6.5 Set Theory

Definition: *A* set *is a group of items, called* elements. *A set has no order.*

The elements of a set are written inside curly braces: { }. Sets are usually called with capital letters, like A or B. Sets can have numbers or anything else for elements.

Definition: *The* cardinality *of a set is the number of elements. The cardinality of a set A is written as $|A|$ or as $n(A)$.*

Examples of sets include: $A = \{1, 2, 3, 4\}$, $B = \{x : x > 0\}$, and $C = \{1, \text{Elephant}, \heartsuit\}$. In these cases, $|A| = 4$, $|B|$ is infinite, and $|C| = 2$.

The symbol $\in$ means 'in,' and $\notin$ means 'not in.' For example, using A above, we could say $1 \in A$, 1 is in A, but $6 \notin A$, 6 is not in A.

Definition: *The* union *of 2 sets is a set that includes all elements that are in* either *of the 2 sets.*

Union is written as $\cup$. For example, if $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, then

$$A \cup B = \{1, 2, 3, 4, 5, 6\}.$$

Definition: *The* intersect *of 2 sets is a set that includes all elements that are in* both *of the 2 sets.*

Intersect is written as $\cap$. Using the same $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$,

$$A \cap B = \{3, 4\}.$$

A useful formula for cardinality is:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Ex 1: *If 12 students say they like Fiddy Q, and 7 students say they like Banana, and there are 15 students total, how many like both Fiddy Q and Banana?*

Solution: We let A be the set of students who like Fiddy Q, and B be the set of students who like Banana. The data tells us that the total number of students, $|A \cup B| = 15$, and what we want to find is the number of students who like *both* Fiddy Q and Banana, $|A \cap B|$.

$$\begin{aligned} |A \cup B| = |A| &= |B| - |A \cap B| \\ 15 &= 12 + 7 - |A \cap B| \\ 15 &= 19 - |A \cap B| & |A \cap B| &= 4 \end{aligned}$$

4 students like both Fiddy Q and Banana. ✠

Definition: *The* null set *or* empty set *is the set with no elements. It is written as $\varnothing$.*

If $C = \{1, 2, 3\}$ and $D = \{4, 5, 6\}$, then $C \cap D = \varnothing$.

Definition: *The* complement *of a set is everything* not *in the set.*

The complement of A is written as A^C or A'.

De Morgan's Laws:

$$(A \cap B)' = A' \cup B' \qquad \text{and} \qquad (A \cup B)' = A' \cap B'$$

Distributive Laws also hold:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Exercises

6.5.1: (NECTA 2002) In a certain school, there is an equal number of boy and girl students. 1/4 of the boys and 1/10 of the girls go to school on foot. 1/3 of the boys and 1/2 of the girls go to school by bicycle, and the rest go by daladala. Find the proportion of students:
(a) That are girls who go by daladala.
(b) That go by daladala. (10 marks)

6.5.2: (NECTA 2001) 64 Students were questioned about their favourite subject from Geography, Mathematics, and Chemistry. 40 liked Geography, 36 like Mathematics, 30 liked Chemistry, and 10 liked all three subjects. If 4 liked both Geography and Mathematics, and assuming that each student liked at least one subject, how many liked Chemistry only? (2 marks)

6.5.3: (NECTA 2001) Draw a Venn diagram and shade the portion corresponding to the set

$$(S \cap T') \cup (S \cap T)$$

(2 marks)

6.5.4: (NECTA 2000) By using set operations, simplify the following $(A \cup B)' \cap (A \cap B)'$. (2 marks)

6.5.5: (NECTA 2000) Out of 130 students of a certain school, 10 of them study economics and mathematics while 20 study neither of these two subjects. Those who study economics alone are three times as many as those who study mathematics only. How many students study economics? (4 marks)

6.6 Interest and Exponential Functions

Often, when you put money in a bank, or if you borrow money from a bank, there is interest. The interest (hopefully) makes up for inflation, and can make it nice for whoever is getting it.

Definition: *In a loan or bank deposit, the* principle *is the initial amount of money. The principle is the starting amount.*
The amount *is the current amount of money.*
The interest rate *is a percentage per time. Usually the annual rate is given, meaning per 1 year. In* simple interest*, the interest rate is the percentage of the principal that is added to the amount. For* compound interest*, the interest rate is the percentage of amount that is added to the amount.*

For simple interest, only the principal earns interest. Thus the amount $A = P + Pni$ where P is the principle, i is the interest as a decimal, not a percentage, and n is the number of interest time periods that have passed. This can be simplified as:

$$\text{Simple Interest}: \qquad A = P(1 + in)$$

Ex 1: *You take out a loan of* $600,000/=$ *for 2 years at* 8% *annual simple interest. After these 2 years, how much do you need to pay the bank?*

Solution: 'Annual' means 1 time per year. (Semi-annual means 2 times per year, because 'semi' means half.) Thus $P = 600,000$, and expressing the interest rate as a decimal we get $i = 0.08$, and after 2 years means $n = 2$. Therefore

$$\begin{aligned} A &= P(1 + in) \\ &= 600,000(1 + 0.08 \cdot 2) \qquad\qquad = 696,000 \end{aligned}$$