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Existence of equations governing right triangle within the smallest acute angle of the right triangle

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Abstract

We prove that equations governing right triangle whose hypotenuse side and non-hypotenuse side (adjacent side) differ by any positive integer greater than zero within the smallest acute angle of the right triangle do exist.

Keywords. *Mambosasa triangle;Mambosasa triangle order; Right triangle; Pythagoras theorem; Trigonometric functions*

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1 Introduction

In 1995 C.C. Chan and T.A. Peng proved the existence of infinitely many right triangles with integral sides in which lengths of two non-hypotenuse sides differ by 1 as shown in [2]. *But does the right triangle whose hypotenuse side and non-hypotenuse side (adjacent side) differ by any positive integer greater than zero within the smallest acute angle of the right triangle exist?* Let us call such triangle a *Mambosasa triangle*, also let the difference between hypotenuse side and non-hypotenuse side (adjacent side) be a *Mambosasa triangle order. But do equations governing Mambosasa triangle exist?* Answers to both questions will be shown in this paper.

2 Preliminaries

In this section we recall some definitions and basic notations of the Mambosasa triangle in **fig 1**.

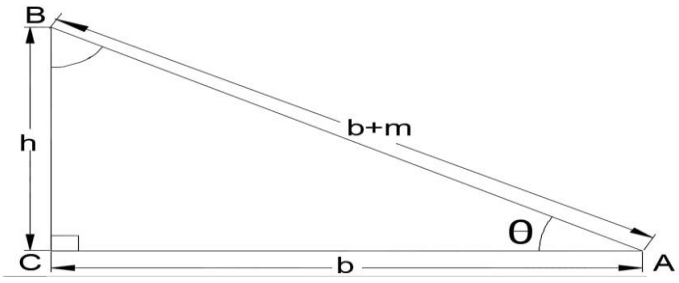

Fig 1. Mambosasa Triangle (see definition 3)

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Definition 1.Let b, c and m to denote non-hypotenuse side (adjacent side), hypotenuse side and Mambosasa triangle order of the right triangle respectively such that

$$m = c - b$$

if and only if

$$0 < m \leq \infty.$$

Definition 2.Minimum angle is the smallest acute angle of the right triangle such that if (θ)*is an acute angle then minimum angle is*

$$0° < \theta \leq 45°$$

within the right triangle.

Definition 3.Mambosasa triangle is the right triangle whose hypotenuse side and non-hypotenuse side (adjacent side) differ by any positive integer greater than zero within the smallest acute angle of the right triangle.

3.0 Main Results

Theorem 3.1 *If the difference between hypotenuse side and non-hypotenuse side (adjacent side) is the Mambosasa triangle order then*

$$b = c - m \tag{1}$$

Proof. In fact, it follows that definition (1) satisfies Theorem (3.1) □

Theorem 3.2 *If* θ *satisfies definition (2) then the adjacent side* (b) *of the right triangle is*

$$\frac{mCos\theta}{1 - Cos\theta}$$

if and only if m satisfies definition (1).

Proof. Let us first recall that $Cos\theta = \frac{adjacent}{Hypotenuse}$ as in [1, 4]

$$Cos\theta = \frac{b}{c} \tag{2}$$

We substitute equation (1) into equation (2)

$$Cos\theta = \frac{b}{b + m}$$

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$$(b+m)\times Cos\theta = \frac{b}{b+m}\times(b+m)$$

$$(b+m)Cos\theta = b$$

$$bCos\theta + mCos\theta = b$$

$$mCos\theta = (b - bCos\theta)$$

$$mCos\theta = b(1 - Cos\theta)$$

we thus get

$$b = \frac{mCos\theta}{1-Cos\theta} \tag{3}$$

which proves Theorem 3.2.Now we show examples in **table 1.** □

Theorem 3.3 *If B is the isosceles triangle base then*

$$B = 2(c-m)$$

if and only if m satisfies definition (1).

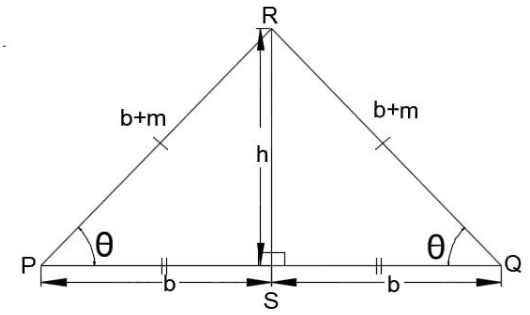

Fig 2. Isosceles triangle

Proof. Our proof starts with the observation that b is the adjacent side of the right triangle **QRS** in **fig 2**, then let

$$b + b = 2b. \tag{4}$$

Then let us substitute equation (1) into equation (4) which gives

$$2b = 2(c-m).$$

Then let

$$2b = B,$$

therefore

$$B = 2(c-m) \tag{5}$$

which completes the proof. □

Theorem 3.4 *If hypotenuse side of the right triangle satisfies definition (1) then*

$$c = \frac{m}{1-Cos\theta}$$

if and only if m satisfies definition (1) and θ satisfies definition (2).

Proof. Let $Cos\theta = \frac{adjacent}{hypotenuse}$ as in [1, 4]

$$Cos\theta = \frac{b}{c}$$

But

$$b = c - m \text{ (see equation 1)}$$

We thus get

$$Cos\theta = \frac{c-m}{c}$$

$$c \times Cos\theta = \frac{(c-m)}{c} \times c$$

$$cCos\theta = c - m$$

$$c - cCos\theta = m$$

$$c(1 - Cos\theta) = m$$

$$c = \frac{m}{1-Cos\theta} \quad (6)$$

which completes the proof. □

Theorem 3.5 *If h is the height of the right triangle, m is the Mambosasa triangle order and c is the hypotenuse side of the same right triangle then*

$$h^2 = 2mc - m^2$$

if and only if both h and m satisfy definition (1).

Proof. Let us first recall *Pythagoras theorem* ($h^2 + b^2 = c^2$) as in [1,3, 5]. (7)

Then we substitute equation (1) into equation (7)

we thus get

$$h^2 + (c-m)^2 = c^2$$

$$h^2 + (c-m)(c-m) = c^2$$

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$$h^2 + (c^2 - cm - cm + m^2) = c^2$$

$$h^2 + (c^2 - 2cm + m^2) = c^2$$

$$h^2 + c^2 - 2cm + m^2 = c^2$$

$$h^2 - 2cm + m^2 = c^2 - c^2$$

$$h^2 - 2cm + m^2 = 0$$

then we obtain that

$$h^2 = 2cm - m^2 \tag{8}$$

therefore the proof is complete. □

Theorem 3.6 *If both h and m satisfy definition (1) then*

$$h = \frac{mSin\theta}{1-Cos\theta}$$

if and only if θ satisfies definition (2)

Proof. Let $Sin\theta = \frac{Opposite}{Adjacent}$ $as\ in$ [1, 4]

$$Sin\theta = \frac{h}{c} \tag{9}$$

then we substitute equation (6) into equation (9)

thus we obtain that

$$h = \frac{m}{1 - Cos\theta} \times Sin\theta$$

$$h = \frac{mSin\theta}{1-Cos\theta} \tag{10}$$

which completes the proof of theorem 3.6 □

Theorem 3.7 *If c is the hypotenuse side of the right triangle then area of the right triangle (A) is*

$$\frac{(c-m)\sqrt{2mc - m^2}}{2}$$

if and only if both m and c satisfy definition (1).

Proof. Let area of the right triangle be

$\frac{base(b)\times height(h)}{2}$ as in [1]. (11)

Then we substitute equation (1) and (8) into equation (11)

which gives

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$$A = \frac{(c-m)\sqrt{2mc-m^2}}{2}$$

which completes the proof. □

Theorem 3.8 *If θ satisfies definition (2) then area (A) of the right triangle is*

$$\frac{0.5m^2 Sin\theta Cos\theta}{Cos^2\theta - 2Cos\theta + 1}$$

if and only if m satisfies definition (1).

Proof. Let area $= \frac{base \times height}{2}$ as in [1] (12)

But

$$b = \frac{mCos\theta}{1-Cos\theta} \quad \text{(see equation 3)}$$

and

$$(h) = \frac{mSin\theta}{1-Cos\theta} \quad \text{(see equation 10)}$$

Then equation (12) become

$$\frac{\left(\frac{mCos\theta}{1-Cos\theta}\right) \times \left(\frac{mSin\theta}{1-Cos\theta}\right)}{2}$$

then we obtain that

$$A = \frac{0.5m^2 Sin\theta Cos\theta}{Cos^2\theta - 2Cos\theta + 1}$$

which completes the proof. □

Theorem 3.9 *If P is the Perimeter of the right triangle, then*

$$P = (2c - m) + \sqrt{(2mc - m^2)}$$

if and only if both c and m satisfy definition(1)

Proof. Let perimeter (P) of the right triangle be $a + b + c$ as in [1]. (13)

Then let

$$h = \sqrt{2mc - m^2} \text{ (see equation 8)}$$

and

$$b = c - m \qquad \text{(see equation 1)}$$

We substitute both equations (1 and 8) into equation (13).

Thus we obtain that

$$P = \sqrt{2mc - m^2} + (c - m) + c$$

$$\mathrm{P} = (2\mathrm{c} - \mathrm{m}) + \sqrt{2\mathrm{mc} - \mathrm{m}^2}$$

which completes the proof. □

Theorem 4.0 *If P is the Perimeter of the right triangle and m satisfies definition (1) then*

$$P = \frac{m(Sin\theta + Cos\theta + 1)}{1 - Cos\theta}$$

if and only if θ satisfies definition (2).

Proof. Let the perimeter of the right triangle be $h + b + c$ as in [1] (14)

But

$$h = \frac{mSin\theta}{1 - Cos\theta} \quad \text{(see equation 10)}$$

$$b = \frac{mCos\theta}{1 - Cos\theta} \quad \text{(see equation 3)}$$

and

$$c = \frac{m}{1 - Cos\theta} \text{ (see equation 6)}$$

Then we obtain that

$$P = \left(\frac{mCos\theta}{1 - Cos\theta} + \frac{mCos\theta}{1 - Cos\theta} + \frac{m}{1 - Cos\theta}\right)$$

$$P = \frac{m(Sin\theta + Cos\theta + 1)}{1 - Cos\theta}$$

$$\mathrm{P} = \frac{\mathrm{m(Sin\theta + Cos\theta + 1)}}{\mathrm{1 - Cos\theta}}$$

which completes the proof of Theorem 4.0, further examples are shown in **table 1.** □

Table1. First seven Mambosasa triangle orders within a minimum angle of 36.87°.

Mambosasa Triangle order(m)	Minimum angle(θ)	Height(h) in centimetre	Hypotenuse(c) in centimetre	Base(b) in centimetre
1	36.87°	2.99999	4.99997	3.99997
2	36.87°	5.99998	9.999946	7.999946

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3	36.87°	8.99997	14.99991	11.99991
4	36.87°	11.99996	19.99988	15.99988
5	36.87°	14.99995	24.99985	19.99985
6	36.87°	17.99994	29.99982	23.99982
7	36.87°	20.99993	34.99979	27.99979

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