n \rho_w$$

$$n = 0.50$$

$$\Rightarrow \rho_{sat} = \mathbf{1350 \text{ kgm}^{-3}} + 0.50((1000) \text{ kgm}^{-3})$$

$$\rho_{sat} = \mathbf{1850 \text{ kgm}^{-3}}$$

ALTERNATIVELY:

$$\rho_{sat} = \frac{(G_s + e S_r) \rho_w}{1 + e}$$

$$\Rightarrow \rho_{sat} = \frac{(G_s + e) \rho_w}{1 + e}; S_r = 1$$

Therefore

$$\rho_{sat} = \frac{(2.70 + 1.0)(1000)kgm^{-3}}{1 + 1.0} = \frac{3700kgm^{-3}}{2} = \mathbf{1850kgm^{-3}}$$

OR

$$\rho_{sat} = [(1 - n)G_s + n]\rho_w$$

$$n = 0.50$$

$$\rho_{sat} = [(1 - 0.50)2.70 + 0.50]1000kgm^{-3}$$

$$\begin{aligned}\rho_{sat} &= [(0.50 \times 2.70) + 0.50]1000kgm^{-3} = [1.35 + 0.50]1000kgm^{-3} \\ &= 1.85 \times 1000kgm^{-3} = \mathbf{1850kgm^{-3}}\end{aligned}$$

OR

$$\rho_{sat} = \frac{(1 + m_{sat})G_s\rho_w}{1 + e}; m_{sat} = ?$$

$$S_r e = m_{sat} G_s; S_r = 1$$

$$\Rightarrow m_{sat} = \frac{e}{G_s} = \frac{1.0}{2.7} = \frac{10}{27} = 0.3704 = 37.04\%$$

$$\begin{aligned}\Rightarrow \rho_{sat} &= \frac{(1 + m_{sat})G_s\rho_w}{1 + e} = \frac{\left(1 + \frac{10}{27}\right)2700kgm^{-3}}{1 + 1} \\ &= \frac{2700kgm^{-3} + 1000kgm^{-3}}{2} = \frac{3700kgm^{-3}}{2} = \mathbf{1850kgm^{-3}}\end{aligned}$$

Saturated UNIT WEIGHT:

$$\gamma_{sat} = \rho_{sat}g$$

$$\gamma_{sat} = \mathbf{18.50kNm^{-3}}$$

(v) Total weights **W**; and dry weights **W_s** for each case

Total weights

$$V = 1\text{m}^3; \gamma = \gamma = 16.20\text{kNm}^{-3}$$

$$\frac{W}{V} = \gamma \Rightarrow W = \gamma V$$

$$W = 16.20\text{kNm}^{-3} \times 1\text{m}^3 = 16.20\text{kN}$$

$$\mathbf{W = 16.20kN}$$

Likewise; for

$$V = 10\text{m}^3; W = 10 \times 16.20\text{kN} = \mathbf{162kN}$$

$$V = 15\text{m}^3; W = 15 \times 16.20\text{kN} = \mathbf{243kN}$$

Dry weights:

$$V = 1\text{m}^3; \gamma_d = 13.50\text{kNm}^{-3}$$

$$W_s = \gamma_d V$$

$$W_s = 13.50\text{kNm}^{-3} \times 1\text{m}^3 = 13.50\text{kN}$$

$$\mathbf{W_s = 13.50kN}$$

Likewise; for

$$V = 10\text{m}^3; W_s = 10 \times 13.50\text{kN} = \mathbf{135kN}$$

$$V = 15\text{m}^3; W_s = 15 \times 13.50\text{kN} = \mathbf{202.50kN}$$

ALTERNATIVELY:

All the values of W_s corresponding to the three above values of V can also be found by using the relationship

$$W_s = \frac{W}{1 + m}$$

where $m = 0.2$ is the moisture content

(vi) total mass M and dry mass M_s for each case

Total masses:

$$V = 1\mathbf{m}^3; W = 16.20\mathbf{kN}$$

$$Mg = W \Rightarrow M = \frac{W}{g}$$

$$M = \frac{16.20\mathbf{kN}}{10\mathbf{ms}^{-2}} = \frac{16200\mathbf{N}}{10\mathbf{ms}^{-2}} = 1620\mathbf{kg}$$

$$\mathbf{M = 1620kg}$$

Likewise for:

$$V = 10\mathbf{m}^3; M = 10 \times 1620\mathbf{kg} = \mathbf{16200kg}$$

$$V = 15\mathbf{m}^3; M = 15 \times 1620\mathbf{kg} = \mathbf{24300kg}$$

Dry masses

$$V = 1\mathbf{m}^3; W_s = 13.50\mathbf{kN}$$

$$M_s g = W_s \Rightarrow M_s = \frac{W_s}{g}$$

$$M_s = \frac{13.50\mathbf{kN}}{10\mathbf{ms}^{-2}} = \frac{13500\mathbf{N}}{10\mathbf{ms}^{-2}} = 1350\mathbf{kg}$$

$$\mathbf{M_s = 1350kg}$$

Likewise for:

$$V = 10\mathbf{m}^3; M_s = 10 \times 1350\mathbf{kg} = \mathbf{13500kg}$$

$$V = 15\mathbf{m}^3; W_s = 15 \times 1350\mathbf{kg} = \mathbf{20250kg}$$

Note that all the values of M_s corresponding to the above values of V could have also been found by using the relationship

$$M_s = \frac{M}{1 + m}$$

where m is the moisture content

(vii) Degree of Saturation, S_r

From the relationship $S_r e = m G_s$ we have

$$S_r = \frac{m G_s}{e}$$

$$\Rightarrow S_r = \frac{0.20(2.70)}{1} = 0.54 = 54\%$$

$$\mathbf{S_r = 54\%}$$

(viii) Volumes of water in each of the three (total volume) cases

We know that:

$$S_r = \frac{V_w}{V_v}$$

$$\Rightarrow V_w = V_v S_r$$

$$V = 1m^3; V_v = 0.50m^3; S_r = 0.54$$

$$\Rightarrow V_w = 0.50m^3(0.54) = 0.27m^3$$

Therefore

$$\mathbf{V_w = 0.27m^3; V = 1m^3}$$

Likewise;

$$\mathbf{V_w = 10 \times 0.27m^3 = 2.70m^3; V = 10m^3}$$

$$\mathbf{V_w = 15 \times 0.27m^3 = 4.05m^3; V = 15m^3}$$

(ix) volume occupied by air V_a , air content, N_a and the air voids ratio A_v

From the relationship $V_v = V_a + V_w$ we have

$$V_a = V_v - V_w$$

However, we note from above that

$$V = 1m^3; V_v = 0.50m^3; V_w = 0.27m^3$$

$$\Rightarrow V_a = 0.50m^3 - 0.27m^3 = 0.23m^3$$

$$\mathbf{V_a = 0.23m^3}$$

Then;

$$N_a = \frac{V_a}{V}$$
$$\Rightarrow N_a = \frac{0.23m^3}{1m^3} = 0.23 = 23\%$$
$$N_a = \mathbf{23\%}$$

And finally;

$$A_v = \frac{V_a}{V_v} = \frac{0.23m^3}{0.50m^3} = 0.46 = 46\%$$

Likewise; for:

$$V = \mathbf{10m^3}; V_a = 10 \times 0.23m^3 = 2.30m^3$$

$$V_a = \mathbf{2.30m^3} \text{ and}$$

$$N_a = \mathbf{23\%}; A_v = \mathbf{46\%} \text{ as before}$$

$$V = \mathbf{15m^3}; V_a = 15 \times 0.23m^3 = 3.45m^3$$

$$V_a = \mathbf{3.45m^3} \text{ and}$$

$$N_a = \mathbf{23\%}; A_v = \mathbf{46\%} \text{ as before}$$

The ratios N_a and A_v remain the same in all cases because they represent the same soil, i.e.

$$N_a = \frac{0.23m^3}{1m^3} = \frac{10 \times 0.23m^3}{10m^3} = \frac{15 \times 0.23m^3}{15m^3} = 0.23m^3 = \mathbf{23\%}$$

and

$$A_v = \frac{V_a}{V_v} = \frac{0.23m^3}{0.50m^3} = \frac{10 \times 0.23m^3}{10 \times 0.50m^3} = \frac{15 \times 0.23m^3}{15 \times 0.50m^3} = 0.46 = \mathbf{46\%}$$

ALTERNATIVELY:

For $V = 10m^3$ and $V = 15m^3$, we could also, have used the following procedure in obtaining the values of V_a , though a somehow waste of time, unnecessary and tedious procedure, i.e.,

$$V = 10m^3; V_v = 5.0m^3; V_w = 10 \times 0.27m^3$$

$$V_a = V_v - V_w$$

$$\Rightarrow V_a = 5.0m^3 - 2.70m^3 = 2.30m^3$$

$$V = 15m^3; V_v = 7.50m^3; V_w = 15 \times 0.27m^3$$

$$V_a = V_v - V_w$$

$$\Rightarrow V_a = 7.50m^3 - 4.05m^3 = 3.45m^3$$

Here again, all the *three* volumes could have led to the same ratios $N_a = 23\%$ and $A_v = 46\%$

Note also here that:

$$A_v = 46\%; S_r = 54\%$$

So that

$$A_v + S_r = 100\% \text{ or } A_v + S_r = 1$$

(c) Moisture content at complete saturation and saturated weights for each case

Moisture content:

$$S_r e = m G_s$$

$$\Rightarrow e = m_{sat} G_s; S_r = 1$$

$$\Rightarrow m_{sat} = \frac{e}{G_s} = \frac{1.0}{2.7} = 0.3704 = 37.04\%$$

$$m_{sat} = 37.04\%$$

Saturated weight:

$$W_{sat} = \gamma_{sat} V$$

$$V = 1m^3; \gamma_{sat} = 18.50kNm^{-3}$$

$$W_{sat} = 18.50kNm^{-3} \times 1m^3 = 18.50kN$$

$$W_{sat} = \mathbf{18.50kN}$$

Likewise; for:

$$V = 10m^3; W_{sat} = 10m^{-3} \times 18.50kNm^{-3} = \mathbf{185kN}$$

$$V = 15m^3; W_{sat} = 15m^{-3} \times 18.50kNm^{-3} = \mathbf{277.5kN}$$

(d) Amount of water (required to be) added

(i) From $m = 0$ to $m = 0.20$

$$\rho_d = 1350kgm^{-3}; m = 0$$

$$\rho = 1620gm^{-3}; m = 0.20$$

$$\Rightarrow \rho_d + X_m(\text{water per cubic meter}) = \rho$$

where

$$X_m = \text{amount of water added per cubic meter}$$

$$\Rightarrow X_m = \rho - \rho_d$$

$$X_m = 1620gm^{-3} - 1350kgm^{-3} = 270kgm^{-3}$$

$$X_m = 270kgm^{-3}$$

$$\Rightarrow W'_w = \mathbf{270kg \text{ in } 1m^3}$$

$$\Rightarrow 270kg \text{ of water should be added for every } 1m^3$$

This indicates that, $W'_w = 270kg$ of water are needed for every $1m^3$ in order to raise the moisture content of this type of soil from $m = 0$ to $m = 0.2$

Likewise;

$W'_w = 270 \times 10 = \mathbf{2700kg}$ of water are needed for a $\mathbf{10m^3}$ volume of this type of soil in order to raise its moisture content from $m = 0$ to $m = 0.2$, and finally;

$W'_w = 270 \times 15 = \mathbf{4050kg}$ of water are needed for a $\mathbf{15m^3}$ volume of soil for the same purpose as above

(ii) From $m = 0$ to $m_{sat} = \mathbf{0.3704}$

$$\rho_d = 1350kgm^{-3}; m = 0$$

$$\rho_{sat} = 1850kgm^{-3}; m_{sat} = 0.3704$$

Again as before, let

$X_{sat} = \text{amount of water added (per cubic meter) to saturation}$

$$\Rightarrow X_{sat} = \rho_{sat} - \rho_d$$

$$X_{sat} = 1850kgm^{-3} - 1350kgm^{-3} = 500kgm^{-3}$$

$$X_{sat} = 500kgm^{-3}$$

$$\Rightarrow W'_w = \mathbf{500kg \text{ in } 1m^3}$$

This indicates that, $W'_w = 500kg$ of water are needed for every $1m^3$ in order to raise the moisture content of this type of soil from $m = 0$ to saturation, i.e. $m_{sat} = 0.3704$

Likewise;

$$W'_w = 10 \times 500kg = \mathbf{5000kg \text{ for } 10m^3 \text{ volume of soil}}$$

$$W'_w = 15 \times 500kg = \mathbf{7500kg \text{ for } 15m^3 \text{ volume of soil}}$$

(iii) From $m = \mathbf{0.20}$ to $m_{sat} = \mathbf{0.3704}$

$$\rho = 1620kgm^{-3}; m = 0.20$$

$$\rho_{sat} = 1850kgm^{-3}; m_{sat} = 0.3704$$

Again as before, let

X_{sat} = amount of water added (per cubic meter) to saturation

$$\Rightarrow X_{sat} = \rho_{sat} - \rho$$

$$X_{sat} = 1850 \text{kgm}^{-3} - 1620 \text{kgm}^{-3} = 230 \text{kgm}^{-3}$$

$$X_{sat} = 230 \text{kgm}^{-3}$$

$$\Rightarrow W'_w = 230 \text{kg in } 1 \text{m}^3$$

Likewise;

$$W'_w = 10 \times 230 \text{kg} = \mathbf{2300 \text{kg}} \text{ for a } \mathbf{10 \text{m}^3} \text{ volume of soil}$$

$$W'_w = 15 \times 230 \text{kg} = \mathbf{3450 \text{kg}} \text{ for a } \mathbf{15 \text{m}^3} \text{ volume of soil}$$

(e) What volume of water was added in each of the cases in (d) above

Additional volume of water

(iv) From $m = 0$ to $m = 0.20$

$$W'_w = 270 \text{kg in } 1 \text{m}^3$$

$$\Rightarrow V_w = \frac{W'_w}{\gamma_w} = \frac{M'_w}{\rho_w}$$

$$V'_w = \frac{270 \text{kg}}{1000 \text{kgm}^{-3}} = 0.27 \text{m}^3$$

Therefore additional volume of water is

$$\mathbf{V'_w = 0.27 \text{m}^3 ; V = 1 \text{m}^3}$$

Hence

$$\mathbf{V'_w = 10 \times 0.27 \text{m}^3 = 2.70 \text{m}^3 ; V = 10 \text{m}^3}$$

$$\mathbf{V'_w = 15 \times 0.27 \text{m}^3 = 4.05 \text{m}^3 ; V = 15 \text{m}^3}$$

(v) From $m = 0$ to $m_{sat} = 0.3704$

$$W'_w = 500 \text{kg in } 1 \text{m}^3$$

$$V'_w = \frac{500kg}{1000kgm^{-3}} = 0.50m^3$$

Therefore additional volume of water is

$$V'_w = 0.50m^3 ; V = 1m^3$$

Hence

$$V'_w = 10 \times 0.50m^3 = 5m^3 ; V = 10m^3$$

$$V'_w = 15 \times 0.50m^3 = 7.50m^3 ; V = 15m^3$$

(vi) From $m = 0.2$ to $m_{sat} = 0.3704$

$$W'_w = 230kg \text{ in } 1m^3$$

$$V'_w = \frac{230kg}{1000kgm^{-3}} = 0.23m^3$$

Therefore additional volume of water is

$$V'_w = 0.23m^3 ; V = 1m^3$$

Hence

$$V'_w = 10 \times 0.23m^3 = 2.30m^3 ; V = 10m^3$$

$$V'_w = 15 \times 0.23m^3 = 3.45m^3 ; V = 15m^3$$

(f) Boundary values of m

The minimum value of m is obtained when $\rho = \rho_d$ and the maximum when $\rho = \rho_{sat}$

Under this particular case therefore

$$\rho = \rho_d \Rightarrow m = 0$$

and

$$\rho = \rho_{sat} \Rightarrow m = m_{sat} = 0.3704$$

Hence, the boundary values of m are

$$0 \leq m \leq m_{sat} \Rightarrow 0 \leq m \leq 0.3704$$

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