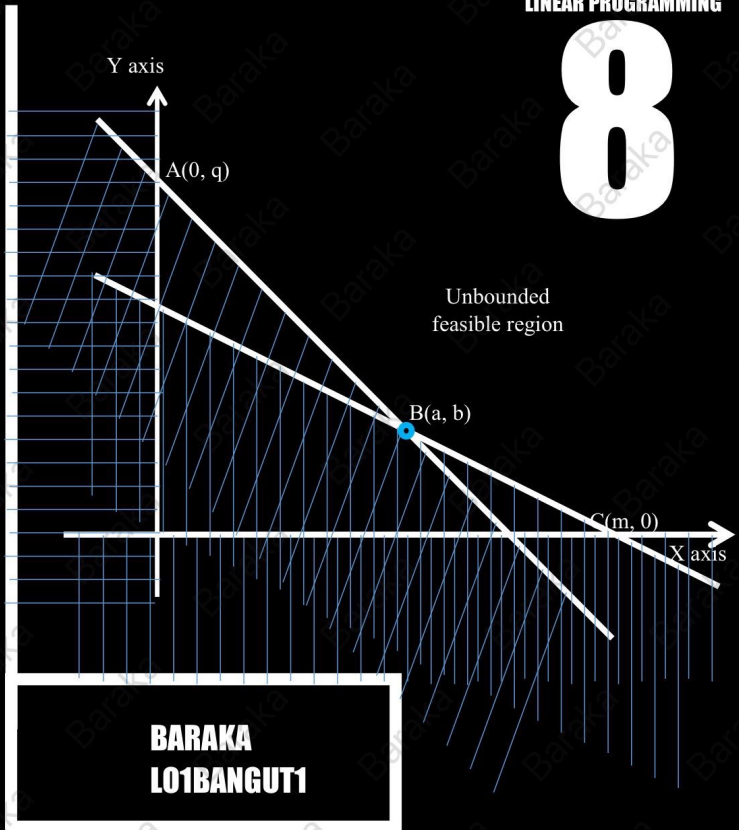


ADVANCED MATHEMATICS

LINEAR PROGRAMMING

8



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To my daughters Gracious and Grace

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LINEAR PROGRAMMING

Chapter

8

Linear programming was first introduced in the by the Russian mathematician K. Leonid in 1930's – 1940's. The aim was to optimize the resources during the war (man and material).

Definitions of terms in linear programming

- (a) **Programming** mean plan on how to use the available resources at an optimal situation
- (b) **Optimal problem**: This is a problem which involves the maximization or minimization of the intended function subject to the certain conditions
- (c) **Objective function**: Is a function of several variables (here two variable) which determine the optimal value or point of the linear programming problem
- (d) **Constraints**: These are linear inequalities of the linear programming which are satisfied accordingly
- (e) **Optimal solution**: Is a point of the feasible region which gives an optimal value
- (f) **Optima value**: Is a maximum or minimum value obtained from feasible region of a linear programming problem
- (g) **Feasible region**: The unshaded region of a linear programming graph.
- (h) **Feasible solution**: Any solution which also satisfies the non-negativity restrictions of the problem is called a feasible solution.
- (i) **Bounded feasible region**: Is a feasible region which is between the lines of the inequalities
- (j) **Unbounded feasible region**: Is a feasible region which is outside the inequalities and the axes
- (k) **Solution**: A set of values of decision variables satisfying all the constraints of a linear programming problem.



- (l) **Optimal feasible solution:** Any feasible solution, which maximizes or minimizes the objective function, is called an optimal feasible solution.
- (m) **Corner point:** A corner point of a feasible region is a point in the feasible region that is the intersection of two boundary lines

Steps to solve a linear programming problem

The following steps can be followed where necessary

- Introduce the decision variables and explain your variables x and y
- Form the constraints if not given add the non-negativity restrictions on the decision variables.
- Write the objective function
- Sketch the model and shade unrequired region hence show the feasible region
- List the corner points and use the objective function to optimize it
- Draw a valid conclusion

Keywords used in linear programming with their signs

The word: -

- | | |
|---------------------|----------------------|
| (a) At least $\geq$ | (f) Demand $>$ |
| (b) At most $\leq$ | (g) Available $\leq$ |
| (c) Required $\geq$ | (h) Budget $<$ |
| (d) Maximum $\leq$ | (i) Less $\leq$ |
| (e) Minimum $\geq$ | |

Note that, in linear programming, the shaded part is the part which does not satisfy the inequality/constraint. The inequalities with the sign $>$ or $<$ are drawn using the dotted lines, while the inequalities with the sign $\geq$ or $\leq$ are drawn full lined.

Not negativity constraints are $x \geq 0$ and $y \geq 0$, these inequalities are always in any linear programming problem, to help avoiding getting the negative values or having a negative plan.

Questions

- Sketch and shade the region represented by the inequalities $3x + 5y \geq 15$, $x - 2y \leq 2$ and $2x + y \leq 5$
- By shading the required region of the following inequalities find the corner points of the feasible region obtained from these inequalities $x + y \geq 10$, $y \leq 2$, $7x + 4y \leq 28$, $-3x + 2y \geq 4$ and $5x - y \geq 30$ hence find which point gives a maximum and minimum value of the objective function $z = x + y$ (Correct to one decimal place where necessary).

Example 1

Given the constraints

$$x + y \leq 5, \quad x + y \geq 2, \quad x \leq 3, \quad y \leq 3, \quad x \geq 0, \quad y \geq 0$$

Optimize the function $f(x, y) = 100x + 300y - 10$

Solution

To sketch the graph of inequalities we first draw equation of the line representing the inequality and then test, to get the region which is satisfied by the inequality. To test, randomly choose a point (coordinate) either above or below the line but not on the line. Take the abscissa and the ordinate of the chosen point and fix in the appropriate inequality and shade the false region. Remember if the coordinate chosen is above the inequality and the result in testing is true, then shade below the line which is false automatically, vice versa is true.

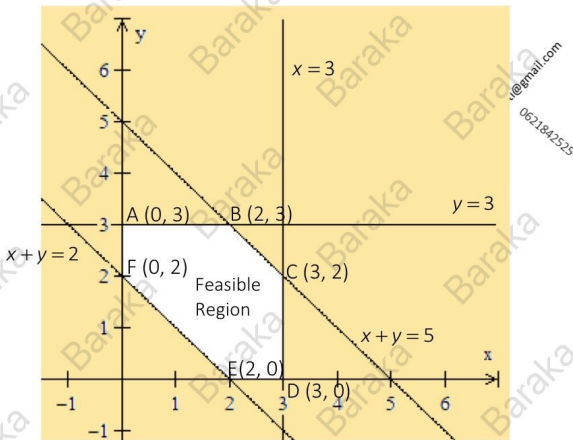


Table of results

Corner points	Objective function $f(x, y) = 100x + 300y - 10$
A (0, 3)	890
B (2, 3)	1090
C (3, 2)	890

D (3, 0)	290
E (2, 0)	190
F (0, 2)	590

The maximum value of the objective function is 1090 at point B (2, 3) and the minimum value is 190 at point E (2, 0).

Example 2

The school calendar shows that a mathematics teacher works at most 24 hours in a week. He earns Tshs. 10,000/- for teaching additional mathematics and Tshs. 7000/- for teaching basic mathematics per week. He wishes to spend at least 4 hours but not more than 9 hours a week in teaching additional mathematics. The teacher also wishes to use at least 12 hours in a week for teaching. How would he program his work so as to obtain maximum income?

Solution

Let x be hours of teaching additional mathematics

Let y be hours of teaching basic mathematics

The statement "The school calendar shows that a mathematics teacher works at most 24 hours in a week" mathematically become: $x + y \leq 24$. (the word at most is used here, to show the maximum time).

The statement "He wishes to spend at least 4 hours but not more than 9 hours a week in teaching additional mathematics", mathematically become: $x \geq 4$, and $x \leq 9$. Lastly the statement "The teacher also wishes to use at least 12 hours in a week for teaching", mathematically is written as $x + y \geq 12$.

The statement "He earns Tshs. 10,000/- for teaching additional mathematics and Tshs. 7000/- for teaching basic mathematics per week" is the objective function, mathematically this is: $f(x,y) = 10000x + 7000y$, and goes with "How would he program his work so as to obtain maximum income?" which needs to maximize the objective function.

Without forgetting the non-negative constraints: $x \geq 0$ and $y \geq 0$

Collectively, the constraints are:-

$$\begin{cases} x + y \leq 24, & x \geq 4 \\ x \leq 9 & \text{and } x + y \geq 12 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

The objective function is

$$f(x, y) = 10000x + 7000y$$

(maximum)

Graphically

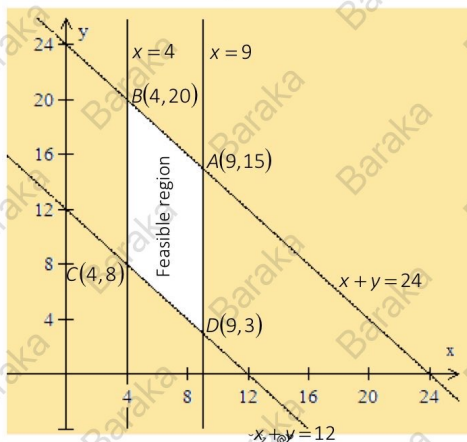
x and y intercepts method,

$x + y = 24$ and $x + y = 12$

The lines $x = 4$ and $x = 9$

are the vertical lines.

Table of results



Corner points

Objective function

$$f(x, y) = 10000x + 7000y$$

A (9, 15)	195,000
B (4, 20)	180,000
C (4, 8)	96,000
D (9, 3)	110,000

Conclusion: From the table above, to maximize the revenue the teacher need to program for 9 lessons for additional mathematics and 15 lessons for basic mathematics. The maximum revenue is Tshs. 195,000/-

Example 3

Silprosa wants to invest at most Tshs. 15,000/- in bank A and B. According to the rules she has to invest at least Tshs. 4,000/- in bank A and at least Tshs. 5000 in bank B. If the rate of interest on bank A is 9% per annum and on bank B is 12% per annum,

- How should she invest her money for a maximum interest?
- What is the maximum interest per annum?
- What is total amount of money received at the end of one year?

Solution

Let x be money invested in bank A

Let y be money invested in bank B

Bank	Interest	Investment (at least)
A	9%	4000
B	12%	5000
Total		15000

Total investment: $x + y \leq 15000$ (available for investment)

Investment on bank A: $x \geq 4000$ (the minimum amount to invest)

Investment on bank B: $y \geq 5000$ (the minimum amount to invest)

Objective function: $f(x, y) = \frac{9}{100}x + \frac{12}{100}y$ (maximum)

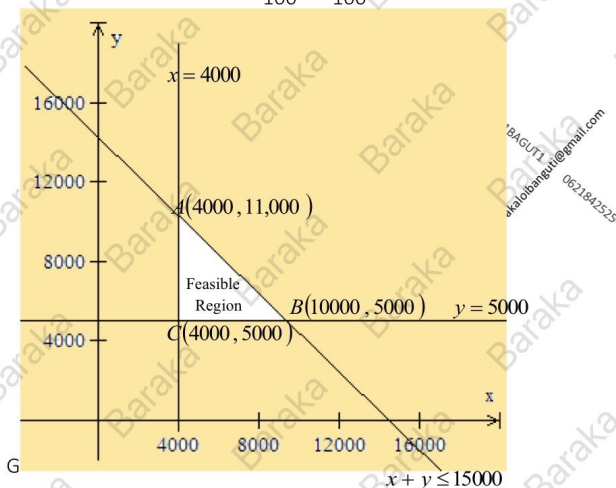


Table of results

Corner points	Objective function $f(x, y) = \frac{9}{100}x + \frac{12}{100}y$
A (4000, 11,000)	1680
B (10000, 5000)	1500
C (4000, 5000)	960

- (a) She should invest Tshs. 4000/- in bank A and 11,000 in bank B
- (b) The maximum interest per annum is Tshs. 2,680/-
- (c) The total amount is = money invested + interest
Total amount Tshs. 16680

EXERCISE 1

1. Two carpentry centre X and Y received an order of making at least 70 chairs and 56 tables for St. Paul school. Carpentry X and Y spent 15/= and 14/= per day to make a chair and a table respectively, X can make 7 chairs and 2 tables per day, while Y can make 2 chairs and 7 tables per day.
 - (a) How many days shall each work to minimum cost and meet the requirements?
 - (b) If X make a sale of Tshs. 25/- and Y make a sale of Tshs. 27/- determine: -
 - (i) The maximum revenue of X and Y at a minimum cost.
 - (ii) The net profit for the obtained collectively.
2. A certain district was hit by heavy flood, bridges and roads were destroyed. It was urgently necessary to send in at least 105 rescue workers and at least 112 tons of food, tents and other goods. Two helicopters A and B were available. A can carry 6 people and 4 tons of goods, while B can carry 5 people and 8 tons of goods per trip.
 - (a) How many trips should each helicopter make in order to transport all the people and all the goods and so that the total number of trips will be minimum.
 - (b) If the transportation cost of helicopter A and B is Tshs 400/- and 450/- respectively, what is the minimum cost of transportation?
3. The manager of a car park allows 9m^2 of parking space for each car and 18m^2 for each lorry. The total space available is 315m^2 . He decides that the maximum number of vehicles at any time must not exceed 31 and he also insists that number of cars must be at least half number of lorries. If the number of cars is X and the number of lorries is Y.
 - (a) Write down the inequalities to represent this information
 - (b) Draw a graph of the inequalities
 - (c) If the parking charge is Tshs. 1000/- for each car and Tshs. 5000/- for each lorry. Find how many vehicles of each kind he should admit to maximize his income and calculate his income.
 - (d) If the charges are charged to sh.2000 for a car and sh.3000 for a lorry, find how many of each kind he now admits to maximize his income.

4. A doctor prescribes a minimum of 40 units of vitamin A and 57 units of vitamin C. His patient shall have portions of food 1 and food 2. The following table shows the units of vitamin A and C in each portion

	A	C
FOOD 1 (KG)	3	4
FOOD 2 (KG)	2	7

The doctor also suggested that the amount of kg of food 2 taken by the patient should be greater than or equal the amount of kg of food 1 taken. Let food 1 be x and food 2 be y

- (i) Form linear programming constraints
 - (ii) Maximize $x + y$
 - (iii) Determine the intermediate value of $x + y$
5. A manufacture produces nuts and bolts. It takes an hour of work on machine A and 3 hours on machine B to produce a package of nuts while it takes 1 hour on Machine A and 7 hours in machine B to produce a package of bolts. He earns a profit of Tshs.10.00/- per package on nuts and Tshs.14.00/- per package on bolt. How many packages of each should be produced per day so as to maximize his profit if he operates machines A for at most 11 hours per week and machine B for at most 49 hours per week?
6. The officers of a high school senior class are planning to rent buses and vans for a class trip. Each bus can transport 30 students requires 4 chaperones and costs Tshs. 90,000/- to rent. Each van can transport 12 students require 2 chaperones and costs Tshs. 50,000/- to rent. Since there are 240 students in the senior class that may be eligible to go on the trip, the officers must plan to accommodate at least 240 students. Since only 36 parents have volunteered to serve as chaperones.
- (a) How many vehicles of each type the officers rent in order to minimize the transportation costs?
 - (b) What are the minimal transportation costs?
7. Two mills produce the same three types of plywood. The table given below gives the production, demand and cost data.

Plywood	Mill 1 per day	Mill 2 per day	Six – month demands
A	100 sheets	20 sheets	2000 sheets
B	40 sheets	80 sheets	3200 sheets
C	60 sheets	60 sheets	3600 sheets
Daily costs	3,000,000	2,000,000	Cost in Tshs

- Find the number of days that each mill should operate during the 6 months in order to supply the required sheets in the most economical way.
8. A tailor has the following materials available 16 square metre of cotton, 11 square metre of silk and 15 square of wool. If a trouser requires 2 square of cotton, 1 square of silk and 1 square of wool while a coat requires 1 square of cotton, 2 square of silk and 3 square of wool, and if a trouser sells for Tshs.30/= and a coat sells for Tshs.25/=. How many trousers and coats should the tailor make to obtain the maximum amount of money?
 9. A factory manager wishes to install two types of machines, small machine and large machine. Small machines needs 2 operators and occupy 4m^2 of floor space; large machines needs 3 operators and occupy 8m^2 of floor space. There are 56 operators available and 136m^2 of floor space. The sales per week is 30/= on a small machine and 50/= on a large. If the cost of running both machines for a week is 365/=. find the greatest weekly net profit.
 10. A calculator company produces a non-programmable calculator and a programmable calculator. Long-term projections indicate an expected demand of at least 100 non-programmable calculator and 80 programmable calculator each day. Because of limitations on production capacity, no more than 200 non-programmable calculator and 170 programmable calculator can be made daily. To satisfy a shipping contract, a total of at least 250 calculators must be shipped each day. If each non-programmable calculator sold results in a loss of 200, but each programmable calculator produces a 500 profit, how many of each type should be made daily to maximize net profit?
 11. A farmer has 10 acres of land on which he can grow either maize or Wheat. He has 59 working days available during the cultivation season. Maize requires 5 days per acre of labour while wheat requires 8 days of labour per acre. Net profit from maize per acres is Tshs 30000/= and that of wheat is Tshs 45000/=. How many acres of each crop should he plant to maximize his profit? What is the maximum profit?
 12. Bwiru Ufundi make product A and B. A requires 2 hours of processing at work centre1 (WC1) and 1 hour of processing at work centre 2 (WC2). B requires 1 hours of processing at (WC1) and 2 hours of processing at WC2. WC1 and WC2 are in operation 280 and 330 hours per month respectively. Bwiru Ufundi makes a profit of Tshs. 1000/= per unit of products A and sh. 1500/= per units of product B. How many hours should be used to produce each product to maximize profit?

13. A farm is to be planted with sorghum and maize while observing the following

	Sorghum	Maize	Maximum total
Days labour per hectare	3	3	18
Labour cost per hectare	2000	3000	16000
Cost of fertilizer per hectare (Tshs)	200	100	1000

If sorghum yields a profit of Tshs. 800/- shillings per hectare while maize yield Tshs. 600/- shillings per hectare. How many hectares should be planted with each crop for maximum profit?

14. A furniture company manufacturing dining tables and chairs. Each table requires 8 hours in the assembly department and 2 hours in the finishing department and contributes a profit of 2500/=-, each chair required 2 hours in the assembly department and 1 hour in the finishing department and contribute a profit of 900/=-. The maximum labour hours available each day in the assembly and finishing departments are 400 and 120 respectively. How many tables and chairs should be manufactured each day to maximize the daily profit?
15. A nutritionist prescribes a special diet for patients containing the following number of units of vitamins A and B per kg of two food types F_1 and F_2

	Vitamin A	Vitamin B
F_1	20	14
F_2	15	14

If the minimum daily intake is 120 units of A and 70 units B, what is the least total mass of food a patient must have so as to have enough of these vitamins?

16. A farmer wants to plant tomatoes and potatoes. Tomatoes needs 3 men per hectare and potatoes need also 3 men per hectare. He has 48 hired labourers available. To maintain a hectare of tomatoes needs 250 shillings while a hectare of potatoes costs him 100 shillings. Find the greatest possible area of land he can saw provided the farmer must grow the two crops, if he is prepared to use more 25,000 shillings for farming.
17. A technical school is planning to buy two types of machines. A lather machine needs a minimum of $3m^2$ of floor space and a drill machine need not less than $2m^2$. The cost of one lather machine is 6,000 shillings and a drill machine costs 7,000 shillings, and the school can spend no more than 74,000 shillings. If the school can buy at least 8 machines. Find the greatest number of machines the school can buy.
18. BARICK Company owns two small gold mines each of which produced three grades G1, G2 and G3. The first mine at Geita costs Tshs 80,000/=- per day to run and produces 300g of G1, 100g of G2 and 200g of G3 per day. The second mine at Buhemba costs 60,000/=- per day to run and produce 100g of G1, 100g of G2 and

600g of G3 per day. The company received an order for 600g of G1, 400g of G2 and 1200g of G3 per week. How many days per week should each mine work to fulfil the order as cheaply as possible?

19. The Rangi Company owns a small paint factory that produces both interior and exterior house paints for whole sale distribution. Two basic raw materials A and B are used to manufacture the paints. The maximum availability of A is 10 tons a day that of B is 12 tons in day. The daily requirement of materials per ton of interior and exterior paints are summarized in the table.

	Raw materials A	Raw materials B
Exterior (per ton)	1	2
Interior (per ton)	2	1

The daily demand for interior paint should not exceed that of exterior paint by more than 1 ton. The interior paint should be at most 2 tons daily. The selling price for exterior is 300/= per ton and that of interior paint is 200/= per ton. How much interior and exterior paint should the company produce in order to maximize its gross income and what is this gross income?

20. Sara had 280 shillings to buy erasers and pencils. An eraser cost 20 shillings while a pencil costs 30 shillings. The number of erasers bought is at least twice the number of pencils and then the number of erasers bought is at least 6 and she must buy at least 2 pencils. The discount is each eraser is 1 shilling while the discount for each pencil is 3 shillings then;-
- Formulate the inequalities that represent this information.
 - Determine how many erasers and pencils she bought to obtain the maximum discount, and what is the maximum discount?
21. Products A and B are to made from chemicals P, Q and R. Product A requires 2kg, 1kg and 1kg whereas product B requires 1kg, 2kg and 3kg of chemicals P, Q and R respectively. How many units of each product be made to maximize profit if the profit on A is Tshs 3500/= per unit and on B Tshs 4000/= per unit given that the amount that were available for chemicals P, Q and R were 160kg, 110kg and 150kg respectively.
22. GRACIOUS investments intend to put on the market two items, B and M. The cost of materials is the same for each but B requires 10 hours in the first stage of its preparation and 20 hours in the final stage where item M requires 15 hours in the first stage and 10 hours in the final stage. The amount of time available in any stage is 150 hours. First stage can be repeated for at most 11 hours per day and final stage can be repeated for a maximum of 18 hours per day. The profit on item B is T.sh. 200/= per kg and that on item M is Tshs. 150/= per kg. How many kg of item B and M should be made to obtain the highest profit?

23. A retail shop receive orders from two customers A and B for the following food packages. The package for A should contain 20 kg of beans, 20 kg of rice and 20 kg of maize flour, while that for B should contain 10kg of beans, 30 kg of rice and 20 kg of maize flour. The shop has only 340 kg of beans, 540 kg of rice and 380 kg of maize flour. If a unit of package A cost Tshs. 1200/- and package B costs Tshs. 900/-
- (a) How many packages should he supply to each of his customers so as to realize the maximum sales?
 - (b) How much of each commodity does the retailer remain with after meeting the order?
24. A company own 2 mines. Mine A produces 2 tons of high grade, 3 tons of medium grade and 5 tons of low grade ores each day. Mine B produces 2 tons grades of three grade ores each day. The company needs 80 tons of high grade ores, 160 tons of medium grade ores and 200tons of low grade ores. How many days should each mine be operated to minimize cost if it costs 200,000/= per day to operate each mine?
25. GRACE workshop makes tables and chair which are possessed through assembly and finishing section. If total of 72 hours per week are available in the assembly section and 48 hours per week are available in finishing section and to manufacture 1 table requires 3 hours in assembly and 3 hours in finishing section while chair requires 4 hours in assembly and 2 hours in finishing section. If the profit of each item is 100/=. How many chairs and tables must be constructed per week so that he can get maximum profit?
26. A hotel has 150, 120 and 150 units of ingredients A, B and C respectively. A loaf of bread requires 1, 1 and 2 units of A, B and C respectively. A cake requires 5, 2 and 1 units of A, B and C respectively. If a bread cost 150/= and a cake cost for 400/=: how many of each should the hotel manufacture to minimize the cost and what is the minimum cost?
27. The Appliance Barn has 2400 cubic feet of storage space for refrigerators. Large refrigerators come in 60-cubic-feet packing crates and small refrigerators come in 40-cubic-feet crates. Large refrigerators can be sold for 2500 profit and the smaller ones can be sold for 1500 profit. How many of each type of refrigerator should be sold to maximize profit and what is the maximum profit if at least 50 refrigerators must be sold each month.
28. A small manufacturing plant produces two types of toys: P and Q. At full capacity the plant use 1000 man-hours in a week in its machining department, 400 man-hours in the assembly department, 250 man-hours in the painting department, to produce each type of toy in each department requires the following man-hours per week.

Department toys	Machining	Assembly	Painting
P	2	2	1
Q	5	1	1

How many of each type should the plant produce per week so as to maximize sales if type P sales at 3,500/= and type Q sales at 2,200?

29. The school of St. Jude decided to buy books from two suppliers, Kimahama books supplier and Kase store books supplier. The book from Kimahama supplier costs \$3 and a book from Kase store costs \$2. The school can spend at least \$30 for books and also decided that books from Kimahama supplier should not exceed 6 and that of Kase store should not exceed 12 books also books in total should not exceed 16.

- Summarize this information mathematically
- Assume the discount is per book, and if the discount from Kimahama supplier to the school is \$ 0.7 and from Kase store to school is \$ 0.3, how much books the school should buy from each supplier to maximize the discount?

30. A company makes two types of soft-drinks, Coca-Cola and Fanta. The contribution of each of the product is Tshs. 150 per bottle and 140 per bottle respectively. Both products are processed by three machine M1, M2 and M3. The time required, in hours, per week on each machine is as follows:

MACHINE	COCACOLA	FANTA	AVAILABLE
M1	3	3	36
M2	2	6	60
M3	4	2	44

- How should the company schedule its production in order to maximize contribution?
 - Find the maximum contribution
31. A company produces two different products. One of them needs $\frac{1}{4}$ of an hour of assembly work per unit, $\frac{1}{8}$ of an hour in quality control work and Tshs. 120/- in raw materials. The other product requires $\frac{1}{3}$ of an hour of assembly work per unit, $\frac{1}{3}$ of an hour in quality control work and Tshs. 200/- in raw materials. Given the current availability of staff in the company, each day there is at most a total of 200 hours available for assembly and 80 hours for quality control, there is at least Tshs. 100,000/- for raw materials. The first product described has other production cost of Tshs. 800/- per unit and the second is Tshs. 700/- per units. In addition, the minimum amount of daily sales for the first product is estimated to be 100 units, without there being a maximum limit of daily sales for the second product.

- (a) Formulate and solve graphically a linear programming model that will allow the company to minimize the cost.
- (b) What is the minimum cost?
32. A diet is to contain at least 400 units of carbohydrates, 500 units of fat and 600 units of protein. Two foods A and B are available. Food A cost 2 US dollars per units and food B cost 4 US dollars per unit. A units of food A contains 8 units of carbohydrates, 20 units of fat and 15 units of protein. A unit of food B contains 25 units of carbohydrates, 10 units of fat and 20 units of protein. Formulate a linear programming problem so as to find the minimum cost for a diet that consists of a mixture of these two foods and also meet the minimum requirements.
33. In the production of two types of toys, a factory uses three machines A, B and C. The time required to produce the first type of toy is 6 hours, 5 hours and 12 hours in machine A, B and C respectively. The time required to make the second type of toys is 8 hours, 4 hours and 4 hours in machine A, B and C respectively. The maximum available time in minutes for the machine A, B and C are 22800, 12000 and 24000 respectively. The profit on the first type of toy is 5 dollars while that on the second type of toys is 3 dollars. Find the number of toys of each type that should be produced to get maximum profit.
34. Sketch the graph of $\begin{cases} x + y \leq 50 \\ x - y \leq 10, \quad y - x \geq 10 \\ x \leq 5, \quad y \leq 25, \quad x \geq 0 \end{cases}$ show the feasible region and find the minimum and maximum value of $f(x, y) = 100(x + y)$
35. Sketch the graphs of $\begin{cases} -x + y < 10 \\ x + y > 10 \\ 2x - y \leq 14 \end{cases}$

TRANSPORTATION PROBLEMS

In a process of trying to program a transportation schedule of materials or goods from two or more the depots to the sites, one must think of how to minimize the transportation cost. This idea can be done well using linear programming processes.

Two – sources and two – destinations

Source is a place where materials or goods where deposited/manufactured while destination is where the goods are to be taken. In this book we use a rectangle to represent source and an oval represent a destination. Other producers of solving a linear programming remains the same.

Example 4

The transport manager has two deposit D1 and D2 holding 120 tonnes of cement and 40 tonnes of cement respectively. He has two customers C1 and C2 have ordered 80 tonnes and 50 tonnes of cement respectively, C1 is 20km from D1 and 40km from D2; C2 from D1 is 15km and 30km from D2 and delivery costs are proportional to the distance travelled such that 1km = Tshs. 1/-. How the manager will distribute the cement at minimum cost to his customers?

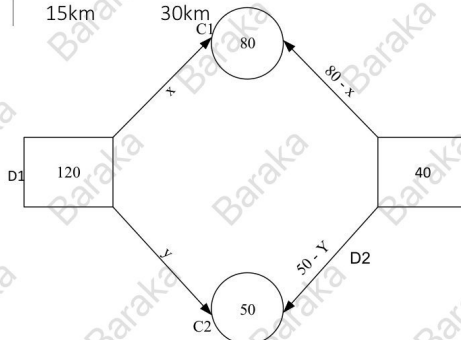
Solution

Two deposits named D1 and D2 and two customers named C1 and C2 are given.
Let

- The cements taken from D1 to C1 be x
- The cements taken from D1 to C2 be y
- The cements taken from D2 to C1 will be $80 - x$
- The cements taken from D2 to C2 will be $50 - y$

Transportation table

From / to	D1	D2
C1	20km	40km
C2	15km	30km



Constraints

$$x + y \leq 120$$

(Number of cements taken from D1 to C1 and C2 cannot exceed the amount available in a depot)

$(80 - x) + (50 - y) \leq 40$ (Number of cements taken from D2 to C1 and C2 cannot exceed the amount available in a depot)

$$80 - x + 50 - y \leq 40 \text{ collecting like terms } -x - y \leq 40 - 130 \text{ therefore } x + y \geq 90$$

From $80 - x \leq 80$ then $x \geq 0$

From $50 - y \leq 50$ then $y \geq 0$

Then $x \geq 80$ and $y \geq 50$

$$\text{Collectively } \begin{cases} x + y \leq 120, & x + y \geq 90 \\ x \geq 0, & y \geq 0 \\ x \leq 80, & y \leq 50 \end{cases}$$

The objective function, $f(x, y) = 20x + 15y + 40(80 - x) + 30(50 - y)$

$$f(x, y) = 4700 - 20x - 15y \text{ (Minimum)}$$

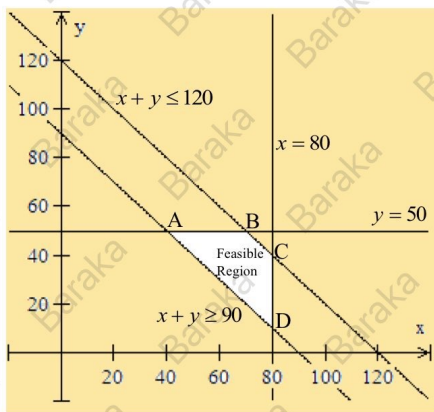


Table of results

Corner points	Objective function $f(x, y) = 4700 - 20x - 15y$
A (40, 50)	3150
B (70, 50)	2550
C (80, 40)	2500
D (80, 10)	2950

The minimum transport cost is Tshs. 2500/-. He should schedule:- 80 tonnes from D1 to C1, 40 tonnes from D1 to C1 and 10 tonnes from D2 to C2.

Example 5

A company produced two product has two factories F1 and F2 for product storage. F1 has 90 tonnes of the product, and 80 tonnes of the product in F2. Two customers A and B place orders for 40 and 70 units respectively. Distance from A to F1 and F2 are 10km and 8km respectively, while the distance from B to F1 and F2 are 9 km to 9 km. The transport costs of 1 tonne from each factory to each of the customer are tabulated below.

Factory	Transport costs	
	A	B
F1	80/=	120/=
F2	105/=	130/=

How many tonnes of the product should the company deliver to each customer from each factory in order to minimize the total transport cost?

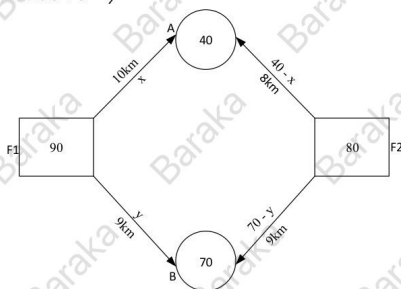
Solution

Let x be tonnes taken from F1 to A

Let y be tonnes taken from F1 to B

Tonnes taken from F2 to A will be $40 - x$

Tonnes taken from F2 to B will be $70 - y$



Constraints:

$$x + y \leq 90 \quad \dots (i)$$

$$x \leq 40 \quad \dots (ii)$$

$$y \leq 70 \quad \dots (iii)$$

$$(40 - x) + (70 - y) \leq 80$$

$$110 - x - y \leq 80$$

$$x + y \geq 30 \quad \dots (iv)$$

$$40 - x \leq 40$$

$$x \geq 0 \quad \dots (v)$$

$$70 - y \leq 70$$

$$y \geq 0 \quad \dots (vi)$$

Objective function $f(x, y) = 115500 - 40x - 90y$

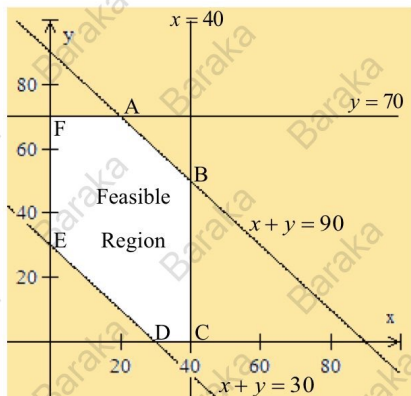


Table of results

Corner points	Objective function
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$$f(x, y) = 115500 - 40x - 90y$$

A (20, 70)	108,400
B (40, 50)	109,400
C (40, 0)	113,900
D (30, 0)	114,300
E (0, 30)	112,800
F (0, 70)	109,200

To minimize the transport cost the company should

- Take 20 tonnes from F1 to A
- Take 70 tonnes from F1 to B
- Take 20 tonnes from F2 to A

The minimum cost is Tshs. 108, 400/-

Two sources and three destinations

Example 6

Kilimanjaro Drinking Water Company has two deposits P and Q which hold of 700 litres and 400 litres respectively. The company is to supply water to 3 sub-depots A, B and C whose requirements are 450 litres, 300 litres and 350 litres respectively. The distance (in km) from P to A, B and C are 7km, 6km and 3km respectively, the distance from Q to A, B and C are 3km, 4km and 2km. The transport cost of 10 litres of water is Tshs. 1 per km. (litres are given to nearest thousands)

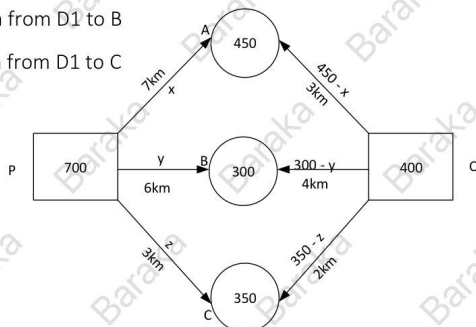
- How should the schedule be made in order to minimize the transport cost?
- Determine the minimum cost

Solution

Let x be litres taken from D1 to A

Let y be litres taken from D1 to B

Let z be litres taken from D1 to C



Constraints

$$x + y + z \leq 700$$

...(i)

$$x \leq 450$$

...(ii)

$$y \leq 300$$

...(iii)

$$z \leq 350$$

...(iv)

$$450 - x + 300 - y + 350 - z \leq 400$$

$$1100 - (x + y + z) \leq 400$$

$$x + y + z \geq 700$$

...(v)

$$450 - x \leq 450$$

$$x \geq 0$$

...(vi)

$$300 - y \leq 300$$

$$y \geq 0$$

...(vii)

$$350 - z \leq 350$$

$$z \geq 0$$

...(viii)

Eliminate z from these equations

From $x + y + z \leq 700$ (equation (i)) make z the subject as an equation and substitute in all other inequalities containing z .

$$z = 700 - (x + y)$$

Substitute z in (4) and (8) above

$$z \leq 350$$

$$700 - (x + y) \leq 350 \text{ then } x + y \geq 350$$

$$\text{Also } z \geq 0$$

$$700 - (x + y) \geq 0 \text{ then } x + y \leq 700$$

Objective function,

It is stated that 10 litres of water per 1 km

From P to A, B and C

- The cost is $\frac{1}{10}(7x + 6y + 3z)$

From Q to A, B and C

- The cost is $\frac{1}{10}(3(450 - x) + 4(300 - y) + 2(350 - z))$

Collecting the two equations above, makes an objective function

$$f(x, y, z) = \frac{1}{10}(7x + 6y + 3z + 3(450 - x) + 4(300 - y) + 2(350 - z))$$

$$= \frac{1}{10}(7x + 6y + 3z + 1350 - 3x + 1200 - 4y + 700 - 2z) = \frac{1}{10}(4x + 2y + z + 3250) \quad \text{But}$$

$$z = 700 - (x + y)$$

$$\text{The objective function is } f(x, y) = \frac{1}{10}(3x + y + 3950)$$

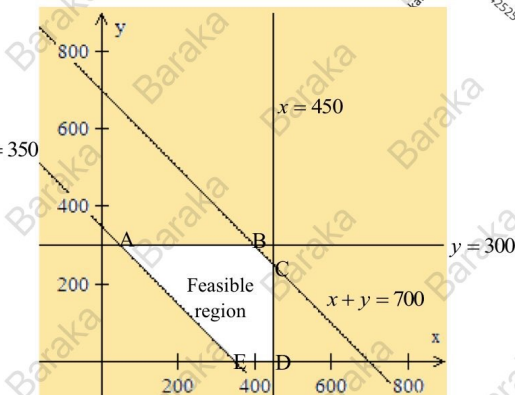


Table of results

Corner points	Objective function $f(x, y) = 1/10(3x + y + 3950)$
A (50, 300)	440
B (400, 300)	545
C (450, 250)	555
D (450, 0)	530
E (350, 0)	500

The corresponding value of z is given by $z = 700 - (x + y)$

(a) To minimize the transport cost the company should

- Take 50 litres from P to A
- Take 300 litres from P to B
- Take 350 litres from P to C
- Take 400 litres from Q to A

(b) The minimum cost is 440 (to nearest thousands)

Example 7

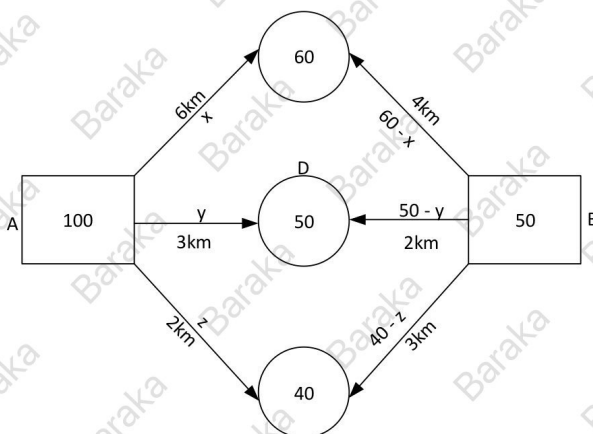
A supplier has two godowns A and B in which has storage capacity of 100 tonnes and 50 tonnes of maize respectively. The three customers C, D and E placed their order 60, 50 and 40 tons respectively. The distance from A to C, D and E are 10km, 6km and 4 km respectively, while from B to C, D and E are 8km, 5km and 7km respectively, the cost of transporting 1 tonne per 1 km (in nearest thousands) from the godowns to the customers are given in the table below

From/to	Godown A	Godown B
C	6	4
D	3	2
E	2	3

- How should the supplier fulfil the needs of his customers at minimum cost?
- What is the minimum cost obtained?
- If the profit of one tonne from godown A and B is Tshs. 70000 and Tshs. 100000 respectively, calculate the net profit of the supplier assuming the transportation cost is upon the supplier.

Solution

- Let x be tonnes from A to C
- Let y be tonnes from A to D
- Let x be tonnes from A to E



Constraints

$$x + y + z \leq 100$$

$$x \leq 60$$

$$y \leq 50$$

$$z \leq 40$$

$$60 - x + 50 - y + 40 - z \leq 50$$

$$x + y + z \geq 100$$

$$60 - x \leq 60$$

$$x \geq 0$$

$$50 - y \leq 50$$

$$y \geq 0$$

$$40 - z \leq 40$$

$$z \geq 0$$

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...(ii)

...(iii)

...(iv)

...(v)

...(vi)

...(vii)

...(viii)

From (i) make z the subject and substitute in (iv) and (viii)

$$z = 100 - x - y$$

Substituting in (iv) $z \leq 40$

$$100 - x - y \leq 40$$

$$x + y \geq 60$$

Substituting in (viii)

$$z \geq 0$$

$$100 - x - y \geq 0$$

$$x + y \leq 100$$

The constraints are
$$\begin{cases} x \leq 60, y \leq 50 \\ x \geq 0, y \geq 0 \\ x + y \geq 60, x + y \leq 100 \end{cases}$$

The objective function

$$f(x, y, z) = 60x + 18y + 8z + 32(60 - x) + 10(50 - y) + 21(40 - z)$$

$$f(x, y, z) = 28x + 8y - 13z + 3260$$

But $z = 100 - x - y$

$$f(x, y) = 28x + 8y - 13(100 - x - y) + 3260$$

$$f(x, y) = 41x + 21y + 1960$$

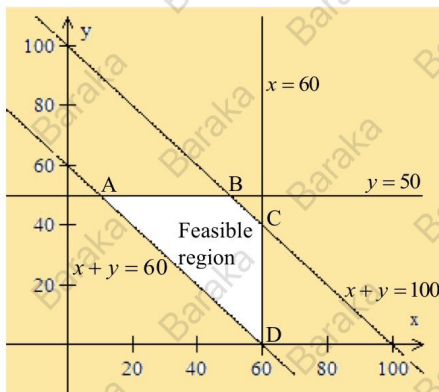


Table of results

Corner points	Objective function $f(x, y) = 41x + 21y + 1960$
A (10, 50)	3420
B (50, 50)	5060
C (60, 40)	5260
D (60, 0)	4420

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(a) To minimize the cost the supplier should

- Take 10 tonnes from A to C, Take 50 tonnes from A to D, Take 40 tonnes from A to E, Take 50 tonnes from B to C

(b) The minimum cost 3420 to nearest thousands

(c) Total tonnes sold from godown A is $10 + 50 + 40 = 100$ tonnes

Total tonnes sold from godown B is 50 tonnes

The profit obtained from godown A is Tshs. $7000 \times 100 = 7,000,000$

The profit obtained from godown B is Tshs. $100,000 \times 50 = 5,000,000$

Total revenue is Tshs. $7,000,000 + 5,000,000 = 12,000,000$

Net profit is Total Revenue – Total Cost

Total cost is Tshs. 3,420,000

Net profit = $12,000,000 - 3,420,000 = 8,580,000$

EXERCISE 2

Magu is supplying water in Arusha town areas, he has two water tanks W1 and W2 with capacity of 300,000 litres and 200,000 litres respectively. He supply water to three small supplies in Sakina, Morombo and Ngaramtoni as 150,000, 200,000, and 150,000 litres of water respectively, the cost of transporting 1000 litres to small supplies from the tanks are given below

	Sakina	Morombo	Ngaramtoni
W1	400	200	300
W2	200	600	400

Magu wants to keep the transportation cost minimum as possible. How should he do it?

- Gracious and Grace Company has two deposits D1 and D2 of wheat supplies flour with capacity of 35,000kg each. Two sellers R1 and R2 placed their order for 30,000kg and 35,000kg respectively. The transportation cost from the deposits to the retailers are tabulated below

	D1	D2
R1	100	90
R2	110	130

Transportation cost is upon the supplies and they want to minimize cost of transportation, find how should the company supply to meet the sellers requirements.

- A certain person got a tender to supply milk to two military camps MC1 and MC2 with the requirements of 100 and 150 litres of milk respectively. This person has two milk refrigeration centres MR1 and MR2 with capacities of 120 and 130 litres of milk respectively. The cost of transporting 20 litres of milk to camps is proportional to distances to camps, the distance from refrigeration centres to camps is as show in the table below

	MR1	MR2
D1	10	9
D2	20	15

Find how should the supplies be made to minimize the cost of transportation

MISCELLANEOUS EXERCISE

1. A farmer has two godowns A and B for storing his farm products. He stored 80 bags in A and 70 bags in B. Two customers C and D placed orders for 35 and 60 bags respectively. The transport cost per bag from each godown to each of the two customers are tabulated below

Godown	C	D
A	8	12
B	10	13

How many bags of products should the farmer deliver to each customer from each godown in order to minimize the total transportation costs?

2. A gravel dealer has two quarries Q1 and Q2 which produce 3000cm^3 and 1500cm^2 of gravel per week respectively. Three builders B1, B2 and B3 require each week 2000cm^3 , 1500cm^3 and 1000cm^3 of gravel respectively. The distance between the quarries and the sites of the builders (in km) are as shown below

	B1	B2	B3
Q1	7	4	3
Q2	3	2	2

How should the gravel dealer supply gravel to the builders as cheaply as possible if the cost is proportional to the distance?

3. There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

Cost in Tshs.			
From/to	A	B	C
P	160	100	150
Q	100	120	100

- (a) How many units should be transported from each factory to each depot in order that the transportation cost is minimum?
 - (b) What will be the minimum transportation cost?
4. A farmer has two factories F1 and F2 which can hold 120 tonnes and 90 tonnes respectively. The requirement of three markets A, B and C are 70, 60 and 80 tonnes respectively. The transportation cost in Tanzanian shillings from the factories to the markets are given in the table below

	F1	F2
A	20	15
B	10	25
C	15	9

- How should the delivery be made so as the transport cost is minimized?
 - Determine the minimum cost
 - If he got a profit of Tshs. 25 per tonne from factory F1 and Tshs. 30 per tonne from factory F2.
 - Calculate the revenue obtained
 - Calculate the net profit obtained
5. Simba cement Company has two depots D1 and D2 with stock 25000 cements packets and 20000 packets respectively. Three sites S3, S4 and S5 needs 15000, 20000 and 10000 cements packets respectively. Transport cost of 1000 cement packages from depots sites are given in table below

	D1	D2
S3	50	60
S4	40	30
S5	35	20

How should Company fulfil the needs at a minimum transport cost?

6. A farmer has two ranches R1 and R2 with 1000 and 800 cows respectively. He receive an order from two meat industries M1 and M2 in town with the requirements 750 and 900 cows for slaughtering in a year term respectively. Transporting a cow from ranch R1 to M1 and M2 are Tshs. 300/- and 400/- respectively, transporting a cow from R2 to M1 and M2 are 200/- and 500/- respectively. The profit of a cow from R1 is Tshs. 1000 and from R2 is Tshs. 1500. Use linear programming technique to:-
- Advise the farmer on how to meet the requirements at a minimum transportation cost?
 - What is the maximum revenue he can earn from the sold cows?
 - Calculate the net profit.

7. A cooperative society has two storage depots. At depot Q there are 300 tonnes of beans and at depot R there are 200 tonnes of beans. The beans has to be sent to three stores A, B and C. The demand at store A is 200 tonnes, at store B is 150 tonnes and at store C is 150 tonnes. How much beans should be sent from each depot to each of the stores in order to minimize the total transport cost. If the transport cost in Tshs. per tonne from a depot to a store is as given in the table below

	A	B	C
Q	80	100	40
R	50	150	80

8. A medical company has two factories in P and Q. From these, supply is made to each of its three agencies situated at A, B and C. The weekly requirement of the agencies are 40, 40 and 50 packets respectively, while the production capacity of factories are 60 and 70 respectively. The transportation cost per packet from the factories to the agencies are in the table below:

	P	Q
A	5	4
B	4	3
C	3	5

- (a) How many packets from each factory be transported to each agency at the least cost of transportation?
 (b) What is the optimal cost?
9. A student is taking a test in which items of type A worth 10 points and items of type B worth 15 points. It takes 3 minutes to answer each item of type A and 6 minutes to answer each item of type B. The total time allowed is 60 minutes and she may not answer more than 16 questions. How many of each type should she answer to get the maximum score?
10. Arusha institute has two storage depots. At depot A there is 200 tons of rice stored and at depot B there is 300 tons. The rice has to be sent to three marketing centres A, B and C. The demand at centres A, B and C are 150, 150 and 200 tons respectively. The transport cost per ton from a deposit to centres is given below

From/To	A	B	C
Depot A	50	100	70
Depot B	80	150	40

- (i) How many numbers of tons of rice should be sent from each depot to each of the marketing centre in order that the transportation cost is minimum?
 (ii) Verify whether the transportation problem above is balanced one or not with reason.
11. A paint manufacturer produces two types of paint, one type of standard quality (X) and the other of top quality (Y). To make these paints, he needs two ingredients, the pigment and the resin. Standard quality paint requires 2 units of pigment and 3 units of resin for each unit made, and is sold at a profit of 1 per unit. Top quality paint requires 4 units of pigment and 2 units of resin for each unit made, and is sold at a profit of 1.50 per unit. He has stocks of 12

units of pigment, and 10 units of resin. Formulate the above problem as a linear programming problem to maximize his profit.

12. A homemaker wishes to mix two types of food F1 and F2 in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food F1 costs E60/Kg and Food F2 costs E80/kg. Food F1 contains 3 units/kg of vitamin A and 5 units/kg of vitamin B while Food F2 contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Formulate this problem as a linear programming problem to minimize the cost of the mixtures.
13. A furniture company produces inexpensive tables and chairs. The production process for each is similar in that both require a certain number of hours of carpentry work and a certain number of labour hours in the painting department. Each table takes 4 hours of carpentry and 2 hours in the painting department. Each chair requires 3 hours of carpentry and 1 hour in the painting department. During the current production period, 240 hours of carpentry time are available and 100 hours in painting is available. Each table sold yields a profit of \$7; each chair produced is sold for a \$5 profit. Find the best combination of tables and chairs to manufacture in order to reach the maximum profit.
14. A small brewery produces Ale and Beer. Suppose that production is limited by scarce resources of corn, hops and barley malt. To make Ale 5kg of Corn, 4kg of hops and 35kg of malt are required. To make Beer 15kg of corn, 4 kg of hops and 20kg of malt are required. Suppose that only 480 kg of corn, 160kg of hops and 1190 kg of malt are available. If the brewery makes a profit of E13 for each kg of Ale and E23 for each kg of Beer, how much Ale and Beer should the brewer produce in order to maximize profit?
15. A patient in a hospital is required to have at least 84 units of drug A and 120 units of drug B each day. Each gram of substance M contains 10 units of drug A and 8 units of drug B, and each gram of substance N contains 2 units of drug A and 4 units of drug B. Now suppose that both M and N contain an undesirable drug C, 3 units per gram in M and 1 unit per gram in N. How many grams of substances M and N should be mixed to meet the minimum daily requirements at the same time minimize the intake of drug C? How many units of the undesirable drug C will be in this mixture?

LINEAR PROGRAMMING

BARAKA

LO1BANGUT1