

Dears Second Year students 2019

IMPORTANT HINTS

- This document is not final, so if you spot something unusual please let me know in our next classes or by email
- The value of g used in all the calculations is 10 metres per second per second. *However in tests and examinations you should always use $g = 9.81\text{m/s}^2$ unless otherwise stated*
- This document is 1.5 spaced and the font used is 14, just for your easy reading in soft copies. So the pages are not as many as they appear to be. Please don't get discouraged going through it to the last page

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UNIVERSITY OF DAR ES SALAAM



DEPARTMENT OF GEOLOGY

SOIL MECHANICS STUDY MANUAL

WORKED EXAMPLES INVOLVING *BASIC RELATIONSHIPS IN SOIL MECHANICS*

$$\rho_{bulk} = \frac{M}{V} = \frac{(G_s + eS_r)\rho_w}{1 + e}; \quad 0 < S_r < 1$$

$$\rho_{dry} = \frac{M_s}{V} = \frac{G_s\rho_w}{1 + e}; \quad S_r = 0$$

$$\rho_{sat} = \frac{M_{sat}}{V} = \frac{(G_s + e)\rho_w}{1 + e}; \quad S_r = 1$$

$$\rho_{sat} = \frac{G_s\rho_w}{1 + e} + \frac{e\rho_w}{1 + e} = \rho_{dry} + n\rho_w; \quad 0 \leq n < 1$$

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REMARKS

This SOIL MECHANICS STUDY MANUAL is intended to make students understand the basic concepts/ knowledge of soil mechanics through self study or in-class problem solving. The manual contains several solved as well as guided-to-solutions problems that are thought to be relevant for the purpose. Students are therefore recommended and encouraged to go through all the problems contained in this manual

MECHANICS BEHIND A THREE-PHASE STATE OF A WET SOIL

INTRODUCTION: BASIC CONCEPTS AND EQUATIONS

Normally a sample of soil may exist in ***two SOLID states*** only, of *either dry or wet* state. When in a wet state, the same soil may exist in ***two phase states***, of *either partially saturated or completely saturated* phase states. In our discussion here we will mainly deal with a wet soil sample under the latter mentioned category of partial and complete saturation.

Usually, a wet soil sample must necessarily contain also some amount of air. Since there are normally, pore spaces existing between soil grains technically known as the *voids*, they constitute the shelter of air and water molecules in it, so that the total volume of the soil is a sum of the volume of the voids and that which is normally occupied by its solid particles

In a situation where the voids contain both air and water, the soil is said to be *partially saturated* but when the voids contain water only, it is said to be *completely saturated*. In our discussion here, we are going to dedicate our time resources looking at the mechanics of both a partially as well a completely saturated wet soil, using what is popularly known as a three-phase soil model

diagram. You may wish to briefly study the following generalized three-phase model diagram shown below

A GENERALISED THREE-PHASE (SOLID PARTICLES, WATER AND AIR) MODEL OF SOIL				
TOTAL WEIGHT, W $W = W_w + W_s$	Weight of water, W_w	<u>AIR</u> <i>Volume of air, V_a</i> <i>Weight of air, $W_a \approx 0$</i> $V_a = V_v - V_w$	Volume of voids, V_v	TOTAL VOLUME, V $V = V_s + V_v$ $V = V_s + V_w + V_a$
	Weight of Solids, W_s	<u>WATER</u> <i>Volume of water, V_w</i> <i>Weight of water, W_w</i> $W_w = mW_s$ $W_w = m\gamma_s V_s$ $W_w = mG_s \gamma_w V_s$ $V_w = mG_s V_s$		
		<u>SOLID</u> <i>Volume of Solid, V_s</i> <i>Weight of Solids W_s</i> $W_s = \gamma_s V_s$ $W_s = G_s \gamma_w V_s$ $V_s = \frac{W_s}{G_s \gamma_w}$	Volume of solids, V_s	

Figure 1: Tabular representation of a three-phase state model diagram of a soil:
A detailed generalized case

From the model sketch, we find that ***the total weight of soil is $W = W_w + W_s$ (Weight of air $W_a \approx 0$) and the total volume is $V = V_s + V_v = V_s + V_w + V_a$*** ; where;

V_s = Volume of soil solids;

V_v = Volume of voids which is a combination of volumes of water, V_w and of that of air V_a ;

W_s = Weight of solids or solid particles in a soil;

G_s = Specific gravity of soil solids

γ_s = Unit weight of solids

W_w = Weight of water contained within that particular soil specimen;

γ_w = Unit weight of water and;

m = Moisture content of the soil

Moreover, in latter sections that follow, you will encounter additional symbols representing the bulk mass M , mass of solids M_s , saturated bulk mass M_{sat} , mass of water M_w , moisture content of soil at complete saturation m_{sat} and the saturated weight W_{sat} ; where $M = M_s + M_w$ or $M_{sat} = M_s + M_w$

WEIGHT-TO-WEIGHT/VOLUME AND VOLUME-TO-VOLUME RELATIONSHIPS OF A SOIL SPECIMEN WITH TOTAL VOLUME, V AND TOTAL WEIGHT, W

PART A: WEIGHT-TO-WEIGHT/VOLUME RELATIONSHIPS

In a three (3) phase state soil (soil containing solid particles, water and air) state, the total volume V of a given soil sample can be expressed as

$$V = V_s + V_v = V_s + V_w + V_a \quad (1a)$$

A GENERALISED AND SUMMARISED THREE-PHASE MODEL OF WET SOIL		
$W = W_w + W_s$	THREE-PHASE SOIL	$V = V_v + V_s$
W_w	<u>AIR</u> $W_a \approx 0$ $V_a = V_v - V_w$	$V_v = V_w + V_a$
	<u>WATER</u> $W_w = mG_s\gamma_w V_s$ $V_w = mG_s V_s$	
W_s	<u>SOLID</u> $W_s = G_s\gamma_w V_s$ $V_s = \frac{W_s}{G_s\gamma_w}$	V_s

Fig. 2: Tabular representation of a three-phase state model diagram of soil: A summarized generalized case

Whereas for all cases the volume of air, V_a is not zero unless the soil is fully saturated (i.e. unless all the voids in a soil are filled with water, so that $V_a = 0$); the weight of air, W_a is normally taken to be negligible (i.e. $W_a \approx 0$) Now, with W_a being zero, the total weight W is always expressed as

$$W = W_s + W_w \quad (1b)$$

Now since $W = Mg$, then it follows from above equation that

$$M = M_s + M_w \quad (1c)$$

I. BASIC RELATIONSHIPS AND DEFINITIONS

(i) Mass M and weight W

Mass is defined as the content of matter in an object/ body. The weight of a body is simply a pull caused by the earth's pull popularly known as the force of gravity. Whereas the mass of a particular body is constant everywhere on the earth's surface, the weight of the same body may however, vary from one location to another. This is because weight is normally dependent on the value of the gravitational force, a pull force towards the centre of the earth **which** varies with location on the earth's surface and this is the phenomenon which explains the main difference between these two quantities i.e. mass and weight.

Therefore, the relationship existing between mass and weight is

$$W = Mg \quad (2a)$$

(ii) The relationship between Bulk Unit Weight, γ and Bulk Density, ρ

Since $W = Mg$ where M is the mass of the soil and g is the acceleration due to gravity. Now, since $\gamma = W/V$ or simply $\gamma = Mg/V$; therefore

$$V = \frac{Mg}{\gamma} \quad (2b)$$

Similarly $\rho = M/V$ implying that

$$V = \frac{M}{\rho} \quad (2c)$$

On equating equations (2b) and (2c) above we get $Mg/\gamma = M/\rho$

$\Rightarrow \gamma = \rho g$, after we have eliminated V . This indicates that **the unit weight γ is simply the product of density and the gravitational acceleration, g**

Therefore

$$\gamma = \rho g \quad (2d)$$

Using these fundamentally defined quantities, we can then define the following (derived) extra parameters or **weight-to-weight/volume** quantities namely; the unit weights (**bulk, dry and saturated unit weights**), the densities (**bulk, dry and saturated densities**), the moisture content and the specific gravity; which is a ratio between two unit-weights/ two densities

(iii) **Bulk Unit Weight, γ and Bulk Density, ρ**

The bulk unit weight, γ is defined as *the ratio of the **total weight W** of the soil to its **total volume, V*** . Hence, the bulk unit weight, γ is given by the expression

$$\gamma = \frac{W}{V} \quad (3a)$$

Moreover, **the bulk density ρ** is simply *the ratio of the **total mass M** of the soil to its **total volume, V*** i.e.

$$\rho = \frac{M}{V} \quad (3b)$$

And from this definition, we can also define the dry unit weight γ_d , as well the saturated unit weight, γ_{sat} .

(iv) **The Dry Unit Weight, γ_d and the Dry Density, ρ_d**

Also, **the dry unit weight** is just the *ratio of **the total dry weight, W_s** (weight of soil without water) to its **total volume, V*** whereas **the dry density ρ_d** is simply *the ratio of the **mass of soil solids M_s** to its **total volume, V*** . Therefore;

$$\gamma_d = \frac{W_s}{V}; \quad (4a)$$

$$\rho_d = \frac{M_s}{V} \quad (4b)$$

We shall later come to deduce that the dry unit weight (and hence the dry density) can also be expressed by the relationships

$$\gamma_d = \frac{W_s}{V} = G_s \gamma_w V_s = G_s \gamma_w (1 - n) \quad (4c)$$

$$\rho_d = \frac{M_s}{V} = G_s \rho_w V_s = G_s \rho_w (1 - n) \quad (4d)$$

where:

n = Soil porosity (to be defined latter in a section that will deal with *volume-to-volume* relationships)

ρ_w = Bulk density of water; and the rest of the parameters are as previously defined

However, it is very important here to note that the expressions appearing in equations (4c) and (4d) above are specially applied when the total volume is unity and so that the volume of solids; $V_s = 1 - n$. Therefore, for cases where the volume is not unity, the magnitude of such a volume should be multiplied with the respective expression used in the calculation. We will be able to clearly see this when we come to the part of the calculations

(v) **The Saturated Unit Weight, γ_{sat} and the Saturated Density, ρ_{sat}**

The **saturated unit weight, γ_{sat}** is the ratio of the total (fully saturated) weight W_{sat} , to the total volume V , of the soil when all the voids are filled with water only, such that the volume of air V_a is zero; that is, $V = V_s + V_w + 0$ since $V_a = 0$, so that $V_v = V_w$

Similarly **the saturated density, ρ_{sat}** is the ratio of the total mass M_{sat} of the soil to its total volume V , when all the voids are filled with water only.

The saturated unit weight is therefore expressed by the relationship

$$\gamma_{sat} = \frac{W_{sat}}{V} = \frac{W_s + W_w}{V_s + V_w} = \frac{W_s + W_w}{V_s + V_v} \quad (5a)$$

or simply

$$\gamma_{sat} = \frac{W_{sat}}{V} = \frac{G_s \gamma_w + e \gamma_w}{1 + e} = \frac{(G_s + e) \gamma_w}{1 + e} \quad (5b)$$

whereas the saturated density is given by the expression

$$\rho_{sst} = \frac{M_{sat}}{V} = \frac{M_s + M_w}{V} = \frac{(G_s + e) \rho_w}{1 + e} \quad (5c)$$

where:

e is the void ratio (also to be defined latter in the later section whose discussion will be on *volume-to-volume* relationships)

Note that:

- a) **Unit weights** are related to the **weight of the soil** whereas **the densities are related to it mass**
- b) at full saturation; where all the voids in the soil become completely occupied by water, the **model state of the soil** is reduced to two phases only (solid particles and water only) instead of the usual three phases mentioned earlier

(vi) The Moisture Content, m

This is simply *the ratio of the weight of water, W_w to the weight of solids W_s* and is normally expressed as a percentage (%)

Therefore m is normally expressed as:

$$m = \frac{W_w}{W_s} \times 100\% \quad (6a)$$

$$m = \frac{M_w}{M_s} \times 100\% \quad (6b)$$

At this point, equations (3a), and (4a), (4b) /or (3b) and (4c) (4d) can be combined to arrive at a relationships

$$\gamma = \gamma_d(1 + m) \text{ or } \gamma_d = \frac{\gamma}{1 + m} \quad (6c)$$

and

$$\rho = \rho_d(1 + m) \text{ or } \rho_d = \frac{\rho}{1 + m} \quad (6d)$$

Note: At this point, you must be able to derive these two expressions by simply eliminating the quantities W, M, W_s, M_s and W_w, M_w

However, the following is one of the several alternative procedures to arrive at one of them. Given that

$$\gamma = \frac{W}{V}, \quad \gamma_d = \frac{W_s}{V} \text{ and } m = \frac{W_w}{W_s}$$

$$\gamma = \frac{W_s + W_w}{V} = \frac{W_s + W_w}{\frac{W_s}{\gamma_d}} = \frac{\gamma_d(W_s + W_w)}{W_s}$$

Therefore by dividing by W_s throughout we get

$$\gamma = \gamma_d(1 + \frac{W_w}{W_s}) = \gamma = \gamma_d(1 + m)$$

since the ratio W_w/W_s is simply the expression for the moisture content, m

Likewise, the following additional expressions may be derived in a similar fashion by directly substituting $\gamma = W/V$; $\gamma_d = W_s/V$ in equation (6c) and $\rho = M/V$; $\rho_s = M_s/V$ in equation (6d)

$$W = W_s(1 + m) \text{ or } W_s = \frac{W}{1 + m} \quad (6f)$$

$$M = M_s(1 + m) \text{ or } M_s = \frac{M}{1 + m} \quad (6e)$$

Hint: From $\rho = M/V$ and $\rho_d = M_s/V \Rightarrow \rho/\rho_d = M/M_s$. Also; from equation (6d) above $\rho/\rho_d = 1 + m \Rightarrow M_s = M/(1 + m)$.

A similar procedure can be applied by substituting the quantities $\gamma = W/V$ and $\gamma_d = W_s/V$ in their respective equation above, to yield the relationship $W = W_s(1 + m)$

In our latter discussions, we will also be able to show that, for a partially saturation case

$$\gamma = \frac{(1 + m)G_s\gamma_w}{1 + e} = \frac{W}{V} \text{ or } \gamma = \frac{(G_s + eS_r)\gamma_w}{1 + e} = \frac{W}{V} \quad (6g)$$

$$\rho = \frac{(1 + m)G_s\rho_w}{1 + e} = \frac{M}{V} \text{ or } \rho = \frac{(G_s + eS_r)\rho_w}{1 + e} = \frac{M}{V} \quad (6h)$$

For cases of dry soils when the saturation is zero, the following corresponding equations will be deduced

$$\gamma_d = \frac{G_s\gamma_w}{1 + e} = \frac{W_s}{V} \text{ or;} \quad (6i)$$

$$\rho_d = \frac{G_s\rho_w}{1 + e} = \frac{M_s}{V} \quad (6j)$$

and for fully (complete) saturated cases;

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e} \text{ or } \gamma_{sat} = \frac{(1 + m_{sat}G_s)\gamma_w}{1 + m_{sat}G_s} \quad (6k)$$

$$\rho_{sat} = \frac{(G_s + e)\rho_w}{1 + e} \text{ or } \rho_{sat} = \frac{(1 + m_{sat}G_s)\rho_w}{1 + m_{sat}G_s} \quad (6l)$$

where S_r is the degree of saturation (the extent to which the voids are filled with water, instead of air), m_{sat} is the moisture content at complete saturation and ρ_w is the density of pure water = 1000 kg/m^3

However, it is very important to note here that the values of γ_d and ρ_d remain unchanged even under saturated conditions because the amount/physical presence of solid soil particles in the soil cannot be affected by the addition of water in it

(vii) The Specific Gravity, G_s

This is simply a **ratio of the unit weight of soil solids, γ_s to the unit weight of water γ_w** or a **ratio of the density of soil solids, ρ_s to the density of water ρ_w** i.e.

$$G_s = \frac{\gamma_s}{\gamma_w} \quad (7a)$$

$$G_s = \frac{\rho_s}{\rho_w} \quad (7b)$$

(viii) **PART A: SUMMARY**

So you will notice that with the exception of the specific gravity G_s , the moisture content m involves a **weight-to-weight** relationships whereas the rest of all other quantities, each involve a **weight-to-volume relationship**. Please try to observe the stated fact below:

- The bulk unit weight, (a **weight-to-volume** relationship);

$$\gamma = \frac{W}{V}$$

$$\gamma = \frac{W_s + W_w}{V_s + V_v}$$

- The dry unit weight (a **weight-to-volume** relationship);

$$\gamma_d = \frac{W_s}{V}$$

$$\gamma_d = \frac{W_s}{V_s + V_v}$$

- The saturated unit weight (a **weight-to-volume** relationship)

$$\gamma_{sat} = \frac{W_{sat}}{V}$$

$$\gamma_{sat} = \frac{W_s + W_w}{V_s + V_w}$$

- The moisture content, (a **weight-to-weight** relationship)

$$m = \frac{W_w}{W_s}$$

- The specific gravity. which is simply a ratio of the unit weight (or density) of the soil to the unit weight ((or density) of water **i.e.**

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{\rho_s}{\rho_w}$$

a ratio between two unit weights

PART B: VOLUME-TO-VOLUME RELATIONSHIPS

I. BASIC RELATIONSHIPS AND DEFINITIONS

Apart from the quantities discussed above, there are also basic ***volume-to-volume relationships*** in Soil Mechanics. These are the void ratio e , the porosity n , and the degree of saturation, S_r

(ix) The Void Ratio, e

The void ratio is defined as the ratio of the volume of voids (space occupied by *water and air* in the soil), V_v to the volume of solids (space occupied *solids* in the soil) V_s and is given by the expression

$$e = \frac{V_v}{V_s} \quad (9)$$

(x) The Porosity, n

The porosity is the ratio of the volume of voids V_v to the total volume V , of the soil and is given by the expression

$$n = \frac{V_v}{V} \quad (10a)$$

As in the previous case of the relationship between γ , γ_d and m , it can also be shown (by eliminating V , V_w and V_s) that:

$$e = \frac{n}{1 - n}; \quad 0 \leq n < 1 \quad (10b)$$

Or

$$n = \frac{e}{1 + e}; \quad 0 \leq n < 1 \quad (10c)$$

Note that equation (10c) above indicates that

- (a) when $n = 0.5$, $e = 1$ and using equation (9) $e = V_v/V_s$ we find that $V_s = V_v$. This suggests that ***in any given volume of a soil sample***, when the porosity is 50% ***the volume of solids is equal to the volume of voids, thereby making the void ratio equal to 1.***

Moreover, you will also notice that equations (10b) and (10c) above which define the relationship between the two quantities (i.e. porosity and void ratio) don't anymore depend on the volume of the soil. ***However, it is important again to note here that they (both) separately, each depend on the volume of the soil.***

- (b) Likewise equation (10b) suggests that the denominator $1 - n$ is zero when $n = 1$ and hence, the void ratio e becomes undefined at this value of n . This indicates therefore that, it is practically impossible to have such a value of e . because of the fact that $0 \leq n < 1$

In summary therefore, in any given volume of a soil, it is impossible to have a value of porosity equal to 1 or 100% because this would make it not the soil again but rather some sort of air/water or a mixture of both ***without any solid soil particles.*** This is outright ridiculous since it is practically impossible to have a soil containing water and/air only or their mixture, but without any solid (soil) particles. A value of $n = 1$ would therefore suggest that the porosity (amount of pore spaces in the soil) is 100% which is practically nonsense, with respect to any given volume of a soil!

- (c) With regard to the relationships involving the void ratio $e = V_v/V_s$; when the volume of solids $V_s = 1$ then $e = V_v$ meaning that ***when the volume of solids is unity (one) then the void ratio is equal to the volume of solids.*** Moreover, the ratio $n = e/(1 + e)$ is always a value greater than zero since $e > 0$. Now when $V_s = 1$ then $V_v = e$ and as

per the details in (iii) above, the total volume is then $V = V_s + V_v$ so that V is greater than 1 in this particular case i.e. $V = 1 + e > 1$

The above situation be related in the following mathematical way

if $V_s = 1$ when $V = 1 + e$;

then;

what is $V_s = ?$ when $V = 1$?

Cross multiplication resulting from the above query will give you the value of the volume of solids as

$$V_s = \frac{1 \times 1}{1 + e} \Rightarrow V_s = \frac{1}{1 + e}$$

Therefore, $V_s = e/(1 + e)$ when $V = 1$. This set of a relationship does not explain two different situations but rather, the relationship is a **two-in one relationship** explainer insisting that the scenario $V_s = 1$, $V_v = e$ and $V = 1 + e$ is equivalent to or same as the scenario , $V_s = 1/(1 + e) = 1 - n$, $V_v = e/(1 + e) = n$ and $V = 1$

Now, refer to information presented in the table above (a generalized and summarized three-phase model of wet soil fig.2) to be used together with an illustration table presented below in figure 3

From the above figure, it has already been shown that $W_w = mG_s\gamma_wV_s$ or $M_w = mG_s\rho_wV_s$ and $W_s = G_s\gamma_wV_s$ or $M_s = G_s\rho_wV_s$. Therefore for a certain volume V of soil the bulk density will be given by the expression

$$\rho = \frac{M}{V} = \frac{M_w + M_s}{V} = \frac{(1 + m)G_s\rho_wV_s}{V}$$

Now let us consider two cases basing on the recent information provided above:

Case 1: $V_s = 1$ when $V = 1 + e$, which indicates that

$$\rho = \frac{(1+m)G_s\rho_w V_s}{V} = \frac{(1+m)G_s\rho_w x(1)}{1+e}$$

$$\rho = \frac{(1+m)G_s\rho_w}{1+e}$$

Case 2: $V_s = 1/(1+e)$ when $V = 1$, therefore

$$\rho = \frac{(1+m)G_s\rho_w V_s}{V} = \frac{(1+m)G_s\rho_w \left(\frac{1}{1+e}\right)}{1} = \frac{(1+m)G_s\rho_w}{1+e}$$

which gives an expression exactly the same as in the first case

Now, combining the above relationships with the relationship $S_r e = mG_s$ we get

$$\rho = \frac{(G_s + S_r e)\rho_w}{1+e}$$

Finally the expressions for the bulk unit weight **for both of these cases** follow directly as:

$$\gamma = \frac{(1+m)G_s\gamma_w}{1+e}$$

$$\gamma = \frac{(G_s + S_r e)\gamma_w}{1+e}$$

and if expressed in terms of the quantities n and/ S_r , we find that

$$\gamma = \frac{\left(G_s + \frac{nS_r}{1-n}\right)\gamma_w}{1 + \frac{n}{1-n}} = \frac{(1-n)\left(G_s + \frac{nS_r}{1-n}\right)\gamma_w}{1-n+n} = (1-n)(G_s + nS_r)\gamma_w$$

so that

$$\gamma = (1-n)(G_s + nS_r)\gamma_w = (1-n)(1+m)G_s\gamma_w$$

Similarly;

$$\rho = (1-n)(G_s + nS_r)\rho_w = (1-n)(1+m)G_s\rho_w$$

	$V = 1$	$V = 1 + e$ $= \frac{1}{1 - n}$	Remarks
V_s	$\frac{1}{1 + e} = 1 - n$	1	Whether $V = 1$ or $V = 1 + e$; the values of the void ratio and the porosity remain unaffected, i.e., they are independent of the magnitude or the value of V . However, V_s and V_v take on different values in each of these cases i.e. both the quantities V_s and V_v are dependent on the magnitude or the value of V since $V_s + V_v = V$
V_v	$\frac{e}{1 + e} = n$	$e = \frac{n}{1 - n}$	
Void Ratio $= \frac{V_v}{V_s}$	$e = \frac{n}{1 - n}$	$e = \frac{n}{1 - n}$	
Porosity $= \frac{V_v}{V}$	$\frac{e}{1 + e} = n$	$\frac{e}{1 + e} = n$	
Density $\rho = \frac{M}{V}$	$= \frac{(1 + m)G_s \rho_w V_s}{V}$ $= \frac{(1 + m)G_s \rho_w}{1 + e} = (1 - n)(1 + m)G_s \rho_w$ $= \frac{(G_s + eS_r)\rho_w}{1 + e} = (1 - n)(G_s + nS_r)\rho_w$		In both cases, the expressions (and not the values) for the (bulk) density ρ and the (bulk) unit weight γ are independent of the magnitude or the value of V i.e. these expressions remain the same irrespective of the magnitude or the value of V
Unit weight $\gamma = \frac{W}{V}$	$= \frac{(1 + m)G_s \gamma_w V_s}{V}$ $= \frac{(1 + m)G_s \gamma_w}{1 + e} = (1 - n)(1 + m)G_s \gamma_w$ $= \frac{(G_s + eS_r)\gamma_w}{1 + e} = (1 - n)(G_s + nS_r)\gamma_w$		

Fig. 3: Tabular representation of fundamental quantities and their derived relationships

Therefore we observe three important things here that:

- (i) when $V_s = 1$, then $V = 1 + e$ and hence, $\rho = G_s \rho_w (1 + m)/(1 + e)$
- (ii) when $V = 1$, then $V_s = 1/(1 + e)$ and the expression for the bulk density $\rho = G_s \rho_w (1 + m)/(1 + e)$ as before
- (iii) from (i) and (ii) above, we note that whether $V_s = 1$ or $V_s = 1/(1 + e)$, **the expressions** for finding the values of the densities and hence, the values of the unit weights, remain identical in both cases, with respect to the quantity in consideration. This is because as previously seen, both of these cases just represent the same situation but with changing ratios within the entire volume of voids, (i.e. is a trade-off of air and water in the volume of voids) Note also with great care that here, it is not the values of the densities (or unit weights) that are identical, but rather, the expressions/ the relationships for finding such values of the stated quantities

The three points stated here are very helpful when calculating the component volumes V_s and V_v for any volume of the soil that is not unity (i.e. when the volume of the soil is different from 1 metric unit such that $V = V_s + V_v \neq 1$ so that either $V = V_s + V_v < 1$ or $V = V_s + V_v > 1$).

The most important thing that must be clearly understood well here is that ***for any volume of the soil that is not unity, V_s and V_v will take on values different from those taken when the volume of the soil is unity.***

Moreover, the expressions for finding the densities (or unit weights) are the same or identical irrespective of the case being considered, since their mathematical meaning is that the density (or unit weight) of soil is a certain given value of mass

in kg (or weight in kN), **in every one cubic meter**. So from the nature of their definitions, their expressions for finding these quantities will never get affected by the situation of having a volume of soil that is not unity. You will be able to clearly observe this when you get to the calculations sections ***(You must be able to observe this scenario in examples numbers 7, 8, 9, 10, 11 and 12 in the calculations section below)***

- (d) in the porosity relationship, $n = V_v/V$; when the total volume is unity i.e. $V = 1$ then the porosity is simply equal to the volume of the voids i.e. $n = V_v$ when $V = 1$. Now with $V = V_s + V_v = 1$ where $V_v = n$; then the volume of solids becomes $V_s = 1 - n$ in this latter case.

In summary therefore::

When $V_s = 1$, then $V = 1 + e = 1/(1 - n)$ which is the same as saying $V_s = 1/(1 + e) = 1 - n$ when $V = 1$; according to point (ii) above.

Mathematically, you can handle the above explained situation as follows:

if $V_s = 1$ when $V = 1 + e$;

then;

what is $V_s = ?$ when $V = 1$?

Cross multiplication will give you a value $V_s = 1/(1 + e) = 1 - n$ after which the expressions of both V and V_s in terms of n are obtained by replacing e by the expression $e = n/(1 - n)$

(xi) The degree of Saturation, S_r

This is the *ratio of the volume of water V_w to the volume of voids V_v* and is given by the expression

$$S_r = \frac{V_w}{V_v} \quad (11)$$

This ratio gives an indication of what fraction of the voids constitutes water and it should not be confused with the moisture content, which is ***the proportion of the amount/weight of water in a given mass of a soil.***

[Note here therefore that ***the moisture content is not*** the same as the amount of water or the weight of water in the soil].

In other words, the degree of saturation in any given volume of a soil, is a ***trade-off between the volume fraction of the voids*** that is filled with water and that which is filled with air. In fact, we are going to show that $A_v + S_r = 1$ where A_v is the air voids ratio i.e. the ratio of the volume of air to the volume of voids.

In a more precise way, the ratio S_r states also the fact that at full saturation (i.e. when all the voids are filled with water, $S_r = 1$ and hence $V_w = V_v$), indicating that ***the volume of water contained by the soil when at its full saturation point is exactly equal to the volume of voids***, since the volume of air becomes zero at that point

This fact will also be discussed in detail using a *two-phase state* (not three-phase state) soil model/ diagram. Therefore at ***full saturation***, only water and solid soil particles are available in the soil under consideration.

Note however that for a completely dry soil, $W_w = 0 \Rightarrow V_w = 0; m = 0$ and hence $S_r = 0$.

(xii) **The air voids ratio, A_v and the air content (percentage air voids) N_a and the relationship between A_v and S_r**

The ***air voids ratio*** is the ratio of the volume of air V_a to the total volume of voids, V_v i.e.

$$A_v = \frac{V_a}{V_v} \quad (12a)$$

whereas air **content** N_a is the ratio of the volume of air V_a to the total volume V of the soil i.e.

$$N_a = \frac{V_a}{V} \quad (12b)$$

Now, from equations (11) above we have $S_r = V_w/V_v$ and since $n = V_v/V$, which implies that $S_r = V_w/n$ when $V = 1$ then

$$\frac{V_w}{V} = nS_r \quad (12c)$$

Similarly; $V_a = V_v - V_w \Rightarrow n - nS_r = n(1 - S_r)$

$$\frac{V_a}{V} = n(1 - S_r) \quad (12d)$$

Finally, the air voids ratio will be given by the expression

$$A_v = \frac{V_a}{V_v} = \frac{n(1 - S_r)}{n}$$

$$A_v = 1 - S_r \text{ or } A_v + S_r = 1 \quad (12e)$$

Similarly, the percentage air voids (or air content) is given by the expression

$$N_a = \frac{V_a}{V} = \frac{V_a}{1}$$

$$N_a = \frac{n(1 - S_r)}{1}$$

Therefore

$$N_a = n(1 - S_r); 0 \leq S_r \leq 1 \quad (12f)$$

Example 1:

A water content (moisture content) test was made on a sample of silty clay. The weight of the wet soil plus container was 17.53 g, and the weight of the dry soil plus container was 14.84 g. Weight of the empty container was 7.84 g. Calculate the water content of the sample.

Solution:

MOISTURE CONTENT DETERMINATION EXPERIMENT	
Weight of container + wet soil (g)	17.53 (A)
Weight of container + dry soil (g)	14.84 (B)
Weight of empty container (g)	7.84 (C)
Weight of wet soil (g) = A - C	9.69 (D)
Weight of dry soil (g) = B - C	7.0 (E)
Weight of water (g) = D - E	2.69 (F)
Moisture content, $m = [(D - E)/(B - C)] \times 100\%$ $m = \left(\frac{E}{F}\right) \times 100$	38.43%

Example 2:

Differentiate between the following terms as applied to soil mechanics

- (i) Mass M and weight W**
- (ii) Moisture/ water content, m and degree of saturation, S_r**
- (iii) Bulk density, ρ , and bulk unit weight, γ**
- (iv) Dry density, ρ_d and dry unit weight, γ_d**
- (v) Volume of solids V_s and volume of voids V_v**
- (vi) Porosity, n and void ratio e**
- (vii) Percentage air voids N_a (air content) and the air voids ratio A_v**
- (viii) Partial saturation and complete saturation of soil**

Example 3:

Plot a grain size distribution curve and determine the soil texture for the following soil sieve analysis data.

Diameter (mm)	Percentage passing
32.00	100
16 .00	100
8.00	100
4.00	98
2.00	95
1.00	80
0.500	60
0.25	42
0.125	35
0.063	30

Example 4:

Plot a grain size distribution curve and determine the soil texture for the following soil sieve analysis data.

Diameter (mm)	Percentage passing
32.00	100
16 .00	100
8.00	100
4.00	95
2.00	92
1.00	80
0.500	70
0.25	65
0.125	40

0.063	20
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Example 5:

A soil sample has a porosity of 41%, moisture content of 15.2% and specific gravity of 2.65. Determine the degree of saturation, the void ratio, the percentage air voids and the air voids ratio

Given:

$$n = 41\%$$

$$m = 15.2\%$$

$$G_s = 2.65$$

$$S_r = ?$$

$$e = ?$$

$$N_a = ?$$

$$A_v = ?$$

- **The degree of saturation**

$$S_r = \frac{mG_s}{e}$$

Since we are given the porosity, then

$$S_r = \frac{mG_s(1 - n)}{n}$$

after we have replaced e with the expression that relates it to the porosity n

Therefore

$$S_r = \frac{0.152 \times 2.65 (1 - 0.41)}{0.41} = \frac{0.4028 (0.59)}{0.41} = 0.58$$

$$S_r = 58\%$$

- **The void ratio**

$$e = \frac{n}{1 - n}$$

$$e = \frac{0.41}{1 - 0.41} = \frac{0.41}{0.59} = 0.695$$

$$e = 0.695$$

- **The air void ratio, A_v**

To get the air voids ratio, we first need to find the volume of air V_a

Obvious this is a partial saturation case since some amount of air is still available in the soil. The volume of air is given by the expression

$$V_a = V_v - V_w = n - mG_s(1 - n)$$

where

$$V_v = n$$

and

$$V_w = mG_s(1 - n)$$

Therefore

$$V_a = n - mG_s(1 - n)$$

$$V_a = 0.41 - (0.152 \times 2.65) \times (1 - 0.41) = 0.41 - (0.4028 \times 0.59)$$

$$V_a = 0.410 - 0.238 = 0.172$$

$$V_a = 0.172 \text{ cubic units}$$

Since $V_a = 0.172 \text{ cubic units}$ and the total volume being not stated in the question, it should be taken as $V = 1 \text{ cubic unit}$, then the **air content (percentage air) N_a** is

$$N_a = \frac{V_a}{1} = V_a$$

$$N_a = 0.172$$

Again, since for partial saturation $V_v = n$, then the **air voids ratio** in this case is

$$A_v = \frac{V_a}{V_v} = \frac{0.172 \text{ cubic units}}{0.410 \text{ cubic units}}$$

$$A_v = 0.42$$

Alternatively:

$$V_a = V_v - V_w = \frac{e}{1+e} - \frac{mG_s}{1+e}$$

$$V_a = \frac{e - mG_s}{1+e}$$

where

$$V_v = \frac{e}{1+e}$$

and

$$V_w = \frac{mG_s}{1+e}$$

Therefore

$$V_a = \frac{e - mG_s}{1+e} = \frac{0.695 - (0.152 \times 2.65)}{1 + 0.695} = \frac{0.695 - (0.4028)}{1.695} = \frac{0.2922}{1.695}$$

$$V_a = 0.172$$

Therefore, again

$$N_a = \frac{0.172}{1} = 0.172$$

and similarly

$$A_v = \frac{V_a}{V_v} = \frac{0.172 \text{ cubic units}}{0.410 \text{ cubic units}} = 0.42$$

$$A_v = 0.42$$

as previously found

(xiii) The relationship existing among unit weights, void ratio e , moisture content m , degree of saturation S_r and specific gravity G_s

Recall from equation (7a) above that $G_s = \gamma_s/\gamma_w$ as well as from the relationships $V_w = W_w/\gamma_w$ and $S_r = V_w/V_v$. So that $V_w = W_w/\gamma_w = S_r V_v$. By using this relationship, we therefore deduce that

$$\frac{W_w}{\gamma_w} = S_r V_v \Rightarrow \frac{m G_s \gamma_w V_s}{\gamma_w} = S_r V_v \quad (13a)$$

Recall also that $m = W_w/W_s$. Now, with $W_s = V_s \gamma_s$ and $\gamma_s = G_s \gamma_w$ so that $W_s = V_s \gamma_s = G_s \gamma_w V_s$

$\Rightarrow W_w = m W_s = m G_s \gamma_w V_s$. With $V_s = 1$ and $V_v = e$; we get

$$\frac{m G_s \gamma_w}{\gamma_w} = S_r V_v \Rightarrow m G_s = S_r e$$

Therefore

$$m G_s = S_r e \quad (13b)$$

A similar expression $m G_s = S_r e$ is obtained by substituting $V_s = 1/(1 + e)$ and $V_v = e/(1 + e)$ in equation (13a) above and in that case, the same equation reduces to:

$$\frac{m G_s}{1 + e} = \frac{S_r e}{1 + e} \Rightarrow m G_s = S_r e \quad (13b)$$

This equation indicates also that, at full saturation when $S_r = 1$

$$m G_s = e \quad (13c)$$

This equation reveals the fact that when ***the soil is at its full saturation point***, the void ratio can be obtained by simply taking the product of the moisture content m and the specific gravity; ***irrespective of the value of V or of V_s***

II. PART B: SUMMARY

All of the three quantities (void ratio, porosity and degree of saturation) discussed under this section/ part obey a *volume-to-volume* rule relationship. You may wish to revise them here below:

- **Void Ratio** (a *volume-to-volume* relationship)

$$e = \frac{V_v}{V_s}$$

- **Porosity** (a *volume-to-volume* relationship);

$$n = \frac{V_v}{V}$$

- **Degree of Saturation** (a *volume-to-volume* relationship);

$$S_r = \frac{V_w}{V_v}$$

- **Air Voids ratio** (a *volume-to-volume* relationship);

$$A_v = \frac{V_a}{V_v}$$

- **Air Content (Percentage Air Content)** (a *volume-to-volume* relationship);

$$N_a = \frac{V_a}{V}$$

$$N_a = V_a \text{ when } V = 1$$

A GENERALISED AND SUMMARISED THREE-PHASE MODEL OF SOIL		
$W = W_w + W_s$	THREE-PHASE SOIL	$V = V_v + V_s$
W_w	<u>AIR</u> $W_a \approx 0$ $V_a = V_v - V_w$	$V_v = V_w + V_a$
	<u>WATER</u> $W_w = mG_s\gamma_w V_s$ $V_w = mG_s V_s$	
W_s	<u>SOLID</u> $W_s = G_s\gamma_w V_s$ $V_s = \frac{W_s}{G_s\gamma_w}$	V_s

Figure 4: Summarized phase state model diagram

III GENERAL SUMMARY TABLE: WEIGHT-TO-VOLUME/WEIGHT RELATIONSHIPS

WEIGHT-TO-VOLUME/WEIGHT RELATIONSHIPS				OTHER RATIO(S)
Bulk unit weight	Dry unit weight	Saturated unit weight	Moisture content	Specific Gravity
$\gamma = \frac{W}{V}$	$\gamma_d = \frac{W_s}{V}$	$\gamma_{sat} = \frac{W_{sat}}{V}$	$m = \frac{W_w}{W_s}$	$G_s = \frac{\gamma_s}{\gamma_w}$
VOLUME-TO-VOLUME RELATIONSHIPS				OTHER RATIO(S)
Void ratio	Porosity	Degree of Saturation	Air content/ air voids ratio	
$e = \frac{V_v}{V_s}$	$n = \frac{V_v}{V}$	$S_r = \frac{V_w}{V_v}$	$N_a = \frac{V_a}{V}$ $A_v = \frac{V_a}{V_v}$	
FUNDAMENTAL RELATIONSHIPS				
Total weight	Total volume			
$W = W_s + W_w$	$V = V_s + V_v$ $= V_s + V_w + V_a$			

IV MAIN SUMMARY OF THE RELATIONSHIPS IN THE TOPIC:

Total volume of the soil of the soil is

$$V = V_s + V_w + V_a$$

$$V = V_s + V_v$$

$$V_v = V_w + V_a$$

Total weight W of the soil is

$$W = W_s + W_w; W_a \approx 0$$

$$W = Mg$$

$$W_s = G_s \gamma_w V_s$$

$$W_w = m G_s \gamma_w V_s, \text{ for a PARTIAL SATURATION case}$$

For a COMPLPLETE SATURATION case the weight of water W_w is given by

$$W_w = n \gamma_w = \left(\frac{e}{1 + e} \right) \gamma_w$$

The total mass M is given by the expression:

$$M = M_s + M_w; M_a \approx 0$$

$$M_s = \frac{G_s \gamma_w V_s}{g}$$

$$M_w = \frac{m G_s \gamma_w V_s}{g}$$

For a COMPLPLETE SATURATION case, the mass of water M_w is given by the expression

$$M_w = \frac{n \gamma_w}{g} = \left(\frac{e}{1 + e} \right) \frac{\gamma_w}{g}$$

The moisture content m is given by the expression:

$$m = \frac{W_w}{W_s} \times 100\% \text{ or } m = \frac{M_w}{M_s} \times 100\%$$

The voids ratio e is given by the expression:

$$e = \frac{V_v}{V_s}$$

The porosity n is given by the expression:

$$n = \frac{V_v}{V}$$

$$e = \frac{n}{1 - n} > 0$$

$$n = \frac{e}{1 + e} ; 0 \leq n < 1$$

Now since $V = V_s + V_v$; suppose the value of $V = 1$, then;

$V_s = 1 - V_v$ or $V_v = 1 - V_s$ and therefore from the definitions of e and n above.

$V_v = n$ and $V_s = 1 - n$ or;

$$V_v = \frac{e}{1 + e} \text{ and } V_s = \frac{1}{1 + e}$$

$$S_r = \frac{V_w}{V_v}$$

$$N_a = \frac{V_a}{V}$$

$$V_a = n(1 - S_r)$$

$$N_a = n(1 - S_r)$$

$$A_v = \frac{V_a}{V_v}$$

$$A_v = 1 - S_r$$

$$V = \frac{M}{\rho}$$

Also;

$$V = \frac{W}{\gamma} = \frac{Mg}{\gamma} = \frac{M}{\rho}$$

Therefore

$$\gamma = \rho g$$

$$\gamma = \frac{W}{V} = (1+m)(1-n)G_s\gamma_w = \frac{(1+m)G_s\gamma_w}{1+e} = \frac{(G_s + eS_r)\gamma_w}{1+e}; 0 < S_r < 1$$

$$\rho = \frac{M}{V} = (1+m)(1-n)G_s\rho_w = \frac{(1+m)G_s\rho_w}{1+e} = \frac{(G_s + eS_r)\rho_w}{1+e}; 0 < S_r < 1$$

$$\gamma_{sat} = \frac{W_{sat}}{V} = \frac{W_s + W_w}{V_s + V_w} = \frac{[(n + (1-n)G_s)\gamma_w]}{1+e} = \frac{(G_s + e)\rho_w}{1+e}; S_r = 1$$

$$\rho_{sat} = \frac{M_{sat}}{V} = \frac{M_s + M_w}{V_s + V_w} = \frac{[(n + (1-n)G_s)\rho_w]}{1+e} = \frac{(G_s + e)\rho_w}{1+e}; S_r = 1$$

$$\gamma_d = \frac{W_s}{V} = G_s\gamma_w(1-n) = \frac{(G_s + eS_r)\gamma_w}{1+e} = \frac{G_s\gamma_w}{1+e}; S_r = 0$$

$$\rho_d = \frac{M_s}{V} = G_s\rho_w(1-n) = \frac{(G_s + eS_r)\rho_w}{1+e} = \frac{G_s\rho_w}{1+e}; S_r = 0$$

$$\gamma = \gamma_d(1+m) \quad \text{or} \quad \gamma_d = \frac{\gamma}{1+m}$$

$$W = W_s(1+m) \quad \text{or} \quad W_s = \frac{W}{1+m}$$

$$\rho = \rho_d(1+m) \quad \text{or} \quad \rho_d = \frac{\rho}{1+m}$$

$$M = M_s(1+m) \quad \text{or} \quad M_s = \frac{M}{1+m}$$

WORKED EXAMPLES/ EXAMPLE CALCULATIONS

[Note: the value of the acceleration due to gravity used in almost all the calculations is $g = 10\text{m/s}^2$. However, do not get confused wherever you may find the used value as being $g = 9.81\text{m/s}^2$]

Example 6:

1. Differentiate between the following terms as applied to soil mechanics

- (i) Mass M and weight W
- (ii) Moisture/ water content, m and degree of saturation, S_r
- (iii) Bulk density, ρ , and bulk unit weight, γ
- (iv) Dry density, ρ_d and dry unit weight, γ_d
- (v) Volume of solids V_s and volume of voids V_v
- (vi) Porosity, n and void ratio e
- (vii) Air content N_a , and the air voids ratio A_v
- (viii) Partial saturation and complete saturation of soil

Example 7:

A soil sample has a weight of 0.70 kg and its volume was found to be $3.5 \times 10^{-4} \text{ m}^3$. After oven drying, the weight was reduced to 0.60 kg. If its particle specific gravity was found to be 2.60; determine the following for this particular sample of the soil:

- (i) Moisture content, m
- (ii) Bulk density, ρ and hence the bulk unit weight, γ
- (iii) Dry density, ρ_d and hence the dry unit weight, γ_d
- (iv) Volume of solids V_s , volume of voids V_v the void ratio e and hence the porosity, n

- (v) The volume of water, V_w and hence the degree of saturation, S_r
- (vi) The volume of air V_a , the air content N_a and hence the air voids ratio A_v

SOLUTION:

Given:

$$V = 3.5 \times 10^{-4} m^3$$

$$M = 0.70 \text{ kg}$$

$$M_s = 0.60 \text{ kg}$$

$$G_s = 2.60$$

[**Note** that this is a problem presented where $V = 3.5 \times 10^{-4} m^3 \neq 1 m^3$, that is $V = V_s + V_v \neq 1$ and this situation therefore, gives us an opportunity to verify the explanations presented in the previously sections, when trying to find the value of V_s]

(i) The moisture content, m

$$m = \left(\frac{W_w}{W_s} \right) \times 100\%$$

$$m = \left(\frac{Mg - M_s g}{M_s g} \right) \times 100\%$$

After cancelling out the gravitational acceleration g , we arrive at the expression

$$m = \left(\frac{M - M_s}{M_s} \right) \times 100$$

And on substituting in the respective values, we get

$$m = \left(\frac{0.70 - 0.60}{0.60} \right) \times 100$$

$$m = 0.167 \times 100\% = 16.7\%$$

Therefore

$$m = 16.7\%$$

Note that in other words, m can be expressed as

$$m = \frac{\text{Weight of water}}{\text{Total **dry** weight (weight of solids)}} = \frac{W_w}{W_s}$$

$$m = \frac{\text{Mass of water}}{\text{Total **dry** mass (mass of solids)}} = \frac{M_w}{M_s}$$

(ii) The bulk density, ρ and bulk unit weight, γ

The bulk density, ρ

$$\rho = \frac{M}{V}$$

$$\rho = \frac{0.70kg}{3.5 \times 10^{-4} m^3} = 2000 kgm^{-3}$$

$$\rho = 2000 kgm^{-3}$$

The bulk unit weight, γ

$$\gamma = \frac{W}{V} = \frac{Mg}{V}$$

By taking $g = 10ms^{-2}$

$$\gamma = \frac{0.70kg \times 10kgms^{-2}}{3.5 \times 10^{-4} m^3} = 20000 Nm^{-3}$$

$$\gamma = 20.0 kNm^{-3}$$

Alternatively;

$$\gamma = \rho g$$

$$\rho = 2000 kgm^{-3}$$

and

$$g = 10ms^{-2}$$

$$\text{Therefore } \gamma = 2000 \text{kgm}^{-3} \times 9.81 \text{ms}^{-2} = 20000 \text{Nm}^{-3}$$

Hence

$$\gamma = 20.0 \text{kNm}^{-3}$$

as found in our recent previous calculations. The *SI units* of unit weight (the weight/force acting per unit volume of a soil) is *newtons or (kilo-newtons) per cubic meter*, and hence the origin of its name **unit weight**

(iii) The dry density, ρ_d and the dry unit weight, γ_d

The dry density, ρ_d

$$\rho_d = \frac{M_s}{V}$$

$$\rho_d = \frac{0.60 \text{kg}}{3.5 \times 10^{-4} \text{m}^3} = 1714.0 \text{kgm}^{-3}$$

$$\rho_d = 1714.3 \text{kgm}^{-3}$$

Alternatively, we can calculate the value of ρ_d by using the relationship

$$\rho = \rho_d(1 + m)$$

since at this point both m and ρ are known

Therefore

$$\rho_d = \frac{\rho}{1 + m}$$

$$\rho_d = \frac{2000}{1 + 0.167} = 1714.0 \text{kgm}^{-3}$$

$$\rho_d = 1714.0 \text{kgm}^{-3}$$

The dry unit weight, γ_d

$$\gamma_d = \rho_d g$$

$$g = 10 \text{ms}^{-2}$$

Therefore

$$\gamma_d = 1714.3 \text{kgm}^{-3} \times 10 \text{ms}^{-2}$$

Note that in other words

$$\gamma_d = \frac{\text{mass} \times \text{acceleration}}{\text{volume}} = \frac{\text{Force in newtons}}{\text{volume in cubic meter}}$$

$$\gamma_d = 1714.3 \text{ kgm}^{-3} \times 9.81 \text{ ms}^{-2}$$

$$\gamma_d = 17143 \text{ kgm}^{-2} \text{ s}^{-2}$$

or simply (with regard to the note above)

$$\gamma_d = 17143 \text{ Nm}^{-3}$$

Therefore

$$\gamma_d = \mathbf{17.14 \text{ kNm}^{-3}}$$

Alternatively the value of γ_d could be calculated directly from the relationship

$$\gamma_d = \frac{W_s}{V}$$

where $W_s = M_s g$ which also gives the same result for $\gamma_d = \mathbf{17.14 \text{ kNm}^{-3}}$ with an answer being consistent with the previous above cases

Again, γ_d can be calculated from the relationship

$$\gamma_d = \frac{\gamma}{1 + m} = \frac{20000 \text{ Nm}^{-3}}{1 + 0.167} = 17.14 \text{ kNm}^{-3}$$
$$\gamma_d = \mathbf{17.14 \text{ kNm}^{-3}}$$

an answer which is again, consistency with the above two calculation procedures!

- (iv) **The volume of solids V_s , the volume of voids V_v , the void ratio e and the porosity n**

The volume of solids V_s

We need to find first the value of the void ratio e from the relationship

$$\gamma_d = \frac{G_s \gamma_w}{1 + e} \Rightarrow e = \frac{G_s \gamma_w}{\gamma_d} - 1$$

or from

$$e = \frac{G_s \gamma_w}{\rho_d} - 1$$

whichever relationship is easier for you.

$$e = \frac{G_s \gamma_w}{\rho_d} - 1 = \frac{2.60 \times 1000 \text{ kg m}^{-3}}{1714.3 \text{ kg m}^{-3}} - 1 = \frac{2600.0}{1714.3} - 1 = 0.517$$

$$e = 0.517$$

Then

$$\begin{aligned} e &= \frac{V_v}{V_s} = \frac{V - V_s}{V_s} \\ &= \frac{3.5 \times 10^{-4} \text{ m}^3 - V_s}{V_s} = 0.517 \Rightarrow 3.5 \times 10^{-4} \text{ m}^3 = 1.517 V_s \\ &\Rightarrow V_s = \frac{3.5 \times 10^{-4} \text{ m}^3}{1.517} = 2.31 \times 10^{-4} \text{ m}^3 \end{aligned}$$

Therefore

$$V_s = 2.31 \times 10^{-4} \text{ m}^3$$

Alternative way 1:

By definition;

$$e = \frac{V_v}{V_s}$$

Since (as per the information given from the question itself) both V_s and V_v are unknown, we need to calculate first V_s after which we shall then be able to get the value its counterpart V_v ; since the total volume is known. So we have

$$V_s = \frac{W_s}{\gamma_s}$$

Also,

$$\gamma_s = G_s \gamma_w$$

So that

$$V_s = \frac{W_s}{G_s \gamma_w}$$

Therefore

$$V_s = \frac{W_s}{G_s \gamma_w} = \frac{0.60}{2.60 \times 1000}$$

$$V_s = 2.31 \times 10^{-4} m^3$$

Alternative way 2:

$$\text{if } V_s = 1 \text{ when } V = 1 + e$$

Then;

$$\text{what is } V_s = ? \text{ when } V = 3.5 \times 10^{-4} m^3$$

By cross multiplying, we get

$$V_s = \frac{3.5 \times 10^{-4} \times 1}{1 + e}$$

From the above previous section, $e = 0.517$. Therefore

$$V_s = \frac{3.5 \times 10^{-4} \times 1}{1.517} = 2.31 \times 10^{-4} m^3$$

$$V_s = 2.31 \times 10^{-4} m^3$$

The volume of voids V_v

Now, since the total volume $V = 3.50 \times 10^{-4} m^3$ then the volume of voids V_v is simply the difference $V - V_s = V_v = 1.19 \times 10^{-4} m^3$

Note that you can also find the **volume of voids** using same, similar approach; as that which was used in finding V_s , that is;

$$\text{if } V_v = n = \frac{e}{1 + e} \text{ when } V = 1$$

then

$$\text{what is } V_v = ? \text{ when } V = 3.5 \times 10^{-4} m^3$$

$$V_v = \frac{3.5 \times 10^{-4} \times \frac{e}{1+e}}{1} = \frac{3.5 \times 10^{-4} \times e}{1+e} = \frac{3.5 \times 10^{-4} \times 0.517}{1.517} = \frac{1.8095 \times 10^{-4}}{1.517}$$

$$= 1.19 \times 10^{-4} m^3$$

$$V_v = 1.19 \times 10^{-4} m^3$$

The void ratio e

$$e = \frac{V_v}{V_s}$$

Therefore

$$e = \frac{V_v}{V_s} = \frac{1.19 \times 10^{-4} m^3}{2.31 \times 10^{-4} m^3}$$

$$e = 0.515$$

Or

$$e = \frac{G_s \gamma_w}{\rho_d} - 1 = \frac{2.60 \times 1000 kg m^{-3}}{1714.3 kg m^{-3}} - 1 = \frac{2600.0}{1714.3} - 1 = 0.517$$

$$e = 0.517$$

The porosity n

$$n = \frac{V_v}{V}$$

$$n = \frac{1.19 \times 10^{-4} m^3}{3.50 \times 10^{-4} m^3} = 0.34$$

$$n = 0.34$$

Alternatively:

$$n = \frac{e}{1+e} = \frac{0.515}{1+0.515} = \frac{0.515}{1.515} = 0.34$$

Therefore the porosity is **$n = 0.34$**

Hence from the calculations above, we find that $V_s = 2.31 \times 10^{-4} m^3$,
 $V_v = 1.19 \times 10^{-4} m^3$, $e = 0.515$ and $n = 0.34$

The volume of water, V_w and degree of saturation S_r

The volume of water, V_w

$$V_w = \frac{W_w}{\gamma_w}$$

$$V_w = \frac{(0.70 - 0.60) kg}{1000 kg m^{-3}} = 1.0 \times 10^{-4} m^3$$

$$V_w = 1.0 \times 10^{-4} m^3$$

The degree of saturation S_r

$$S_r = \frac{V_w}{V_v}$$

$$S_r = \frac{1.0 \times 10^{-4} m^3}{1.19 \times 10^{-4} m^3} = 0.84 \times 100\%$$

$$S_r = 84\%$$

Alternatively:

$$S_r = \frac{mG_s}{e} = \frac{0.167 \times 2.60}{0.515} = \frac{0.167 \times 2.60}{0.515} = 0.84$$

$$S_r = 84\%$$

(v) **The volume of air V_a the air content N_a , and hence the air voids ratio A_v**

Since $V_v = V_w + V_a$ then $V_a = V_v - V_w$

Now, with $V_v = 1.19 \times 10^{-4} m^3$; the volume of air is therefore

$V_a = 1.19 \times 10^{-4} m^3 - 1.0 \times 10^{-4} m^3 = 0.19 \times 10^{-4} m^3$ so that

$$V_a = 0.19 \times 10^{-4} m^3$$

Hence the percentage air voids is,

$$N_a = \frac{V_a}{V} = \frac{0.19 \times 10^{-4} m^3}{3.50 \times 10^{-4} m^3} \times 100\% = 0.054 \times 100\%$$

$$N_a = \mathbf{0.054 \text{ or } 5.40\%}$$

Finally, the air voids ratio A_v is simply the ratio of the **volume of air** V_a to the **total volume of voids**, V_v so that

$$A_v = \frac{V_a}{V_v} = \frac{0.19 \times 10^{-4} m^3}{1.19 \times 10^{-4} m^3} = 0.16$$

Hence, the air voids ratio is

$$A_v = \mathbf{0.16 \text{ or } 16\%}$$

Alternatively:

$$A_v = 1 - S_r$$

$$A_v = 1.00 - 0.84$$

$$A_v = \mathbf{0.16}$$

Example 8:

The previous problem in (2) above reveals a typical case of a **partially saturated/unsaturated soil**, whose moisture content is 16.7% and a degree of saturation of 84%.

How would the values of ALL the quantities calculated in (2) be if some **amount of water were added to the soil** so as to make it COMPLETELY SATURATED?

SOLUTIONS

(i) ***Mass of water M''_w to be added for complete saturation***

As previously seen, the necessary condition for complete saturation is that the volume of air must be zero (i.e. $V_a = 0$) implying that the volume of water alone should equal the volume of voids (i.e. $V_w = V_v$)

Now, from problem (1) above, the volume of voids was found to be

$$V_v = 1.19 \times 10^{-4} m^3$$

However, ***at complete saturation, this becomes the volume of water (since the volume of air is zero)*** so that

$$V_w = 1.19 \times 10^{-4} m^3$$

Therefore the new value of mass of water M'_w in the soil becomes

$$M'_w = V_w \rho_w$$

$$M'_w = 1.19 \times 10^{-4} \times 1000$$

$$M'_w = 0.119 kg$$

Originally (at an unsaturated condition), the mass of water was

$$M_w = 0.70 - 0.60 = 0.100 kg$$

$$M_w = 0.100 kg$$

The mass of water to be added in order to make the soil become completely saturated is

$$M''_w = M'_w - M_w$$

$$M''_w = 0.119 - 0.100$$

$$M''_w = 0.019 kg$$

Therefore the mass of water required to be added for complete saturation of the soil is

$$M''_w = 0.019 kg$$

The new value of moisture content m' at complete saturation

$$m_{sat} = \frac{\text{Total weight of water}}{\text{Weight of solids}} = \frac{M''_w + M_w}{M_s}$$

$$m_{sat} = \frac{M'_w}{M_s} = \frac{0.119}{0.60} = 0.198$$

Therefore moisture content at complete saturation is

$$m_{sat} = \mathbf{19.81\%}$$

Alternatively, at complete saturation

$$e = m_{sat}' G_s$$

So we could find m' using the relationship

$$m_{sat} = \frac{e}{G_s}$$

since $S_r = 1$

$$m_{sat} = \left(\frac{0.515}{2.60} \right) \times 100\% = \left(\frac{0.515}{2.60} \right) \times 100\% = 19.81\%$$

Therefore, again

$$m_{sat} = \mathbf{19.81\%}$$

as previously found

Moreover, **a pure Mathematician (not an Engineer or a Geologist)** could have approached this problem in the following purely mathematical way, by just looking at the relationships between **the degrees of saturation S_r 's and moisture contents m 's** and still, arrive at the same solution in consistency with the above ones, as follows:

$$\begin{array}{ll} m = 16.7\% & S_r = 84\% \\ m_{sat} = ? & S_r = 100\% \end{array}$$

Meaning that

$$\begin{array}{ll} \text{if } m = 16.7\% & \text{when } S_r = 84\% \\ \text{then, what is } m_{sat}; & \text{when } S_r = 100\%? \end{array}$$

By cross multiplication, the unknown m is simply

$$m_{sat} = \frac{16.7 \times 100}{84} = 19.88\%$$

$$\mathbf{m_{sat} = 19.88\%}$$

The minor difference between this solution and the former ones being attributed for by rounding-off mathematical procedures!

At complete saturation however, $V_a = 0$ since all the voids are filled with water

The following expressions for each of the stated quantity below, were deduced for $V = 1$

The SATURATED(bulk) unit weight, γ_{sat}

$$\gamma_{sat} = \frac{W_s + W_w}{V}$$

Where

$$W_s = (1 - n)G_s\gamma_w = (1 - n)G_s\rho_w g;$$

$$W_w = n\gamma_w = n\rho_w g;$$

But you must note that $W_s = (1 - n)G_s\gamma_w$ when $V = 1$ but in our current situation, $V = 3.50 \times 10^{-4} m^3$ and not 1. Therefore in this case

$$W_s = 3.50 \times 10^{-4} (1 - n)G_s\gamma_w$$

Similarly,

$$W_w = 3.50 \times 10^{-4} (n\gamma_w)$$

and as usual

$$\mathbf{V = 3.50 \times 10^{-4} m^3 \text{ with } V_a = 0}$$

Therefore

$$\gamma_{sat} = \frac{G_s\gamma_w (1 - n) + n\gamma_w}{V}$$

$$\gamma_{sat} = \frac{[G_s(1 - n) + n]\gamma_w}{V}$$

$$\gamma_{sat} = \frac{[G_s(1 - n) + n]\rho_w g}{V}$$

$$\gamma_{sat} = \frac{[(2.60(1 - n) + n)x1000kg/m^3x10m/s^2]x3.50x10^{-4}m^3}{3.50x10^{-4}m^3}$$

Since this expression for γ_{sat} is a ratio, it finally simplifies to

$\gamma_{sat} = [2.60(1 - n) + n]x1000kg/m^3x10m/s^2$; an expression similar to the one for which $V = 1$

Note that in all cases where the total volume V becomes involved in the calculations, its value will always cancel out **but only in those cases where the expression involved in a relationship is a ratio, as in the above case of γ_{sat}**

At complete saturation however,

$$V_v = V_w = 1.19x10^{-4}m^3$$

so that

$$n = \frac{V_w}{V} = \frac{1.19x10^{-4}m^3}{3.50x10^{-4}m^3}$$

$$n = 0.34$$

Therefore

$$\begin{aligned}\gamma_{sat} &= [2.60(1 - 0.34) + 0.34]x1000kg/m^3x(10m/s^2) \\ &= [2.60(0.66) + 0.34]x1000kg/m^3x(10m/s^2) \\ \gamma_{sat} &= [1.716 + 0.34]x1000kg/m^3x(10m/s^2) \\ &= 1.750x1000kg/m^3x(10m/s^2) = 17500N/m^3 \\ \gamma_{sat} &= \mathbf{17.50kN/m^3}\end{aligned}$$

Now **the SATURATED bulk density, ρ_{sat}** could easily be calculated using the relationship $\gamma_{sat} = \rho_{sat}g$ so that

$$\rho_{sat} = \frac{17500N/m^3}{10m/s^2} = 1750kg/m^3$$

$$\rho_{sat} = 1750 \text{ kg/m}^3$$

The dry unit weight, γ_d and the dry density ρ_d

$$\gamma_d = \frac{W_s}{V}$$

$$\gamma_d = \frac{3.50 \times 10^{-4} (1 - n) G_s \gamma_w}{3.50 \times 10^{-4}}$$

$$\gamma_d = (1 - n) G_s \gamma_w$$

$$\gamma_d = (1 - n) G_s \rho_w g$$

$$\gamma_d = (1 - 0.34) \times 2.60 \times 1000 \times 10 \text{ kg/m}^3 = (0.66) \times 2.60 \times 1000 \times 10 \text{ kg/m}^3$$

$$\gamma_d = (0.66) \times 2.60 \times 1000 \times 10 \text{ kg/m}^3 = 17160 \text{ N/m}^3$$

$$\gamma_d = 17.16 \text{ kN/m}^3$$

and therefore it follows that

$$\rho_d = 1716 \text{ kg/m}^3$$

Volume of solids V_s , volume of voids V_v the void ratio e and hence the porosity, n

Until now, we have deduced under this SATURATED case that $V_v = V_w = 1.19 \times 10^{-4} \text{ m}^3$ so that V_a is supposed to be zero

Moreover it has been found that the ratio (the porosity)

$$n = \frac{V_v}{V} = 0.34$$

So we can now find the volume of solids (and hence the new void ratio), as follows

$$V_s = V - V_v$$

$$V_s = (3.50 - 1.19) \times 10^{-4} \text{ m}^3 = 2.31 \times 10^{-4} \text{ m}^3$$

$$V_s = 2.31 \times 10^{-4} \text{ m}^3$$

Note that V_s still maintains its former value of $V_s = 2.31 \times 10^{-4} m^3$ and hence its volume is completely unaffected under complete saturation situations and so is the void ratio e . In fact, the total value of the volume of voids V_v is the one which also remains unaffected. What actually happens within the entire volume of the soil, is only a trade-off of values between the volumes of water and that of air, in such a way that the volume lost by air is wholly gained by that of water and hence the volume of voids always remains constant, with the volume of solids being left completely unaffected

Therefore

$$V_v = V_w = 1.19 \times 10^{-4} m^3$$

$$V_s = 2.31 \times 10^{-4} m^3$$

$$n = 0.34$$

$$e = 0.515$$

Finally, the air voids ratio $A_v = 0$ since $S_r = 1$ or $V_a = 0$ so that the relationship

$$A_v = 1 - S_r = 1 - 1 = 0$$

Example 9:

Determine the moisture content, void ratio, porosity, degree of saturation, dry unit weight, γ_d and the dry density, ρ_d , bulk unit weight, γ and the bulk density, ρ of a soil sample that has a total mass of 1013g and 904g as its dry weight; having a volume of $585 cm^3$. Assume the specific gravity of the soil is 2.65.

SOLUTION:

Given information:

Total weight of soil sample, $W = 1013.0g$

Total volume of soil sample, $V = 585.0 \text{ cm}^3$

Specific Gravity, $G_s = 2.65$

Dry weight of soil, $W_s = 904.0\text{g}$

REQUIRED:

- Moisture content, m
- Void ratio, e
- Porosity, n
- Degree of saturation, S_r
- The dry unit weight, γ_d and the dry density, ρ_d
- The bulk unit weight, γ and the dry density, ρ

[**Note** Here again is a typical case $V = 585.0 \text{ cm}^3 \neq 1\text{m}^3$, so that is $V = V_s + V_v \neq 1$]

The moisture content, m

$$\begin{aligned} m &= \left(\frac{W - W_s}{W_s} \right) \times 100\% = \left(\frac{1013.0 - 904.0}{904.0} \right) \times 100\% = \left(\frac{109.0}{904.0} \right) \times 100\% = \\ &= 0.121 \times 100\% \\ \mathbf{m} &= \mathbf{12.1\%} \end{aligned}$$

Void ratio, e

Looking at the information which is provided in the question, we can easily find the value of the void ratio by using the two relationships involving the dry density, that is

$$\frac{M_s}{V} = \rho_d \text{ and } \rho_d = \frac{G_s \rho_w}{1 + e}$$

$$\begin{aligned}\frac{M_s}{V} &= \frac{G_s \rho_w}{1 + e} \Rightarrow e = \frac{G_s \rho_w}{\rho_d} - 1 = \frac{G_s \rho_w V}{M_s} - 1 \Rightarrow e \\ &= \frac{2.65 \times 1 \text{ g/cm}^3 \times 585.0 \text{ cm}^3}{904.0 \text{ g}} - 1 = \frac{1550.25}{904.0} - 1 = 1.1748 - 1 \\ &= 0.715\end{aligned}$$

$$e = 0.715$$

Alternatively:

As usual

$$e = \frac{V_v}{V_s}$$

Under this case, we do not know the value of $V_v = V_w + V_a$ and so we shall need to find these values by using the following procedures

Since the weight of water W_w is known, we can get the volume of water by applying the relationship

$$\begin{aligned}V_w &= \frac{M_w}{\rho_w} \\ V_w &= 109 \text{ g} / 1 \text{ g cm}^{-3}\end{aligned}$$

Therefore

$$V_w = 109 \text{ cm}^3$$

We need also to find the volume of solids, V_s

$$V_s = \frac{W_s}{G_s \gamma_w}$$

so that

$$V_s = \frac{904.0}{2.65 \times 1.0}$$

Note that since the units here are in a cgs system likewise γ_w is newtons per cubic centimeter i.e. **1 newton per cubic centimeter**

$$V_s = \frac{904.0}{2.65}$$

$$V_s = 341.13 \text{ cm}^3$$

Alternatively:

$$\text{if } V_s = 1 \text{ when } V = 1 + e$$

then;

$$\text{what is } V_s = ? \text{ when } V = 585.0 \text{ cm}^3$$

By cross multiplying, we get

$$V_s = \frac{585.0 \times 1}{1 + e}$$

From the above previous section, $e = 0.517$. Therefore

$$V_s = \frac{585.0 \times 1}{1.715} = 341.13 \text{ cm}^3$$

$$V_s = 341.13 \text{ cm}^3$$

You can also find first the volume of voids using a similar approach

$$\text{if } V_v = n = \frac{e}{1 + e} \text{ when } V = 1$$

then

$$\text{what is } V_v = ? \text{ when } V = 585.0 \text{ cm}^3$$

$$V_v = \frac{585.0 \times \frac{e}{1 + e}}{1} = \frac{585.0 \times e}{1 + e} = \frac{585.0 \times 0.715}{1.715} = \frac{418.275}{1.715} = 243.89 \text{ cm}^3$$

$$V_v = 243.89 \text{ cm}^3$$

Then

$$V_s = V - V_v = 585.0 \text{ cm}^3 - 243.89 \text{ cm}^3 = 341.10 \text{ cm}^3$$

Now with V_s being known

$$V_a = V - V_s - V_w$$

$$V_a = 585.0 - 341.1 - 109.0$$

$$V_a = 134.9 \text{ cm}^3$$

Therefore

$$V_v = V_w + V_a = 109.0 + 134.9$$

$$V_v = 243.9 \text{ cm}^3$$

and hence

$$e = \frac{V_v}{V_s} = \frac{243.9 \text{ cm}^3}{341.13 \text{ cm}^3} = 0.715$$

$$e = 0.715$$

The porosity, n

$$n = \frac{e}{1 + e}$$

$$n = \frac{0.715}{1 + 0.715} = 0.417$$

$$n = 0.417 \text{ or } 41.70\%$$

The degree of saturation, S_r

$$S_r = \frac{V_w}{V_v}$$

$$S_r = \frac{109 \text{ cm}^3}{243.9 \text{ cm}^3} = 0.447$$

$$S_r = 44.70\%$$

The dry unit weight, γ_d and the dry density, ρ_d

$$\gamma_d = \frac{W_s}{V} = \frac{M_s g}{V}$$

$$\gamma_d = \frac{904.0 \text{ g} \times 10 \text{ m/s}^2}{585.0 \text{ cm}^3} = 15.453 \text{ gm/s}^2 / \text{cm}^3$$

$$\gamma_d = 15.453 \text{ kN/m}^3$$

Also;

$$\rho_d = \frac{\gamma_d}{g}$$

$$\rho_d = 1545.3 \text{ kg/m}^3$$

The bulk unit weight, γ and the bulk density, ρ

$$\gamma = \frac{W}{V} = \frac{Mg}{V}$$

$$\gamma = \frac{1013.0 \text{ g} \times 10 \text{ m/s}^2}{585.0 \text{ cm}^3} = 17.316 \text{ gm/s}^2 / \text{cm}^3$$

$$\gamma = 17.316 \text{ kN/m}^3$$

Also;

$$\rho = \frac{\gamma_d}{g}$$

$$\rho = 1731.6 \text{ kg/m}^3$$

Example 10:

For a moist soil sample, the following information is given

- Total volume $V = 1.2 \text{ m}^3$
- Total mass $M = 2350 \text{ kg}$
- Moisture content, $m = 8.6\%$
- Specific gravity of soil solids $G_s = 2.71$

From this information, determine the following

- (a) The moist density/the unsaturated bulk density
- (b) The dry density
- (c) The void ratio and porosity
- (d) The volumes of solids and of voids
- (e) The volume of water and hence the degree of saturation
- (f) The volume of air and the air voids ratio

SOLUTIONS:

(a) **The moist density/the unsaturated bulk density ρ**

$$\rho = \frac{M}{V}$$

$$\rho = \frac{2350.0kg}{1.2m^3} = 1958.33kg/m^3$$

$$\rho = 1958.33kg/m^3$$

(b) **The dry density, ρ_d**

$$\rho_d = \frac{M_s}{V}$$

Now since M_s is unknown while M is known, we can replace the value of M_s by the expression

$$M_s = \frac{M}{1 + m}$$

so that

$$\rho_d = \frac{M}{(1 + m)V}$$

$$\rho_d = \frac{2350.0kg}{(1 + 0.086) \times 1.2m^3} = \frac{2350.0kg}{1.086 \times 1.2m^3} = \frac{2350.0kg}{1.3032m^3} = 1803.25kg/m^3$$

$$\rho_d = 1803.25kg/m^3$$

Alternatively,

$$\rho_d = \frac{\rho}{1 + m}$$

$$\rho_d = \frac{1958.33kg/m^3}{1 + 0.086} = \frac{1958.33kg/m^3}{1.086} = 1803.25kg/m^3$$

$$\rho_d = 1803.25kg/m^3$$

(c) **The void ratio e and porosity n**

We can now determine the value of e from the relationship,

$$\rho_d = \frac{G_s \rho_w}{1 + e}$$

since the rest of the quantities are known except e .

From the above equation therefore

$$e = \frac{G_s \rho_w}{\rho_d} - 1$$

$$e = \frac{2.71 \times 1000 \text{ kg/m}^3}{1803.25 \text{ kg/m}^3} - 1$$

$$\mathbf{e = 0.503}$$

and

$$n = \frac{e}{1 + e}$$

$$n = \frac{0.503}{1 + 0.503} = \frac{0.503}{1.503} = 0.335$$

$$\mathbf{n = 0.335}$$

(d) The volumes of solids and of voids

$$\text{if } V_s = 1 \text{ when } V = 1 + e$$

then

$$\text{what is } V_s = ? \text{ when } V = 1.2 \text{ m}^3?$$

By cross multiplying, we get

$$V_s = \frac{1.2 \times 1}{1 + e}$$

From the above previous section, $e = 0.503$. Therefore

$$V_s = \frac{1.2 \times 1}{1.503} = 0.798 \text{ m}^3$$

$$\mathbf{V_s = 0.798 m^3}$$

Alternatively:

$$V_s = \frac{M_s}{G_s \rho_w}$$

$$M_s = \frac{M}{1+m} \Rightarrow V_s = \frac{M}{G_s \rho_w (1+m)}$$

$$V_s = \frac{2350 \text{ kg}}{2.71 \times 1000 \text{ kg m}^{-3} (1 + 0.086)} = \frac{2350}{2710 \times 1.086} \text{ m}^3 = \frac{2350}{2943.06} \text{ m}^3$$

$$= 0.798 \text{ m}^3$$

$$V_s = \mathbf{0.798 \text{ m}^3}$$

Finally, the **volume of voids** is therefore

$$V_v = V - V_s = 1.200 - 0.798 = \mathbf{0.402 \text{ m}^3}$$

$$V_v = \mathbf{0.402 \text{ m}^3}$$

Alternatively:

The **volume of voids** could also be found using a similar approach as that which was used in finding its counterpart volume of solids, that is:

$$\text{if } V_v = n = \frac{e}{1+e} \text{ when } V = 1$$

then

$$\text{what is } V_v = ? \text{ when } V = \mathbf{1.2 \text{ m}^3}$$

$$V_v = \frac{1.2 \times \frac{e}{1+e}}{1} = \frac{1.2e}{1+e} = \frac{1.2 \times 0.503}{1.503} = \frac{0.6036}{1.503} = 0.402 \text{ m}^3$$

$$V_v = \mathbf{0.402 \text{ m}^3}$$

(e) The degree of saturation, S_r

Since up to this stage, the quantities e and m are known, and G_s is given, we can therefore apply the relation for partial/incomplete saturation

$$S_r e = m G_s$$

so that

$$S_r = \frac{m G_s}{e}$$

$$S_r = \frac{0.086 \times 2.71}{0.503} = 0.4633$$

$$S_r = 46.33\%$$

The volume of water, V_w

$$V_w = \frac{M_w}{\rho_w} = \frac{M - M_s}{\rho_w}$$

But, recall also that:

$$M_s = \frac{M}{1 + m}$$

so that

$$V_w = \frac{M - \left(\frac{M}{1 + m}\right)}{\rho_w}$$

$$V_w = \frac{2350.0kg - \left(\frac{2350.0kg}{1 + 0.086}\right)}{1000kg/m^3} = \frac{2350.0kg - 2163.90kg}{1000kg/m^3} = \frac{186kg}{1000kg/m^3}$$

$$= 0.186m^3$$

$$V_w = 0.186m^3$$

(f) The volume of air V_a , the air content N_a and the air voids ratio, A_v

To get the volume of air, we need to find both the values of the volume of solids and the volume of voids.

Now since we know the void ratio $e = 0.503$ and also since

$$e = \frac{V_v}{V_s} = 0.503$$

implying that

$$\frac{V_v}{V_s} = 0.503$$

$$V_v = 0.503 \times V_s$$

We therefore need to find the value of V_s using the expression

$$V_s = \frac{M_s}{G_s \rho_w}$$
$$V_s = \frac{2163.90 \text{ kg}}{2.71 \times 1000 \text{ kg/m}^3}$$
$$V_s = \mathbf{0.798 \text{ m}^3}$$

Hence from above

$$V_v = 0.503 \times V_s = 0.503 \times 0.80 \text{ m}^3$$
$$V_v = \mathbf{0.4024 \text{ m}^3}$$

The volume of air is therefore

$$V_a = V_v - V_w$$
$$V_a = 0.4024 \text{ m}^3 - 0.1860 \text{ m}^3$$
$$V_a = \mathbf{0.2164 \text{ m}^3}$$

and the **air content** is therefore

$$N_a = \frac{V_a}{V}$$
$$N_a = \frac{0.2164}{1.200} = \frac{0.2164}{1.2000} = 0.18$$
$$N_a = \mathbf{0.18 \text{ or } 18\%}$$

or simply

$$N_a = \frac{n(1 - S_r)}{1}$$

$$N_a = 0.3350(1.0 - 0.4633) = 0.3350(0.5367) = 0.3350(0.5367) = 0.18$$

and again, $N_a = \mathbf{0.18}$ or **18%**

The **air voids ratio** is therefore

$$A_v = \frac{V_a}{V_v}$$

$$A_v = \frac{0.2164m^3_a}{0.4024m^3} = 0.54$$

$$A_v = 0.54$$

or simply

$$A_v = 1 - S_r$$

$$A_v = 1.0 - 0.4633 = 0.54$$

$$A_v = 0.54$$

Example 11:

From the information given below,

- (a) Determine the mass of water required to be added to $10m^3$ of soil for complete saturation:
- Porosity, $n = 0.4$
 - Specific gravity, $G_s = 2.68$
 - Moisture content $m = 12\%$
- (b) What is the value of the moisture content at complete saturation?
- (c) What is the dry density of this type of soil?

SOLUTION

The mass of water required to be added to $10m^3$ of soil for complete saturation:

In other words, the question above requires us to find

- (a) the **UNSATURATED**/**THE BULK** /**THE MOIST** density ρ
- (b) the **SATURATED** density ρ_{sat}

- (c) the difference between the amount of water contained in ρ and in ρ_{sat} ; in kg/m^3

Therefore, the amount of water to be calculated in (c) above is the amount contained in $1m^3$ of this type of soil. By multiplying this result by $10m^3$, we get the correct amount of water to be added, that will make a $10m^3$ volume of this type of soil, fully saturated

Now since we are given, n , G and m then;

$$\rho = (1 + m)(1 - n)G_s\rho_w$$

and

$$\rho_{sat} = [G_s(1 - n) + n]\rho_w$$

Therefore

$$\rho = (1 + m)(1 - n)G_s\rho_w$$

$$\begin{aligned}\rho &= (1 + 0.12)(1 - 0.40)2.68 \times 1000 kg/m^3 = (1.12)(0.60) \times 2680 kg/m^3 \\ &= 0.672 \times 2680 kg/m^3 = 1801 kg/m^3\end{aligned}$$

$$\rho = 1801 kg/m^3$$

Similarly

$$\rho_{sat} = [G_s(1 - n) + n]\rho_w$$

$$\begin{aligned}\rho_{sat} &= [2.68(1 - 0.40) + 0.40] \times 1000 kg/m^3 \\ &= [2.68(0.60) + 0.40] \times 1000 kg/m^3\end{aligned}$$

$$\rho_{sat} = [1.608 + 0.40] \times 1000 kg/m^3 = [2.008] \times 1000 kg/m^3 = 2008 kg/m^3$$

$$\rho_{sat} = 2008 kg/m^3$$

Note that, normally ρ_{sat} is always greater than ρ i.e. $\rho_{sat} > \rho$ because both ρ_{sat} and ρ contain same amount of solid soil particles but ρ_{sat} **contains** more water than ρ However, remember also that the two are equal when $e = mG_s$ (i.e. $\rho_{sat} = \rho$)

Hence . $\rho_{sat} > \rho$, Because of additional water in the soil.

Now, let M_1 be the mass of water added per cubic meter, then

$$M_1 = \rho_{sat} - \rho$$

$$M_1 = 2008kg/m^3 - 1801kg/m^3 = 207kg/m^3$$

$$M_1 = 207kg/m^3$$

Therefore, the mass of water required to be added to $1m^3$ of soil is $207kg$.

Hence, the required amount of water M_w , to be added to $10m^3$ of soil is $10m^3 \times M_1$

$$\Rightarrow M_w = 10M_1 = 2070kg$$

Therefore the mass of water required to be added to $10m^3$ of soil for complete saturation is $M_w = 2070kg$

The value of moisture content m , at complete saturation

Here we need to find m when $S_r = 1$ i.e.,

$$\begin{array}{ll} \text{if } m = 12\% & \text{when } S_r = \frac{mG_s}{e} \\ \text{then what is } m' = ? & \text{when } S_r = 1 \end{array}$$

$$\begin{array}{ll} m = 12\% & S_r = \frac{0.12 \times 2.68}{\left(\frac{0.4}{1.0 - 0.4}\right)} \\ m' = ? & S_r = 100\% \end{array}$$

$$\begin{array}{ll} m = 0.12 & S_r = 0.4824 \\ m' = ? & S_r = 1.0 \end{array}$$

$$\begin{aligned} m' &= \frac{0.12 \times 1.0}{0.4824} \\ m' &= 24.9\% \end{aligned}$$

So from above you can easily observe that

- (i) the **moisture content after complete saturation** is almost twice as much (24.9%) as it was before (12%)
- (ii) the **degree of saturation after complete saturation** is almost twice as much (100%) as it was before (48.24%)
- (iii) the two quantities m and S_r in this case have as is usual, obeyed their obvious direct mathematical relationship $S_r e = m G_s$

Alternatively, we could have calculated the weight of solids W_s and then get m' using the relationship

$$m' = \left(\frac{M_w}{M_s} \right) \times 100\%$$

That is

$$M_s = G_s \rho_w V_s$$

Note again that since the **total volume** $V = 10m^3$ then $V_s = 10(1 - n)$ in this particular case

Therefore

$$M_s = G_s \rho_w \times 10(1 - n)$$

So that the **additional moisture** content resulting from the added mass of water M_w is

$$\begin{aligned}
 m'' &= \frac{M_w}{M_s} \\
 m'' &= \frac{M_w}{G_s \rho_w \times 10(1 - n)} \\
 m'' &= \frac{2070}{2.68 \times 1000 \times 10(1.0 - 0.4)} = \frac{2070}{26800(0.6)} = \left(\frac{2070}{16080} \right) \times 100\% \\
 &= 0.1287 \times 100\% \\
 m'' &= 12.9\%
 \end{aligned}$$

Now, since the soil was originally at a moisture content of 12%, the resulting moisture content (resulting from additional water) is therefore

$$m' = 12\% + m''$$

$$m' = 12\% + 12.9\%$$

$$m' = 24.9\%$$

and which is in consistency with the previous answer

The dry density of the soil, ρ_d

$$\rho_d = \frac{M_s}{V}$$

From above,

$$M_s = G_s \rho_w x 10(1 - n)$$

$$M_s = 16080kg$$

and

$$V = 10m^3$$

Therefore

$$\rho_d = \frac{M_s}{V}$$

$$\rho_d = 1608kg/m^3$$

Alternative way 1:

$$\rho_d = \frac{\rho}{1 + m}$$

or

$$\rho_d = \frac{\rho_{sat}}{1 + m'}$$

where

$$\rho = 1801kg/m^3$$

and

$$\rho_{sat} = 2008 \text{ kg/m}^3$$

Both of these two above procedures will give the same result for dry density i.e.

$$\rho_d = 1608 \text{ kg/m}^3$$

Note: m is the given moisture content of **12%** whereas m'' is the calculated moisture content of **24.9%** i.e. the moisture content at complete saturation.

Alternative way 2:

From

$$\begin{aligned}\rho_{sat} &= \rho_d + n\rho_w \Rightarrow \rho_d = \rho_{sat} - n\rho_w \\ \rho_d &= 2008 \text{ kg/m}^3 - 0.4(1000) \text{ kg/m}^3 = 1608 \text{ kg/m}^3 \\ \rho_d &= 1608 \text{ kg/m}^3\end{aligned}$$

Example 12:

A **COMPLETE SATURATED soil** has a **DRY unit weight** of 16.50 kN/m^3 . If its moisture content is 23%

- (a) Determine its
 - (i) saturated unit weight
 - (ii) specific gravity
 - (iii) void ratio
 - (iv) porosity
- (b) Assuming 10 m^3 as the volume of the soil, what is the volume of water in the sample? What is the volume of air?

SOLUTION

Given:

$$m_{sat} = 0.23$$

$$\gamma_d = 16.50 \text{ kN/m}^3$$

(i) **The saturated unit weight, γ_{sat}**

$$\gamma_d = \frac{\gamma_{sat}}{1 + m_{sat}}$$

$$\gamma_{sat} = \gamma_d(1 + m_{sat})$$

$$\gamma_{sat} = 16.50(1 + 0.23) = 16.50(1.23) = 20.30 \text{ kN/m}^3$$

$$\gamma_{sat} = \mathbf{20.30 \text{ kN/m}^3}$$

(ii) **The specific gravity, G_s**

At complete saturation $S_r e = m_{sat} G_s$. with $S_r = 1$ then;

$$e = m_{sat} G_s$$

But also

$$\gamma_d = \frac{G_s \gamma_w}{1 + e}$$

implying that

$$\gamma_d = \frac{G_s \gamma_w}{1 + m_{sat} G_s}$$

and hence

$$G_s = \frac{\gamma_d}{\gamma_w - m_{sat} \gamma_d}$$

$$G_s = \frac{16.50 \text{ kN/m}^3}{10 \text{ kN/m}^3 - (0.23 \times 16.50 \text{ kN/m}^3)}$$

or

$$G_s = \frac{1650 \text{ kg/m}^3}{1000 \text{ kg/m}^3 - (0.23 \times 1650 \text{ kg/m}^3)}$$

from which we get

$$\mathbf{G_s = 2.66}$$

(iii) **The void ratio; e**

At complete saturation

$$e = mG_s$$

$$e = 0.23 \times 2.66$$

$$e = \mathbf{0.61}$$

(iv) **The porosity; n**

$$n = \frac{e}{1 + e}$$

$$n = \frac{0.61}{1.61}$$

$$n = \mathbf{0.38}$$

(g) **The volume of water; V_w**

At complete saturation the volume of water equals the porosity. Now, for a specific case where the total volume $V = 10m^3$ then

$$V_w = 10 \times n$$

$$V_w = 10 \times 0.38$$

$$V_w = \mathbf{3.8m^3}$$

And it follows therefore from above that the **volume of air is zero** since this is a **complete saturation** case.

Example 13:

- (a) A soil sample has a porosity of 41%, moisture content of 15.2% and specific gravity of 2.65. Determine the degree of saturation, the **void ratio** and the **air voids ratio**.
- (b) What would be the value for the moisture content in (a) above assuming complete saturation of the soil?
- (c) What is the value for the volume of water at complete saturation?

Given:

$$n = 41\%$$

$$m = 15.2\%$$

$$G_s = 2.65$$

$$S_r = ?$$

$$e = ?$$

(a)

- **The degree of saturation**

$$S_r = \frac{mG_s}{e}$$

Since we are given the porosity, then

$$S_r = \frac{mG_s(1 - n)}{n}$$

after we have replaced e with the expression that relates it to the porosity n

Therefore

$$S_r = \frac{0.152 \times 2.65 (1 - 0.41)}{0.41} = \frac{0.4028 (0.59)}{0.41} = 0.58$$

$$S_r = 58\%$$

- **The void ratio**

$$e = \frac{n}{1 - n}$$

$$e = \frac{0.41}{1 - 0.41} = \frac{0.41}{0.59} = 0.695$$

$$e = 0.695$$

- **The air void ratio, A_v**

To get the air voids ratio, we first need to find the volume of air V_a

Obvious this is a partial saturation case since some amount of air is still available in the soil. The volume of air is given by the expression

$$V_a = V_v - V_w = n - mG_s(1 - n)$$

where

$$V_v = n$$

and

$$V_w = mG_s(1 - n)$$

Therefore

$$V_a = n - mG_s(1 - n)$$

$$V_a = 0.41 - (0.152 \times 2.65) \times (1 - 0.41)$$

$$V_a = 0.41 - (0.4028 \times 0.59)$$

$$V_a = 0.410 - 0.238$$

$$V_a = 0.172 \text{ cubic units}$$

Now, since for partial saturation $V_v = n$, then the air voids ratio in this case is

$$A_v = \frac{V_a}{V_v} = \frac{0.172 \text{ cubic units}}{0.410 \text{ cubic units}}$$

$$A_v = 0.42$$

Alternatively:

$$V_a = V_v - V_w = \frac{e}{1 + e} - \frac{mG_s}{1 + e}$$

$$V_a = \frac{e - mG_s}{1 + e}$$

where

$$V_v = \frac{e}{1 + e}$$

and

$$V_w = \frac{mG_s}{1 + e}$$

Therefore

$$V_a = \frac{e - mG_s}{1 + e}$$

$$V_a = \frac{0.695 - (0.152 \times 2.65)}{1 + 0.695} = \frac{0.695 - (0.4028)}{1.695} = \frac{0.2922}{1.695} = 0.172$$

$$V_a = 0.172$$

Therefore, again

$$A_v = \frac{V_a}{V_v} = \frac{0.172 \text{ cubic units}}{0.410 \text{ cubic units}}$$

$$A_v = 0.42$$

as previously found

(b) Moisture content at complete saturation

At complete saturation, $S_r = 1$ and therefore

$$S_r = \frac{mG_s}{e} = 1$$

implying that $e = mG_s$ so that

$$m = \frac{e}{G_s}$$

$$m = \frac{0.695}{2.650}$$

$$m = 26.22\%$$

Alternative way2: Calculate the value of m by sing empirical relationship that exists between m and S_r

That is:

if $m = 0.152$ when $S_r = 58\%$

then what is m ? when $S_r = 100\%$?

$$m = \frac{0.152 \times 100}{58} = 0.2621$$

$$m = 26.21\%$$

Alternative way 3: Calculate the value of m using the porosity, n

The volume of water at complete saturation equals the volume of voids (volume of air is zero) and hence the void ratio n , i.e.

$$V_w = n = 0.41$$

The weight of weight is therefore

$$W_w = V_w \gamma_w$$

and that of solids is

$$W_s = G_s \gamma_w (1 - n)$$

So that the moisture content is

$$m = \frac{W_w}{W_s} \times 100\%$$

$$m = \frac{V_w \gamma_w}{G_s \gamma_w (1 - n)} \times 100\%$$

$$m = \frac{V_w}{G_s (1 - n)} \times 100\%$$

$$m = \frac{0.41}{2.65(1 - 0.41)} \times 100\% = \frac{0.41}{2.65(0.59)} \times 100\% = \frac{0.41}{1.5635} \times 100\% = 26.22\%$$

$$\mathbf{m = 26.22\%}$$

Alternative way 4: Calculate the value of m using the void ratio e

At complete saturation,

$$W_w = \frac{e \gamma_w}{1 + e}$$

$$W_s = \frac{G_s \gamma_w}{1 + e}$$

so that

$$m = \frac{W_w}{W_s} = \left(\frac{\frac{e \gamma_w}{1 + e}}{\frac{G_s \gamma_w}{1 + e}} \right) \Rightarrow m = \frac{e}{G_s}$$

an expression which later simplifies (after cancelling out the like terms) to

$$m = \frac{e}{G_s}$$

$$m = \frac{0.695}{2.650} = 0.2622$$

$$\mathbf{m = 26.22\%}$$

(c) **The value for the volume of water at complete saturation**

$$V_w = n = 0.41$$

$$V_w = 0.41 \text{ cubic units}$$

Note: The term **cubic units** refers to a calculated value of **volume units of an unspecified volume of a soil**. If let's say in our current (this) problem, the value of the total volume was specified to be let's say $9m^3$, then the value of the volume of water at complete saturation would be

$$V_w = 0.41 \times 9m^3 = 3.69m^3$$

Example 14:

A soil sample has a porosity of 0.32 and a moisture content of 25%. Given that G_s is 2.70, determine its

- (a) dry density
- (b) bulk density
- (c) saturated bulk density

Given:

$$n = 0.32$$

$$m = 0.25$$

$$G_s = 2.70$$

- (a) The Dry Density, ρ_d

$$\rho_d = \frac{M_s}{V}$$

$$\rho_d = \frac{G_s \rho_w (1 - n)}{V}$$

$$\rho_d = G_s \rho_w (1 - n)$$

$$\rho_d = 2.70 \times 1000 \text{ kg/m}^3 (1 - 0.32) = 2700 \text{ kg/m}^3 (0.68) = 1836 \text{ kg/m}^3$$

$$\rho_d = 1836 \text{ kg/m}^3$$

Alternatively:

$$\rho_d = \frac{M_s}{V}$$

$$\frac{G_s \rho_w}{1 + e}$$

$$e = \frac{0.32}{1.0 - 0.32} = \frac{0.32}{0.68} = \frac{0.32}{0.68} = 0.47$$

Therefore

$$\rho_d = \frac{2.70 \times 1000 \text{ kg/m}^3}{1 + 0.47} = \frac{2700 \text{ kg/m}^3}{1.47} = 1836.7 \text{ g/m}^3$$

$$\rho_d = 1836.7 \text{ g/m}^3$$

(b) The Bulk Density, ρ

$$\rho = \frac{M}{V}$$

$$\rho = \frac{W_s + W_w}{V}$$

$$\rho = \frac{G_s \rho_w (1 - n) + m G_s \rho_w (1 - n)}{V}$$

$$\rho = \frac{(1 - n)(1 + m) G_s \rho_w}{V}$$

$$\rho = \frac{(1.0 - 0.32)(1 + 0.25) 2.70 \times 1000 \text{ kg/m}^3}{1} = (0.68)(1.25) 2700 \text{ kg/m}^3$$

$$= 2295.0 \text{ kg/m}^3$$

$$\rho = 2295.0 \text{ kg/m}^3$$

Alternatively:

$$\rho = \frac{M}{V}$$

$$\rho = \frac{W_s + W_w}{V}$$

$$\rho = \frac{G_s \gamma_w + m G_s \rho_w}{1 + e}$$

$$\rho = \frac{(1 + m) G_s \rho_w}{1 + e}$$

$$\rho = \frac{(1 + 0.25) \times 2.70 \times 1000 \text{ kg/m}^3}{1 + 0.47} = \frac{(1.25) \times 2700 \text{ kg/m}^3}{1.47} = \frac{3375 \text{ kg/m}^3}{1.47}$$

$$= 2296.0 \text{ kg/m}^3$$

$$\rho = 2296.0 \text{ kg/m}^3$$

(c) The Saturated Density, ρ_{sat}

$$\rho_{sat} = \frac{M_{sat}}{V}$$

$$\rho_{sat} = \frac{W_s + W_w}{V_s + V_w}$$

$$\rho_{sat} = \frac{G_s \rho_w (1 - n) + n \rho_w}{(1 - n) + n}$$

$$\rho_{sat} = \frac{G_s \rho_w (1 - n) + n \rho_w}{1}$$

$$\rho_{sat} = G_s \rho_w (1 - n) + n \rho_w$$

$$\rho_{sat} = [G_s (1 - n) + n] \rho_w$$

$$\rho_{sat} = [2.70 \times (1 - 0.32) + 0.32] \times 1000 \text{ kg/m}^3$$

$$= [2.70 \times (0.68) + 0.32] \times 1000 \text{ kg/m}^3$$

$$\rho_{sat} = [1.836 + 0.32] \times 1000 \text{ kg/m}^3 = [2.156] \times 1000 \text{ kg/m}^3 = 2156.0 \text{ kg/m}^3$$

$$\rho_{sat} = 2156.0 \text{ kg/m}^3$$

Alternative way 1:

$$\rho_{sat} = \frac{M_{sat}}{V} = \frac{M_s + M_w}{V_s + V_w} = \frac{G_s \rho_w + e \rho_w}{1 + e} = \frac{(G_s + e) \rho_w}{1 + e}$$

$$\rho_{sat} = \frac{(2.70 + 0.47) \times 1000 \text{ kg/m}^3}{1 + 0.47} = \frac{(2.70 + 0.47) \times 1000 \text{ kg/m}^3}{1 + 0.47}$$

$$= 2156.5 \text{ kg/m}^3$$

$$\rho_{sat} = \mathbf{2156.5 \text{ kg/m}^3}$$

Alternative way 2:

$$\rho_{sat} = \rho_d + n \rho_w$$

$$\rho_d = 1836 \text{ kg/m}^3; n = 0.32; \rho_w = 1000 \text{ kg/m}^3$$

Therefore

$$\rho_{sat} = 1836 \text{ kg/m}^3 + 0.32(1000 \text{ kg/m}^3) = 2156.5 \text{ kg/m}^3$$

$$\rho_{sat} = \mathbf{2156.0 \text{ kg/m}^3}$$

Example 15:

Determine the void ratio of a saturated soil sample that has a mass of 178 g before drying and 139 g after oven drying. Assume the specific gravity of the soil solids is 2.7.

Given:

$$M = 178 \text{ g}$$

$$M_s = 139 \text{ g}$$

$$G_s = 2.70$$

$$S_r = 1 \text{ (This follows directly from the question that the soil is saturated)}$$

$$e = ?$$

SOLUTION:

We need to know the value of the moisture content m so that we can determine the value of e from the relationship $e = mG_s$

$$m = \left(\frac{M_w}{M_s} \right) \times 100\% = \left(\frac{M - M_s}{M_s} \right) \times 100\%$$
$$m = \left(\frac{178 - 139}{139} \right) \times 100\% = \left(\frac{39}{139} \right) \times 100\% = 28.06\%$$

Therefore

$$e = mG_s$$
$$e = 0.2806 \times 2.70$$
$$e = 0.76$$

Example 16:

Differentiate between the following terms as applied to soil mechanics

- (a) Complete saturation and partial saturation of soil
- (b) Moisture content at partial/incomplete saturation and moisture content at complete saturation

Example 17:

The dry density of a soil is 1780 kg/m^3 and its void ratio is 0.55. What is the bulk density if

- (a) $S_r = 50\%$?
- (b) $S_r = 100\%$?
- (c) water is replaced by oil whose specific gravity is $G_{soil} = 0.9$?

Given:

$$\rho_d = 1780 \text{ kg/m}^3$$

$$e = 0.55$$

$$S_{r1} = 0.5$$

$$S_{r2} = 1.0$$

$$G_{soil} = 0.9$$

Solution:

- $\rho_1 = 1957.42 \text{ kg/m}^3$
- $\rho_{sat} = 2134.84 \text{ kg/m}^3$
- $\rho = 2099.35 \text{ kg/m}^3$

SOLUTION:

Case 1: $S_r = 50\% = 0.5$

(a) Case 1: $S_r = 50\% = 0.5$

Let ρ_1 represent the bulk density of the soil at a 50% degree of saturation, Then

$$\rho_1 = \frac{(G_s + eS_{r1})\rho_w}{1 + e}$$

However, the specific gravity of the soil has not been provided but the dry density. Therefore we need to find first the value of this quantity because it is required in calculating the value of ρ_1

$$\rho_d = \frac{G_s \rho_w}{1 + e} \Rightarrow G_s = \frac{\rho_d(1 + e)}{\rho_w} = \frac{1780 \text{ kgm}^{-3}(1 + 0.55)}{1000 \text{ kgm}^{-3}} = 2.759$$

$$G_s = 2.759$$

$$\rho_1 = \frac{(G_s + eS_{r1})\rho_w}{1 + e} = \frac{(2.759 + 0.50(0.55))1000 \text{ kgm}^{-3}}{1 + 0.55} = 1957.42 \text{ kg/m}^3$$

$$\rho_1 = 1957.42 \text{ kg/m}^3$$

(b) Case 2: $S_r = 100\% = 1$ (i.e. complete saturation case)

$$\rho_{sat} = \frac{(G_s + eS_{r2})\rho_w}{1 + e} = \frac{(G_s + e)\rho_w}{1 + e} = \frac{(2.759 + 0.55)1000\text{kgm}^{-3}}{1 + 0.55}$$

$$= 2134.84 \text{ kg/m}^3$$

$$\rho_{sat} = 2134.84 \text{ kg/m}^3$$

(c) How to determine the saturated density of this soil when water is replaced by oil whose specific gravity is $G_{oil} = 0.9$

In the previous part, we were dealing with soil that has been saturated with water. However, if the voids were completely filled with oil only, (*be careful here of the big trick*) it means they would be saturated with oil instead of water;. Therefore under this particular case, we will use the value $S_r = 100\%$ to find the saturated density of the soil if all the voids are filled with oil

We know from the theory that:

$$\rho_{sat} = \frac{(G_s + e)\rho_w}{1 + e} = \frac{G_s\rho_w + e\rho_w}{1 + e} = \frac{G_s\rho_w}{1 + e} + \frac{e\rho_w}{1 + e} = \rho_d + n\rho_w$$

$$\Rightarrow \rho_{sat} = \rho_d + n\rho_w$$

The first term in the above equation of ρ_{sat} i.e. $G_s\rho_w/(1 + e)$ or ρ_d represents the dry density of soil and is actually a contribution to ρ_{sat} from solid soil particles; whereas the second term $e\rho_w/(1 + e)$ or $n\rho_w$ is a contribution to ρ_{sat} from water

These two equations tell us the following two important things that, the quantity:

- (i) $\rho_d = G_s\rho_w/(1 + e)$ will remain the same in all cases regardless of what is mixed with the soil because it is attributed by the presence of solid soil particle within the soil itself. The amount of these solid soil particles present in the soil does not get affected by the action of physically mixing

the soil with any type of a fluid, assuming no any chemical reaction is associated with the mixing process

- (ii) $n\rho_w = e\rho_w/(1 + e)$ is a quantity attributed by the fluid with which the soil gets mixed with. Therefore this quantity can be affected depending on the type of fluid the soil gets mixed with. For example, if the fluid that is mixed with the soil is called x then we must have $n\rho_x = e\rho_x/(1 + e)$ where ρ_x represents the density of the fluid named x that has been mixed with the soil. In usual cases, normally x represents the fluid substance water but may take on any other fluid substance like diesel , oil or any other type of fluid, and under this case, it has been stated to be oil

So, this time the contribution from water is replaced by that from oil so that the term $e\rho_w/(1 + e)$ becomes $e\rho_{oil}/(1 + e) = n\rho_{oil}$.

Now, since we are given specific gravity of oil and not the density of oil, we will be required to express the density of oil in terms of its specific gravity. Therefore from the definition of specific gravity, we have

$$G_{oil} = \frac{\rho_{oil}}{\rho_w}$$

$$\Rightarrow \rho_{oil} = \rho_w G_{oil}$$

Therefore

$$n\rho_{oil} = \frac{e}{1 + e}(\rho_{oil}) = \frac{e}{1 + e}(\rho_w G_{oil}) = \frac{e\rho_w G_{oil}}{1 + e}$$

Note that the density of water still appears in the expression that stands for the contribution from oil. This is because of the use of the specific gravity of oil, originally given from the question itself; since such a quantity is always a function of the density of water. Had it been that we were directly given the density of oil, then the density of water would not have appeared in the said expression because, under such situation, we would directly employ the use of the expression $e\rho_{oil}/(1 + e)$ in solving our problem

Therefore, we finally have

$$\begin{aligned}\rho_{sat} &= \frac{G_s \rho_w}{1 + e} + \frac{e \rho_w G_{oil}}{1 + e} = \frac{(G_s + e G_{oil}) \rho_w}{1 + e} = \frac{(2.759 + 0.55(0.90))1000 \text{kgm}^{-3}}{1 + 0.55} \\ &= \frac{(2.759 + 0.495)1000 \text{kgm}^{-3}}{1.55} = 2099.35 \text{kg/m}^3 \\ \rho_{sat} &= \mathbf{2099.35 \text{kg/m}^3}\end{aligned}$$

Alternatively:

Since the value of ρ_d has been directly given from the question, we can use the relationship

$$\begin{aligned}\rho_{sat} &= \rho_d + \frac{e \rho_w G_{oil}}{1 + e} \\ \rho_{sat} &= 1780 \text{kgm}^{-3} + \frac{0.55(1000 \text{kgm}^{-3})(0.9)}{1 + 0.55} = 1780 \text{kgm}^{-3} + \frac{495 \text{kgm}^{-3}}{1.55} \\ &= 1780 \text{kgm}^{-3} + 319.35 \text{kgm}^{-3} = \mathbf{2099.35 \text{kg/m}^3}\end{aligned}$$

Of foremost importance to note in the above expression of ρ_{sat} here is that G_{oil} does not appear in the first expression $G_s \rho_w / (1 + e) = \rho_d$ because as stated before, this expression is attributed by the contribution of solid soil particles within the soil itself and therefore its amount does not get affected by the fluid with which the soil gets mixed with!

Example 18:

A soil sample has a total volume of $1.587 \times 10^{-3} \text{m}^3$ and specific gravity of 2.68. If the dry weight of the sample is 2.38kg, what is the volume of voids in the sample? [$V_v = 0.6989 \times 10^{-3} \text{m}^3$]

Given:

$$V = 1.587 \times 10^{-3} m^3$$

$$G_s = 2.68$$

$$M_s = 2.38 kg$$

$$V_v = ?$$

SOLUTION:

Looking at the information provided in the question, we note among the information given is the total volume V and asked to find the volume of voids V_v . But we know from theory that the missing quantity to be sought and the one provided is governed by the relationship $n = V_v/V \Rightarrow V_v = nV$. In other words, this question asks us that, what is the value of nV with respect to the information provided in the question?. So we need to first find the value of n and finally, multiply it with the total volume, to get the value of the required quantity

Now using relationships $M_s/V = \rho_d$ and $\rho_d = G_s \rho_w / (1 + e) = G_s \rho_w (1 - n)$ (by equating them) we can determine this unknown value, that is:

$$\frac{M_s}{V} = G_s \rho_w (1 - n) \Rightarrow n = 1 - \frac{M_s}{V G_s \rho_w}$$

Then

$$\begin{aligned} V_v = nV &= V \left(1 - \frac{M_s}{V G_s \rho_w} \right) = V - \frac{M_s}{G_s \rho_w} \\ \Rightarrow V_v &= V - \frac{M_s}{G_s \rho_w} \end{aligned}$$

$$\begin{aligned} V_v &= 1.587 \times 10^{-3} m^3 - \frac{2.38 kg}{2.68 (1000 kg m^{-3})} \\ &= 1.587 \times 10^{-3} m^3 - \left(\frac{2.38 kg}{2.68} \times 10^{-3} m^3 \right) \\ &= 1.587 \times 10^{-3} m^3 - (0.8881 \times 10^{-3} m^3) = 0.699 \times 10^{-3} m^3 \\ \mathbf{V_v} &= \mathbf{0.6989 \times 10^{-3} m^3} \end{aligned}$$

Alternatively:

From the same equations $M_s/V = \rho_d$ and $\rho_d = G_s \rho_w / (1 + e)$,; if we again equate these equations we remain with only one unknown quantity, the void ratio e , that is

$$\frac{M_s}{V} = \frac{G_s \rho_w}{1 + e} \Rightarrow e = \left(\frac{G_s \rho_w V}{M_s} \right) - 1$$

But also;

$$e = \frac{n}{1 - n} \Rightarrow \left(\frac{G_s \rho_w V}{M_s} \right) - 1 = \frac{n}{1 - n} \Rightarrow n = 1 - \frac{M_s}{V G_s \rho_w}$$

and finally $V_v = nV$ as before and hence

$$V_v = 0.699 \times 10^{-3} m^3$$

Example 19:

If a soil sample has a dry unit weight of 19.5 kN/m^3 , moisture content of 8% and a specific gravity of solids particles is 2.67. By taking $g = 10 \text{ ms}^{-2}$, calculate the following:

- (a) The mass of water to be added per cubic meter of soil to reach 80% saturation
- (b) The original degree of saturation i.e. the degree of saturation at 8% moisture content
- (c) The moisture content at complete saturation
- (d) The void ratio
- (e) The porosity
- (f) The saturated unit weight

$$[M_w = 59.67kg ; S_{8\%} = 57..85\%; m_{sat} = 13.83\%; e = 0.3692; n = 0.2697; \gamma_{sat} = 22.20kNm^{-3}]$$

Given:

$$\gamma_d = 19.5kN/m^3 \Rightarrow \rho_d = 1950kg/m^{-3}$$

$$m = 8\%$$

$$G_s = 2.67$$

$$S_r = 0.8$$

SOLUTION:

$$e = (G_s \gamma_w / \gamma_d) - 1$$

$$m_{sat} = e / G_s$$

$$\gamma_{sat} = (G_s + e) \gamma_w / (1 + e)$$

$$M_w = ?$$

(a) The mass of water to be added per cubic meter of soil to reach 80% saturation

From the relationship $m = M_w / M_s$, we observe that the initial mass of water in the soil (before addition) is $M_{w1} = 0.08M_s$

However the mass of water at complete (100%) saturation is $M'_w = m_{sat}M_s$ and in a similar fashion, the mass of water at 80% saturation will be

$$M_w = 0.80M'_w$$

But from above,

$$M'_w = m_{sat}M_s$$

Therefore

$$M_w = 0.80(m_{sat}M_s) = 0.80 \times 13.83\%M_s = 0.80 \times 0.1383M_s$$

$$M_w = 0.1106M_s$$

Therefore, additional mass of water **per cubic meter needed for 80% saturation** is

$$M_w = M'_w - M_{w1}$$

$$M_w = 0.1106M_s - 0.08M_s$$

$$M_w = 0.0306M_s$$

Now, since $\rho_d = \gamma_d/g$ then $M_s/V = \gamma_d/g$

$$M_s = V\gamma_d/g$$

When $V = 1m^3$ $M_s = \gamma_d/g$ and hence $M_w = 0.0306\gamma_d/g$ is the mass of **ADDITIONAL WATER per meter cubic** needed for 80% saturation is

$$M_w = \frac{0.0306 \times 19.5 \times 1000 N/m^3}{10 m/s^2}$$

$$M_w = 59.67 kg/m^3$$

Alternative way1:

First determine the value of the ***moisture content at 80% saturation*** by using the following procedure

$$\text{if } m_{8\%} = 8\% ; \quad \text{when } S_r = m_8 G_s / e$$

$$\text{then what is } m_? = ? ; \quad \text{when } S_{80} = 80\% ?$$

From this relationship therefore

$$m_? = \frac{S_{80} \times m_8}{S_r}$$

$$m_? = \frac{80\% \times 8\%}{(\frac{m G_s}{e})} = (0.8 \times 0.08) / (\frac{0.08 \times 2.67}{0.3692}) = 0.064 / (\frac{0.08 \times 2.67}{0.3692}) = 0.1106$$

$$\Rightarrow m_? = m_{11.06\%} = 11.06\%$$

Then the required mass of additional water that will raise the moisture content of soil from 8% to 11.06 % (or 80% degree of saturation) is

$$M_w = (m_{11.06\%} - m_{8\%})M_s$$

$$M_w = (11.06\% - 8\%)M_s = 3.06\%M_s = 0.0306M_s$$

$$M_w = 0.0306V\gamma_d/g = 0.0306 \times 1950 \gamma_d/g$$

$$M_w = 59.67 \text{ kg/m}^3$$

as before

Alternative way 2(a): Use of bulk densities

We have seen from above that, moisture contents is 11.06% at 80% degree of saturation and the original moisture content was 8%

So we can use the two values of moisture contents to first determine their respective bulk densities (i. e. bulk densities at both 8% and 11.06% moisture content values) after which we shall take the difference of the densities, which represents the mass of water added per cubic meter (*for this method, you may also refer to a similar approach in example 11 part (a) above*) That is:

$$\rho(m = 11.06\%) - \rho(m = 8\%) = M_w$$

= amount of water added per cubic meter

$$\rho = \frac{(1 + m)G_s\rho_w}{1 + e}$$

$$\rho(m = 11.06\%) = \frac{(1 + m_{11.06\%})G_s\rho_w}{1 + e}; \rho(8\%) = \frac{(1 + m_{8\%})G_s\rho_w}{1 + e}$$

$$\rho(m = 11.06\%) = \frac{(1 + 0.1106)G_s\rho_w}{1 + e} = 1.1106 \left(\frac{G_s\rho_w}{1 + e} \right) = 1.1106\rho_d$$

$$\Rightarrow \rho(m = 11.06\%) = 1.1106\rho_d; S_r = 80\%$$

Similarly;

$$\rho(m = 8\%) = 1.080\rho_d$$

Therefore

$$M_w = \rho(m = 11.06\%) - \rho(m = 8\%) = 1.1106\rho_d - 1.080\rho_d = 0.0306\rho_d$$

$$M_w = 0.0306\rho_d = 0.0306 \times 1950 \text{ kgm}^{-3} = 59.67 \text{ kgm}^{-3}$$

$$M_w = 59.67 \text{ kgm}^{-3}$$

Alternative way 2(b):

$$\rho(m = 11.06\%) - \rho(m = 8\%) = M_w$$

= amount of water added per cubic meter

$$\rho(m = 11.06\%) = \rho_d(1 + 0.1106); \rho(m = 8\%) = \rho_d(1 + 0.08)$$

$$\Rightarrow M_w = (0.1106 - 0.08)\rho_d = 0.0306\rho_d = 0.0306(1950\text{kgm}^{-3}) \\ = 59.67\text{kgm}^{-3}$$

$$\mathbf{M_w = 59.67\text{kgm}^{-3}}$$

Alternative way 3: Use of degrees of saturations

First find the original degree of saturation at 8% moisture content; that is:

$$m = 8\% \Rightarrow S_r = \frac{mG_s}{e}; e = \frac{G_s\rho_w}{\rho_d} - 1$$

$$\Rightarrow S_r = \frac{mG_s\rho_d}{G_s\rho_w - \rho_d} = \frac{0.08 \times 2.67 \times 1950\text{kgm}^{-3}}{2.67 \times 1000\text{kgm}^{-3} - 1950\text{kgm}^{-3}} = \frac{416.52\text{kgm}^{-3}}{720\text{kgm}^{-3}} \\ = 0.5785 = 57.85\%$$

$$S_r = 57.85\%$$

Then

$$M_w = \rho(S_r = 80\%) - \rho(S_r = 57.85\%)$$

$$\rho = \frac{(G_s + S_r e)\rho_w}{1 + e}$$

$$\rho(S_r = 80\%) = \frac{(G_s + 0.80e)\rho_w}{1 + e}; \rho(S_r = 57.85\%) = \frac{(G_s + 0.2422e)\rho_w}{1 + e}$$

$$M_w = \frac{(G_s + 0.80e)\rho_w}{1 + e} - \frac{(G_s + 0.5785e)\rho_w}{1 + e} = 0.2215 \left(\frac{e\rho_w}{1 + e} \right)$$

$$e = \frac{G_s\rho_w}{\rho_d} - 1 = \frac{G_s\rho_w - \rho_d}{\rho_d}$$

$$\begin{aligned}
\Rightarrow M_w &= 0.2215 \left(\frac{G_s \rho_w - \rho_d}{\rho_d} \right) \left(\frac{\rho_w \rho_d}{G_s \rho_w} \right) = 0.2215 \left(\frac{G_s \rho_w - \rho_d}{G_s} \right) \\
&= 0.2215 \left(\frac{2670 \text{ kg m}^{-3} - 1950 \text{ kg m}^{-3}}{2.67} \right) = \frac{0.2215 (720 \text{ kg m}^{-3})}{2.67} \\
&= 59.73 \text{ kg/m}^3
\end{aligned}$$

$$M_w = 59.73 \text{ kg/m}^3$$

The small difference between this value and the previous ones is attributed for by mathematical rounding-off procedures

(b) The original degree of saturation i.e. the degree of saturation at 8% moisture content

The original degree of saturation has already been calculated from above using the relationship

$$S_r = \frac{m G_s \rho_d}{G_s \rho_w - \rho_d} = 57..85\%$$

(c) The moisture content at complete saturation

$$\begin{aligned}
m &= \frac{e}{G_s}; \quad e = \frac{G_s \rho_w}{\rho_d} - 1 = \frac{G_s \rho_w - \rho_d}{\rho_d} \\
\Rightarrow m &= \frac{1}{G_s} \left(\frac{G_s \rho_w - \rho_d}{\rho_d} \right) = \frac{G_s \rho_w - \rho_d}{G_s \rho_d} = \frac{2670 \text{ kg m}^{-3} - 1950 \text{ kg m}^{-3}}{2.67 (1950 \text{ kg m}^{-3})} \\
&= \frac{720 \text{ kg m}^{-3}}{2.67 (1950 \text{ kg m}^{-3})} = 0.1383 = 13.83\% \\
m &= 13.83\%
\end{aligned}$$

Alternative way 1(a):

$$\text{if } m = 8\%; \quad \text{when } S_r = 57..85\%$$

then;

what is m =?; when $S_r = 1$?

$$m = \frac{1 \times 0.08}{0.5785} = 0.1383 = 13.83\%$$

$$m = \mathbf{13.83\%}$$

Alternative way 1(b):

if $m = \mathbf{11.06\%}$; when $S_r = \mathbf{80\%}$

then;

what is m =?; when $S_r = 1$?

$$m = \frac{1 \times 0.1106}{0.80} = 0.1383 = 13.83\%$$

$$m = \mathbf{13.83\%}$$

(d) The void ratio

$$e = \frac{G_s \rho_w}{\rho_d} - 1 = \frac{2.67 \times 1000 \text{ kgm}^{-3}}{1950 \text{ kgm}^{-3}} - 1 = 0.3692$$

$$e = \mathbf{0.3692}$$

(e) The porosity

$$n = \frac{e}{1 + e} = \frac{0.3692}{1.3692} = 0.2697$$

$$n = \mathbf{0.2697}$$

Alternatively:

$$e = \frac{G_s \rho_w}{\rho_d} - 1 = \frac{n}{1 - n} \Rightarrow \frac{G_s \rho_w}{\rho_d} = \frac{n}{1 - n} + 1 = \frac{n + 1 - n}{1 - n} = \frac{1}{1 - n}$$

$$\frac{G_s \rho_w}{\rho_d} = \frac{1}{1 - n} \Rightarrow 1 - n = \frac{\rho_d}{G_s \rho_w} \Rightarrow n = 1 - \frac{\rho_d}{G_s \rho_w}$$

$$n = 1 - \frac{1950 \text{ kg m}^{-3}}{2670 \text{ kg m}^{-3}} = 0.2697$$

$$n = 0.2697$$

(f) The saturated unit weight

$$\gamma_{sat} = \gamma_d + n\gamma_w$$

$$\gamma_{sat} = 19.50 \text{ kNm}^{-3} + 0.2697(10 \text{ kNm}^{-3}) = 19.50 \text{ kNm}^{-3} + 2.697 \text{ kNm}^{-3}$$

$$\gamma_{sat} = 22.20 \text{ kNm}^{-3}$$

Example 20:

A soil sample has a void ratio of 0.72, moisture content of 12% and $G_s = 2.72$

Determine the following:

- Dry unit weight and moist unit weight
- Weight of water in kN/m^3 to be added for the saturation to be changed from the current value of 2% to 80% degree of saturation.
- Is it possible to reach a water content of 30% without changing the present void ratio?
- Is it possible to compact the soil sample to a dry unit weight of 23.5 kN/m^3 ?

$$[\gamma_d = 15.81 \text{ kN/m}^3; \gamma = 16.13 \text{ kN/m}^3; W_w = 3.03 \text{ kN/m}^3]$$

Given:

$$e = 0.72$$

$$m = 2\%$$

$$G_s = 2.72$$

$$S_{r2} = 80\%$$

SOLUTIONS:

(a) Dry unit weight and moist unit weight

$$\gamma_d = \frac{G_s \gamma_w}{1 + e}$$

$$\gamma_d = \frac{2.72(10kNm^{-3})}{1 + 0.72} = \frac{27.2kNm^{-3}}{1.72} = 15.81kNm^{-3}$$

$$\gamma_d = \mathbf{15.81kNm^{-3}}$$

$$\gamma = \gamma_d(1 + m)$$

$$\gamma = 15.81kNm^{-3}(1 + 0.02) = 16.13kNm^{-3}$$

$$\gamma = \mathbf{16.13kNm^{-3}}$$

(b) Weight of water in kN/m^3 to be added for the saturation to be changed from the current value of 2% to 80% degree of saturation

Weight of water W_w in kN/m^3 to be added is: $W_w = \gamma(S_r = 80\%) - \gamma(S_r = 2\%)$

$$\gamma(S_r = 2\%) = \gamma_d(1 + 2\%) = \gamma_d(1 + 0.02)$$

$$\gamma(S_r = 80\%) = \gamma_d(1 + 80\%) = \gamma_d(1 + 0.80\%)$$

$$\gamma(S_r = 2\%) = 16.13kN/m^3$$

$$\gamma(S_r = 80\%) = 19.16kN/m^3$$

Therefore, additional weight of water **per meter cubic**, required for 80% saturation of the soil is

$$W_w = 19.16kN/m^3 - 16.13kN/m^3$$

$$W_w = \mathbf{3.03kN/m^3}$$

Alternatively:

Find the weights of water at both 80% and 2% saturation and take the difference between them, i.e.

$$W_w = W_{w2} - W_{w1}$$

where $W_{w1} = 2\%W_s = 0.02W_s$ and W_{w2} is the weight of water at 80% saturation. Therefore, you may need to calculate the value of W_{w2} using the following alternative procedure

$$W_{w1} = 0.02W_s \quad S_r = \frac{m_1 G_s}{e} = \frac{2\% \times G_s}{0.72}$$

$$W_{w2} = ? \quad S_{80} = 80\%$$

$$W_{w2} = \frac{0.02W_s \times 80\%}{\left(\frac{2\% \times G_s}{0.72}\right)}$$

so that the weight of water to be added for the sake of attaining an **80% degree of saturation** will be $W_w = W_{w2} - W_{w1}$ and the amount W_{w2} is calculated as per the procedures below

$$W_{w2} = \frac{0.02W_s \times 0.8}{\left(\frac{0.02 \times G_s}{0.72}\right)} = \frac{0.72W_s \times 0.8}{G_s} = \frac{0.576W_s}{G_s}$$

$$W_s = \gamma_d V$$

Therefore

$$W_{w2} = \frac{0.576\gamma_d V}{G_s}$$

Now, taking $V = 1m^3$ then we get

$$W_{w2} = \frac{0.576\gamma_d}{G_s} = \frac{0.576G_s\gamma_w}{G_s(1+e)} = \frac{0.576\gamma_w}{(1+e)} = \frac{0.576 \times 10kN/m^3}{1+0.72} = \frac{5.76kN/m^3}{1.72}$$

$$W_{w2} = 3.35kN/m^3$$

Similarly;

$$W_{w1} = 0.02W_s$$

$$W_s = \gamma_d V$$

$$W_{w1} = 0.02\gamma_d V = 0.02\gamma_d \times 1m^3 = 0.02\gamma_d = \frac{0.02G_s\gamma_w}{1+e}$$

$$W_{w1} = \frac{0.02 \times 2.72 \times 10 \text{ kN/m}^3}{1 + 0.72} = \frac{0.544 \text{ kN/m}^3}{1.72} = 0.32 \text{ kN/m}^3$$

$$W_{w1} = 0.32 \text{ kN/m}^3$$

Therefore, additional weight of water **per meter cubic**, required for 80% saturation of the soil is

$$W_w = W_{w2} - W_{w1}$$

$$W_w = 3.35 \text{ kN/m}^3 - 0.32 \text{ kN/m}^3$$

$$W_w = 3.03 \text{ kN/m}^3$$

(c) Is it possible to reach a water content of 30% without changing the present void ratio?

Given:

$$e = 0.72$$

$$G_s = 2.72$$

$$m = 30\%$$

In other words, this question is asking us a hidden question that ***“could the relationship $S_r e = m G_s$ still hold even when the value of $m = 30\%$?”***

So let's find the value of S_r when $m = 30\%$ and see if the relationship $S_r e = m G_s$ will still be obeyed. If it will, then our answer will be a big yes;; otherwise, our answer should be no

$$S_r = m G_s / e$$

$$S_r = \frac{0.30 \times 2.72}{0.72} = \frac{0.816}{0.72} = 1.1333$$

$$S_r = 1.1333$$

We know the boundary conditions of S_r that $0 \leq S_r \leq 1$. So the above value of S_r indicates that it is practically impossible to have a value of 30% moisture

content for this type of soil as it would require S_r to have a value greater than 1,; a practically impossible situation

For this type of soil, we can only have a maximum value of m when $S_r = 1$ and its value is calculated as follows

$$m = m_{sat} = \frac{S_r e}{G_s} = \frac{1 \times e}{G_s} = \frac{e}{G_s}$$

$$m_{sat} = \frac{0.72}{2.72} \times 100\% = 26.47\%$$

So the maximum value of moisture content this type of soil can hold is 26.47%. In other words, the boundary values of m corresponding to all possible values of S_r is $0 \leq m \leq 26.47\%$ and not otherwise. That is:

$$0 \leq m \leq 26.47\% \text{ when } 0 \leq S_r \leq 1$$

(d) Is it possible to compact the soil sample to a dry unit weight of 23.5 N/m³?

Not possible because, we have previously seen from above that the (maximum) dry unit weight is $\gamma_d = 15.81 \text{ kN/m}^3$, which indicates that it is impossible for this type of soil to have a value of γ_d larger than this value i.e. under this particular case the only possibility is $0 < \gamma_d \leq 15.81 \text{ kN/m}^3$ and not otherwise

Example 21:

A dry sandy soil is placed in a container having a volume of 0.038 m^3 . The dry weight of the sample is 56.685 kg . Water is carefully added to the container so as not to disturb the condition of the sand. When the container is filled, the combined weight of soil plus water is 73.691 kg . From these data, compute

(a) The moisture content, the void ratio of soil in the container and the specific gravity of the soil particles.

(b) The dry density and the saturated density

$$m = 30\%; e = 0.81; G_s = 2.70; \rho_d = 1491.7 \text{ kg/m}^3; \rho_{sat} = 1939.21 \text{ kg/m}^3,$$

Given:

$$V = 0.038 \text{ m}^3$$

$$M_s = 56.685 \text{ kg}$$

$$M = 73.691 \text{ kg}.$$

SOLUTION::

(a) The moisture content, the void ratio of soil in the container and the specific gravity of the soil particles.

Moisture content:

From $\rho = \rho_d(1 + m)$; $\rho = M/V$ and $\rho_d = M_s/V$ we find that

$$m = \frac{\rho}{\rho_d} - 1 \Rightarrow m = \frac{M}{M_s} - 1$$
$$m = \frac{73.691 \text{ kg}}{56.685 \text{ kg}} - 1 = 0.30 = 30\%$$
$$\mathbf{m = 30\%}$$

Void ratio and Specific gravity:

Since we are also missing the specific gravity, we will need to find simultaneously, the values of both the void ratio and of the specific gravity. The problem at hand is also a complete saturation case since the container is fully filled with water and solid soil particles only, implying the all the voids are filled with water.

Now, from $\rho_d = M_s/V$ and $\rho_d = G_s\rho_w(1 + e)$, we form a simultaneous equation in e and G_s as follows:

$$\frac{M_s}{V} = \frac{G_s\rho_w}{1 + e} \Rightarrow M_s(1 + e) = G_s\rho_w V \Rightarrow G_s\rho_w V - eM_s = M_s$$

Therefore

$$\begin{aligned} G_s\rho_w V - eM_s &= M_s \Rightarrow 1000kgm^{-3}(0.038m^3)G_s - (e)56.685kg \\ &= 38G_s - 56.685e = 56.685 \end{aligned}$$

Therefore

$$\mathbf{38G_s - 56.685e = 56.685} \quad (i)$$

Also;

$$\begin{aligned} \rho_{sat} &= \frac{M}{V} = \frac{(G_s + e)\rho_w}{1 + e} \Rightarrow M(1 + e) = (G_s + e)\rho_w V \Rightarrow G_s\rho_w V + e\rho_w V - eM \\ &= M \\ &= 1000kgm^{-3}(0.038m^3)G_s + e(1000kgm^{-3}(0.038m^3) \\ &\quad - (e)73.691 = 73.691 \\ &\Rightarrow 38G_s + 38e - 73.691e = 73.691 \Rightarrow 38G_s - 35.691e = 73.691 \end{aligned}$$

Therefore

$$\mathbf{38G_s - 35.691e = 73.691} \quad (ii)$$

Subtracting equation (i) from (ii) we get

$$\begin{aligned} 38G_s - 35.691e - (38G_s - 56.685e) &= 73.691 - 56.685 \Rightarrow 20.994e \\ &= 17.006 \Rightarrow e = \frac{17.006}{20.994} = 0.81 \\ \mathbf{e} &= \mathbf{0.81} \end{aligned}$$

Finally, substitute this value of e in any of the above equations to get the value of G_s .

From let's say

$$38G_s - 35.691e = 73.691 \Rightarrow 38G_s = 73.691 + 35.691(0.81) = 102.60$$

$$\Rightarrow 38G_s = 102.60 \Rightarrow G_s = 2.70$$

Therefore

$$e = 0.81; G_s = 2.70$$

Note: This part of the problem is similar to the last part of **EXAMPLE 26** below

Dry density:

$$\rho_d = \frac{G_s \rho_w}{1 + e}$$

$$\rho_d = \frac{2.70(1000 \text{kgm}^{-3})}{1 + 0.81} = \frac{2700 \text{kgm}^{-3}}{1.81} = 1491.71 \text{kgm}^{-3}$$

$$\rho_d = 1491.71 \text{kgm}^{-3}$$

Saturated density:

$$\rho_{sat} = \rho_d + n\rho_w$$

$$\rho_{sat} = 1491.71 \text{kgm}^{-3} + \frac{0.81}{1 + 0.81} (1000 \text{kgm}^{-3})$$

$$= 1491.71 \text{kgm}^{-3} + 447.51 \text{kgm}^{-3} = 1939.22 \text{kgm}^{-3}$$

Therefore

$$\rho_d = 1491.71 \text{kgm}^{-3}; \rho_{sat} = 1939.22 \text{kgm}^{-3}$$

Example 22:

Dry soil with $G_s = 2.7$ is mixed with water to produce **20% water content** and compacted to produce a cylindrical sample of 40 mm diameter and 80mm long with 5% air content. Calculate the following:

(a) Void ratio of the sample

(b) Mass of dry soil that will be required to be mixed with water

(c) Dry, saturated and moist unit weights

(d) The amount of water to be added for full saturation of the soil

Given:

$$G_s = 2.7$$

$$m = 20\%$$

$$V = \frac{\pi d^2 h}{4}$$

$$d = 40 \times 10^{-3} \text{ m}$$

$$h = 80 \times 10^{-3} \text{ m}$$

$$N_a = 5\%$$

Solution:

The volume of the sample is

$$V = \frac{\pi d^2 h}{4}$$

$$V = \frac{\pi \times (40 \times 10^{-3} \text{ m})^2 \times 80 \times 10^{-3} \text{ m}}{4} = \frac{\pi \times 1600 \times 10^{-6} \text{ m}^2 \times 80 \times 10^{-3} \text{ m}}{4}$$

$$= \pi \times 400 \times 80 \times 10^{-9} \text{ m}^3$$

$$V = 32000 \times \pi \times 10^{-9} \text{ m}^3 = 1.0 \times 10^{-5} \times 10^{-9} \text{ m}^3 = 1.0 \times 10^{-4} \text{ m}^3$$

$$V = 1.0 \times 10^{-4} \text{ m}^3$$

(a) The void ratio:

Air content is related to the degree of saturation by the relationship

$$N_a = n(1 - S_r)$$

such that

$$S_r = 1 - \left(\frac{N_a}{n} \right); n = \frac{e}{1 + e}$$

Therefore

$$S_r = 1 - N_a \left(\frac{1+e}{e} \right) \quad (1)$$

Again, the degree of saturation is related to both the moisture content and the specific gravity by the expression

$$S_r = \frac{mG_s}{e} \quad (2)$$

Upon equating equations (1) and (2) above and eventually making the subject the only unknown quantity e , we obtain

$$\frac{mG_s}{e} = 1 - N_a \left(\frac{1+e}{e} \right) \Rightarrow G_s = e - N_a(1+e) \quad (3)$$

which finally simplifies to an expression

$$e = \frac{mG_s + N_a}{1 - N_a}$$

Therefore

$$e = \frac{(0.2 \times 2.7) + 0.05}{1 - 0.05} = \frac{0.54 + 0.05}{0.95} = \frac{0.59}{0.95} = 0.621$$

$$\mathbf{e = 0.621}$$

(b) Dry mass of sample:

$$\rho_d = \frac{M_s}{V}$$

$$\rho_d = \frac{G_s \rho_w}{1+e}$$

$$\Rightarrow \frac{M_s}{V} = \frac{G_s \rho_w}{1+e} \Rightarrow M_s = \frac{V G_s \rho_w}{1+e}$$

$$M_s = \frac{1.0 \times 10^{-4} \times 2.7 \times 1000}{1 + 0.621} = \frac{1.0 \times 10^{-1} \times 2.7}{1.621} = \frac{2.7 \times 10^{-1}}{1.621} = \frac{0.27}{1.621} = 0.166 \text{ kg}$$

$$\mathbf{M_s = 0.166 kg}$$

(e) Dry, saturated and moist unit weights

Dry unit weight of sample:

$$\gamma_d = \frac{G_s \gamma_w}{1 + e}$$
$$\gamma_d = \frac{2.7 \times 10 \text{ kN/m}^3}{1 + 0.621} = \frac{27 \text{ kN/m}^3}{1.621} = 16.66 \text{ kN/m}^3$$
$$\gamma_d = \mathbf{16.66 \text{ kN/m}^3}$$

Saturated unit weight

$$\gamma_{sat} = \frac{(G_s + e) \gamma_w}{1 + e}$$
$$\gamma_{sat} = \frac{(2.7 + 0.621) \times 10 \text{ kN/m}^3}{1 + 0.621} = \frac{3.321 \times 10 \text{ kN/m}^3}{1.621} = \frac{33.21 \text{ kN/m}^3}{1.621}$$
$$= \mathbf{20.49 \text{ kN/m}^3}$$
$$\gamma_{sat} = \mathbf{20.49 \text{ kN/m}^3}$$

Moist unit weight:

$$\gamma_{moist} = \gamma = \frac{(1 + m) G_s \gamma_w}{1 + e}$$
$$\gamma = \frac{(1 + 0.20) 2.7 \times 10 \text{ kN/m}^3}{1 + 0.621} = \frac{1.20 \times 27 \text{ kN/m}^3}{1.621} = \frac{32.40 \text{ kN/m}^3}{1.621}$$
$$= 19.99 \text{ kN/m}^3$$
$$\gamma = \mathbf{19.99 \text{ kN/m}^3}$$

(d) Amount of water to be added for full saturation of the soil:

We have already found from above that

$$\gamma = 19.99kN/m^3 \Rightarrow \rho = 1999kg/m^3$$

$$\gamma_{sat} = 20.49kN/m^3 \Rightarrow \rho_{sat} = 2049kg/m^3$$

Therefore, mass of water added per cubic meter is

$$M_w = \rho_{sat} - \rho = 2049kg/m^3 - 1999kg/m^3 = 50kg/m^3$$

Note the simple trick in this question that the value calculated above, represents the amount of water that would be required to be added to $1m^3$, in order to make this type of soil saturate. However, in our current situation, **the volume is NOT $1m^3$** but $1.0 \times 10^{-4}m^3$ as per the above calculations. So under this situation therefore, we eventually have to obtain our solution using our popular mathematical procedure that:

$$\text{if } M_w = 50kg \quad \text{when } V = 1m^3$$

then;

$$\text{what is } M_w = ? \quad \text{when } V = 1.0 \times 10^{-4}m^3$$

$$\Rightarrow M_w = \frac{50kg \times 1.0 \times 10^{-4}m^3}{1m^3} = 0.005kg = 5.0g$$

Therefore

$$M_w = 5.0g$$

Alternatively:

We can compare the amount of moisture contents (original moisture content and moisture content at full saturation) since at full saturation $e = mG_s$ so that

$$m = \frac{e}{G_s}$$

$$m = \frac{0.621}{2.7} \times 100\% = \frac{0.621}{2.7} \times 100\% = 23\%$$

$$m = 23\%$$

Hence, water is needed to raise the moisture content from 20% to 23%. Now

Since

$$M_s = 0.166kg$$

then mass of water at partial saturation or mass of water before full saturation is

$$M_{w1} = m_1 M_s = 0.20 M_s$$

and the mass of water after full saturation is

$$M_{w2} = m_2 M_s = 0.23 M_s$$

Therefore the amount of water to be added for full saturation of the soil is

$$M_w = 0.23 M_s - 0.20 M_s$$

$$M_w = 0.03 M_s = 0.03 \times 0.166kg = 0.00498kg$$

$$M_w = 4.98g$$

Example 23:

Dry soil with $G_s = 2.71$ is mixed with 16% of water by weight and compacted to produce a cylindrical sample of 36 mm diameter and 76mm long with 6% air content. Calculate the mass of the mixed soil (i.e. the combined mass of water and soil) that will be required and the void ratio of the sample.

$$[M = 177.274g; e = 0.5251]$$

Given:

$$G_s = 2.71$$

$$m = 16\%$$

$$d = 38 \times 10^{-3}m$$

$$h = 76 \times 10^{-3}m$$

$$N_a = 6\%$$

SOLUTION:

The volume of the sample is

$$V = \frac{\pi d^2 h}{4}$$

and the mass of dry soil is

$$M_s = V \left(\frac{G_s \rho_w}{1 + e} \right) = \frac{\pi d^2 h}{4} \left(\frac{G_s \rho_w}{1 + e} \right)$$

whereas the mass of water is

$$M_w = m M_s$$

So that the total mass is

$$M = M_s + M_w$$

Up to this point, the equations necessary for calculating the value of M contain only one unknown which is the void ratio e , and so before we continue, we will need to get this value beforehand

Now, from the relationships $S_r = m G_s / e$; $S_r = 1 - (N_a / n)$ and $e = n / (1 - n)$; we can deduce that (as we did in our previous problem above)

$$e = \frac{m G_s + N_a}{1 - N_a}$$

when S_r is eliminated from these equations

Therefore

$$e = \frac{m G_s + N_a}{1 - N_a} = \frac{0.16(2.71) + 0.06}{1 - 0.06} = \frac{0.4336 + 0.06}{0.94} = \frac{0.4936}{0.94} = 0.5251$$

$$\mathbf{e = 0.5251}$$

The volume of the sample is

$$\begin{aligned} V &= \frac{\pi d^2 h}{4} = \frac{1}{4} \pi (38 \times 10^{-3} \text{ m})^2 (76 \times 10^{-3} \text{ m}) = 2.7436 \pi \times 10^{-2} \text{ m}^3 \\ &= 8.6 \times 10^{-5} \text{ m}^3 \end{aligned}$$

$$V = 8.6 \times 10^{-5} m^3$$

Then;

$$M_s = V \left(\frac{G_s \rho_w}{1 + e} \right) = 8.6 \times 10^{-5} m^3 \left(\frac{2.71 \times 1000 kg m^{-3}}{1 + 0.5251} \right) = \frac{0.23306 kg}{1.5251}$$

$$= 0.15281 kg$$

$$M_s = 0.15281 kg$$

The mass of water is,

$$M_w = m M_s = 0.16 \times 0.15281 kg = 0.02445 kg$$

$$M_w = 0.02445 kg$$

So, the total mass is

$$M = M_s + M_w = 0.15281 kg + 0.02445 kg = 0.17727 kg$$

Therefore

$$M = 177.27 g; e = 0.5251$$

Example 24:

A moist clayey soil has initial void ratio of 1.5, dry mass of 80gm, and specific gravity of solid particles of 2.5. The sample is exposed to atmosphere so that the sample volume decreases to one half of its initial volume . Calculate the following:

a) The new void ratio.

b) Mass of water if degree of saturation became 25 %.

$$[e_f = 0.25; M_w = 2g]$$

Given:

$$e_i = 1.5$$

$$M_s = 80g$$

$$G_s = 2.5$$

$$V_i = V$$

$$V_f = \frac{1}{2}V$$

$$S_r = 25\%$$

SOLUTION:

Note: Depending on your approach while solving this problem, you can also calculate the initial volume V_i but solving this problem does not necessarily require you to know the exact value of V_i

You can therefore find the initial volume V_i using the relationships

$$\rho_d = \frac{M_s}{V_i} = \frac{G_s \rho_w}{1 + e_i} \Rightarrow \frac{M_s}{V_i} = \frac{G_s \rho_w}{1 + e_i}$$

From which we finally get

$$V_i = \frac{M_s(1 + e_i)}{G_s \rho_w}$$

$$V_i = \frac{80.0g(1 + 1.5)}{2.50 \times 1.0g/cm^3} = \frac{200}{2.50} = 80cm^3$$

And the final volume is therefore

$$V_f = \frac{1}{2}V_i = 40m^3$$

The initial void ratio is 1.5. Therefore, the volume of solids (which normally does not get affected by the compaction or shrinkage processes), can be calculated as follows:

$$e_i = \frac{V_v}{V_s} = 1.5 \Rightarrow \frac{V_i - V_s}{V_s} = 1.5 \Rightarrow V_i = 2.5V_s$$

It is also given from the question that

$$V_f = \frac{1}{2} V_i = \frac{1}{2} (2.5V_s) \Rightarrow V_f = 1.25V_s$$

(a) The final (new void ratio) is therefore

$$e_f = \frac{V_f - V_s}{V_s} = \frac{1.25V_s - V_s}{V_s} = 0.25$$

$$e_f = 0.25$$

(b) the mass of water when $S_r = 25\%$ is therefore

$$S_r e = mG_s = \frac{M_w}{M_s} G_s \Rightarrow M_w = \frac{S_r e M_s}{G_s}$$

$$M_w = \frac{0.25 \times 0.25 \times 80.0g}{2.50} = 2.0g$$

$$M_w = 2.0g$$

Example 25:

By using the details provided and/ data obtained by calculation in Example 24 above, determine:

- (a) The total volume as well as the volume of voids and of solids before and after exposure of the soil to the atmosphere.
- (b) How do the values in (a) above compare? Why?

Given:

$$e_i = 1.5$$

$$e_f = 0.25$$

$$V_i = 2.5V_s$$

$$V_f = 1.25V_s$$

$$M_s = 80g$$

$$G_s = 2.5$$

SOLUTION:

The volume of solids

(a) Case 1: Before exposure to the atmosphere

$$V_s = \frac{M_s}{G_s \rho_w} = \frac{80g}{2.5 \times 1gcm^{-3}} = 32cm^3$$

$$V_i = 2.5V_s \Rightarrow V_i = 2.5 \times 32cm^3 = 80cm^3$$

$$V_i = V_s + V_v \Rightarrow 80cm^3 = 32cm^3 + V_v \Rightarrow V_v = 48cm^3$$

Therefore, before exposure to the atmosphere:

$$V = 80cm^3; V_s = 32cm^3; V_v = 48cm^3$$

Check here to see that the initial void ratio $V_v/V_s = 48/32 = 1.5$ as originally stated in the question

Case 2: After exposure to the atmosphere

The final volume is

$$V_f = \frac{1}{2} \times 80cm^3 = 40cm^3$$

However, under this case, there is a change of void ratio only attributed by the changes in the amount of water formerly present in the soil but which later escaped into the atmosphere through radiation; thereby affecting the void ratio but without having any effect on the volume of solids. Therefore

$$V_v = V_f - V_s = 40cm^3 - 32cm^3 = 8cm^3$$

Hence after exposure to the atmosphere:

$$V = 40cm^3; V_s = 32cm^3; V_v = 8cm^3$$

Check again here to see that the final void ratio $V_v/V_s = 8/32 = 0.25$ as previously calculated in question number 24

- (b) The value of the volume of solids remains unaffected in both cases whereas the rest of the values change. This is because the volume of solids are normally unaffected by such a situation as exposure of the soil to the atmosphere whereas the rest of its parameters do

Alternatively:

We can use the **total volumes** (before and after exposure), to calculate the value of V_s or V_v in the following manner; that is:

(a) Before exposure

$$\text{if } V_s = 1 \quad \text{when } V = 1 + e_i = 2.5$$

then;

$$\text{what is } V_s? \quad \text{when } V = 80\text{cm}^3$$

$$V_s = \frac{1 \times 80}{1 + e_i} = \frac{80}{1 + 1.50} = 32\text{cm}^3$$

$$\Rightarrow V_s = 32\text{cm}^3; V_v = 80\text{cm}^3 - 32\text{cm}^3 = 48\text{cm}^3$$

(b) After exposure

$$\text{if } V_s = 1 \quad \text{when } V = 1 + e_f = 1.25$$

then;

$$\text{what is } V_s? \quad \text{when } V = 40\text{cm}^3$$

$$V_s = \frac{1 \times 40}{1 + e_i} = \frac{40}{1 + 0.25} = 32\text{cm}^3$$

$$\Rightarrow V_s = 32\text{cm}^3; V_v = 40\text{cm}^3 - 32\text{cm}^3 = 8\text{cm}^3$$

Example 26:

A 110 cm^3 sample of wet soil has a mass of 212 g and a degree of saturation of 100%. When oven-dried, the mass is 162 g. Determine its dry density, water content, void ratio, and the specific gravity of the soil particles.

$$[\rho_d = 1472.72 \text{ kg/m}^3; m = 30.86\%; e = 0.83; G_s = 2.70]$$

Given:

$$V = 110 \text{ cm}^3$$

$$M = 212 \text{ g}$$

$$M_s = 162 \text{ g}$$

$$S_r = 100\% \text{ or } S_r = 1$$

SOLUTION:

Dry density:

$$\rho_d = \frac{M_s}{V}$$

$$\rho_d = \frac{162.0 \text{ g}}{110 \text{ cm}^3} = 1472.72 \text{ kg/m}^3$$

$$\rho_d = 1472.72 \text{ kg/m}^3$$

Water content:

$$m = \frac{M_w}{M_s}$$

$$m = \frac{M_w}{M_s} = \frac{212.0 - 162.0}{162.0} = \frac{50}{162} = 0.3086 = 30.86\%$$

$$m = 30.86\%$$

Void ratio and Specific gravity:

From the relationship (for dry density)

$$\rho_d = \frac{M_s}{V} = \frac{G_s \rho_w}{1 + e}$$

$$\Rightarrow \frac{162}{110} = \frac{1 \times G_s}{1 + e} \quad (i)$$

Also; from the relationship for saturated density

$$\rho_{sat} = \frac{M}{V} = \frac{(G_s + e) \rho_w}{1 + e}$$

$$\frac{212}{110} = \frac{1 \times G_s}{1 + e} + \frac{1 \times e}{1 + e} \quad (ii)$$

This is a simultaneous equation in e and G_s . Subtracting equation (i) from (ii), we get

$$\frac{212}{110} - \frac{162}{110} = \left(\frac{G_s}{1 + e} + \frac{e}{1 + e} \right) - \frac{G_s}{1 + e} \Rightarrow \frac{50}{110} = \frac{e}{1 + e} \Rightarrow \frac{5}{11} = \frac{e}{1 + e}$$

$$\Rightarrow 5(1 + e) = 11e \Rightarrow 6e = 5 \Rightarrow e = \frac{5}{6} = 0.83$$

$$\mathbf{e = 0.83}$$

Going back to any of our equations above and substitute this value of e we get

$$\frac{162}{110} = \frac{G_s}{1 + \frac{5}{6}} \Rightarrow \frac{81}{55} = \frac{6G_s}{6 + 5} \Rightarrow G_s = \frac{81 \times 11}{6 \times 55} = \frac{27}{2 \times 5} = 2.70$$

Therefore

$$\mathbf{G_s = 2.70; e = 0.83}$$

Note: This part of the problem is similar to sub-part (a) in **EXAMPLE 21** above.

Alternatively:

You can make G_s a subject in relationship (i) above

$$G_s = \frac{M_s(1 + e)}{\rho_w V}$$

$$G_s = \frac{162(1 + e)}{1 \times 110}$$

Then thereafter, substitute this value of G_s in the expression $e = mG_s$ to get

$$e = m \left(\frac{162(1 + e)}{1 \times 110} \right) \Rightarrow e = 0.16 \left(\frac{162(1 + e)}{1 \times 110} \right)$$

With only one unknown we find this value of e , and then we go back to the relationship

$$G_s = \frac{162(1 + e)}{1 \times 110}$$

where we can insert in this value, and obtain the remaining value G_s

Example 27:

A sample of moist soil was found to have the following characteristics:

- Volume 0.01456 m^3 (as sampled)
- Mass 25.74 kg (as sampled)
- 22.10 kg (after oven drying)
- Specific gravity of solids: 2.69

a) Find the density, dry unit weight, void ratio, porosity, degree of saturation for the soil.

b) What would be the moist unit weight when the degree of saturation is 80%?

$$[\rho = 1767.86 \text{ kg/m}^3; \gamma_d = 15.18 \text{ kN/m}^3; e = 0.7722; n = 0.4356; S_r = 57.37\%; \gamma_{80} = 18.66 \text{ kN/m}^3]$$

Given:

$$V = 0.01456 \text{ m}^3$$

$$M = 25.74 \text{ kg}$$

$$M_s = 22.10 \text{ kg}$$

$$G_s = 2.69$$

SOLUTION:

- a) Density, dry unit weight, void ratio, porosity, degree of saturation for the soil.

Density:

$$\rho = \frac{M}{V}$$
$$\rho = \frac{25.74kg}{0.01456 m^3} = 1767.86kgm^3$$
$$\rho = 1767.86kgm^3$$

Dry unit weight:

$$\gamma_d = \rho_d g = \frac{M_s g}{V}$$
$$\gamma_d = \frac{22.10kg \times 10ms^2}{0.01456 m^3} = 15.18kNm^{-3}$$
$$\gamma_d = 15.18kNm^{-3}$$

Void ratio:

$$e = \left(\frac{G_s \gamma_w}{\gamma_d} \right) - 1$$
$$e = \left(\frac{2.69 \times 10kNm^{-3}}{15.18kNm^{-3}} \right) - 1 = 0.7722$$

Porosity:

$$n = \frac{e}{1 + e}$$
$$n = \frac{0.7722}{1 + 0.7722} = 0.4356$$
$$n = 0.4356$$

Degree of saturation:

$$S_r = \frac{mG_s}{e} = \left(\frac{M - M_s}{M_s} \right) \left(\frac{G_s}{e} \right)$$
$$S_r = \left(\frac{25.74 \text{ kg} - 22.10 \text{ kg}}{22.10 \text{ kg}} \right) \left(\frac{2.69}{0.7722} \right) = \left(\frac{3.64 \text{ kg}}{22.10 \text{ kg}} \right) \left(\frac{2.69}{0.7722} \right) = \frac{9.7916}{17.06562}$$
$$= 0.5737 = 57.37\%$$
$$S_r = 57.37\%$$

b) The moist unit weight when the degree of saturation is 80%

$$\gamma(S_r = 80\%) = \frac{(G_s + 0.8e)\gamma_w}{1 + e} = \gamma_d + 0.8n\gamma_w$$
$$\gamma = \gamma_d + 0.8n\gamma_w$$
$$\gamma = 15.18 \text{ kNm}^{-3} + 0.8(0.4356)(10 \text{ kNm}^{-3}) = 15.18 \text{ kNm}^{-3} + 3.4848 \text{ kNm}^{-3}$$
$$= 18.66 \text{ kNm}^{-3}$$
$$\gamma = 18.66 \text{ kNm}^{-3}$$

Example 28

Given that mass of wet sample = 254g, void ratio = 0.6133, volume of air = 1.9 cm³, mass of solid = 210g. Determine the degree of saturation, air content and the dry unit weight.

$$[S_r = 95.86\%; N_a = 1.574\%; \gamma_d = 17.40 \text{ kN/m}^3]$$

Given:

$$M = 254 \text{ g}$$

$$e = 0.6133$$

$$V_a = 1.9 \text{ cm}^3$$

$$M_s = 210g$$

SOLUTION:

$$S_r = \frac{V_w}{V_v}; V_w = \frac{M_w}{\rho_w} = \frac{M - M_s}{\rho_w}; V_v = V_w + V_a = \frac{M - M_s}{\rho_w} + V_a \Rightarrow V_w + V_a = \frac{M - M_s + \rho_w V_a}{\rho_w}$$

$$\Rightarrow S_r = \frac{\left(\frac{M - M_s}{\rho_w}\right)}{V_w + V_a} = \frac{\left(\frac{M - M_s}{\rho_w}\right)}{\left(\frac{M - M_s + \rho_w V_a}{\rho_w}\right)} = \frac{M - M_s}{M - M_s + \rho_w V_a}$$

$$S_r = \frac{M - M_s}{M - M_s + \rho_w V_a}$$

$$S_r = \frac{254g - 210g}{254g - 210g + 1gcm^{-3}(1.9cm^{-3})} = 0.9586 = 95.86\%$$

$$S_r = \mathbf{95.86\%}$$

Alternative way1:

First, find the value of the total volume and thereafter, that of the specific gravity.

$$V = V_s + V_v = V_s + V_w + V_a$$

$$V_a = 1.90cm^3$$

$$V_w = \frac{M_w}{\rho_w} = \frac{254g - 210g}{1gcm^{-3}} = 44cm^3$$

$$V_w = \mathbf{44cm^3}$$

Also,

$$\frac{V_v}{V_s} = e; V_v = V_w + V_a; \Rightarrow V_s = \frac{V_v}{e} = \frac{V_w + V_a}{e}$$

$$\Rightarrow V_s = \frac{V_w + V_a}{e}$$

$$V_s = \frac{44 + 1.90}{0.6133} = 74.84 \text{ cm}^{-3}$$

$$\mathbf{V_s = 74.84 \text{ cm}^3}$$

Then, the total volume is

$$V = V_s + V_w + V_a = 1.90 \text{ cm}^3 + 44 \text{ cm}^{-3} + 74.84 \text{ cm}^3 = 120.74 \text{ cm}^3$$

$$\mathbf{V = 120.74 \text{ cm}^3}$$

The dry unit weight is given by the relationship:

$$\rho_d = \frac{G_s \rho_w}{1 + e} = \frac{M_s}{V} \Rightarrow G_s = \frac{M_s}{V} \left(\frac{1 + e}{\rho_w} \right)$$

$$G_s = \frac{210 \text{ g}}{120.74 \text{ cm}^3} \left(\frac{1 + 0.6133}{1 \text{ g cm}^{-3}} \right) = 2.806$$

$$\mathbf{G_s = 2.806}$$

Now, having these two values known, the degree of saturation can then be calculated using the popular relationship $S_r e = m G_s$, that is:

$$S_r = \frac{m G_s}{e}; \quad m = \frac{M - M_s}{M_s} \Rightarrow S_r = \left(\frac{M - M_s}{M_s} \right) \left(\frac{G_s}{e} \right)$$

$$S_r = \left(\frac{254 - 210}{210} \right) \left(\frac{2.806}{0.6133} \right) = \left(\frac{44}{210} \right) \left(\frac{2.81}{0.6133} \right) = 0.9278 = 96.0\%$$

$$\mathbf{S_r = 95.86\%}$$

Alternative way2:

With a known total volume of soil xxx, the equations below to can be used to determine the degree of saturation of the soil

$$N_a = \frac{V_a}{V}$$

Also;

$$N_a = n(1 - S_r) = \frac{e}{1 + e}(1 - S_r) \Rightarrow \frac{V_a}{V} = \frac{e}{1 + e}(1 - S_r) \Rightarrow S_r$$

$$= 1 - \left(\frac{V_a}{V}\right)\left(\frac{1 + e}{e}\right)$$

From above, $V = 120.74cm^3$

$$\Rightarrow S_r = 1 - \left(\frac{1.90cm^3}{120.74cm^3}\right)\left(\frac{1 + 0.6133}{0.6133}\right) = 1 - \frac{3.06527cm^3}{74.05m^3} = 0.9586$$

$$= 95.86\%$$

$$S_r = 95.86\%$$

Air content:

$$N_a = n(1 - S_r) = \frac{e}{1 + e}(1 - S_r)$$

$$N_a = \frac{0.6133}{1.6133}(1 - 0.9586) = 0.015738 = 1.574\%$$

$$N_a = 1.574\%$$

Alternatively:

$$N_a = \frac{V_a}{V}$$

$$N_a = \frac{1.9cm^-}{120.74cm^3} \times 100\% = 1.574\%$$

$$N_a = 1.574\%$$

Dry unit weight:

At this stage, we already know the total volume as $V = 124.04cm^3$ because it was calculated from above.

The dry unit weight is therefore

$$\gamma_d = \frac{M_s}{V} = \frac{210g}{120.74cm^3} = 1.7393gcm^3 = 17.40kNm^{-3}$$

$$\gamma_d = 17.40kNm^{-3}$$

Example 29:

A soil specimen is 38 mm in diameter and 76 mm long and weighs 168 g in its natural condition. When dried completely in an oven the specimen weighs 130.5 g. If $G_s = 2.73$, what is the degree of saturation of the specimen? [$S_r = 97.59\%$]

Given:

Specimen diameter, $d = 38mm$

Specimen height, $h = 76mm$

$M = 168g$

$M_s = 130.5g$

$G_s = 2.73$

SOLUTION:

The degree of saturation is given by the relationship

$$S = \frac{V_w}{V_v}$$

Therefore, in this problem, we will need to know both the volumes of voids (water or air) and solids, as well as the total volume of the sample

The total volume of the ***cylindrical sample*** is given by the relationship

$$V = \pi r^2 h; \quad r = \frac{d}{2} \Rightarrow V = \frac{1}{4} \pi d^2 h$$

$$V = \frac{1}{4} \pi (38 \times 10^{-3} \text{ m})^2 (76 \times 10^{-3} \text{ m}) = 8.6227 \times 10^{-5} \text{ m}^3$$

$$\mathbf{V = 8.6227 \times 10^{-5} m^3}$$

The volume of water in the sample is

$$V_w = \frac{M_w}{\rho_w} = \frac{M - M_s}{\rho_w}$$

$$V_w = \frac{0.168 \text{ kg} - 0.1305 \text{ kg}}{1000 \text{ kg m}^{-3}} = \frac{0.0375}{1000} \text{ m}^3 = 3.75 \times 10^{-5} \text{ m}^3$$

$$\mathbf{V_w = 3.75 \times 10^{-5} m^3}$$

The volume of solids is

$$V_s = \frac{M_s}{G_s \rho_w} = \frac{0.1305 \text{ kg}}{2.73(1000 \text{ kg m}^{-3})} = 4.78 \times 10^{-5} \text{ m}^3$$

$$\mathbf{V_s = 4.78 \times 10^{-5} m^3}$$

The volume of voids is therefore $V_v = V - V_s$ and finally:

$$S_r = \frac{V_w}{V_v} = \frac{V_w}{V - V_s}$$

$$S_r = \frac{3.75 \times 10^{-5} \text{ m}^3}{8.6227 \times 10^{-5} \text{ m}^3 - 4.78 \times 10^{-5} \text{ m}^3} = \frac{3.75}{3.8427} = 0.97586 = 97.59\%$$

$$\mathbf{S_r = 97.59\%}$$

Alternatively:

$$S_r = \frac{m G_s}{e}$$

$$e = \frac{G_s \rho_w}{\rho_d} - 1; \rho_d = \frac{M_s}{V} \Rightarrow e = \frac{V G_s \rho_w}{M_s} - 1 = \frac{V G_s \rho_w - M_s}{M_s}$$

$$e = \frac{V G_s \rho_w - M_s}{M_s}; m = \frac{M - M_s}{M_s}$$

$$S_r = \frac{m G_s}{e} \Rightarrow S_r = \left(\frac{M - M_s}{M_s} \right) \left(\frac{M_s}{V G_s \rho_w - M_s} \right) G_s = \frac{(M - M_s) G_s}{V G_s \rho_w - M_s}$$

Therefore

$$S_r = \frac{(M - M_s) G_s}{V G_s \rho_w - M_s}$$

$$S_r = \frac{(0.168kg - 0.1305kg)(2.73)}{(8.6227 \times 10^{-5} m^3) 2.73 \times 1000 kg m^{-3} - 0.1305kg}$$

$$= \frac{0.0375kg(2.73)}{0.2354kg - 0.1305kg} = \frac{1.02375kg}{0.1049kg} = 0.9759 = 97.59\%$$

$$S_r = 97.59\%$$

Note: It is not necessary that you should use this approach of directly substituting in the given values in this later expression of S_r because it is also possible that you can **separately, first calculate each unknown individual value required in getting S_r , as follows:**

$$e = \frac{G_s \rho_w}{\rho_d} - 1;$$

$$m = \frac{M - M_s}{M_s}$$

Then after getting e and m , you finally substitute them in the expression $S_r = m G_s / e$

Example 30:

A soil sample has a dry weight of 2.42kg and its specific gravity is 2.70. What is the volume of solids in the soil? [$V_s = 8.963 \times 10^{-4} m^3$ or $896.3 cm^3$]

Hint:

$$V_s = W_s / G_s \rho_w$$

Example 31:

A 100% saturated soil has a density of 2050 kg/m³ and water content of 25%. What are the dry density and density of the solids (ρ_s) of the soil?

$$[\rho_d = 1640 kg/m^3; \rho_s = 2779.66 kg/m^3]$$

Given:

$$\rho = 2050 kg/m^3$$

$$m = 25\%$$

$$S_r = 1 \text{ (Soil fully saturated)}$$

SOLUTION:

$$\rho_d = \rho / (1 + m)$$

$$\rho_d = \frac{2050 kgm^{-3}}{1 + 0.25} = 1640 kgm^{-3}$$

$$\rho_d = 1640 kgm^{-3}$$

In order to get the value of the density of solids, we first need to know the value of the specific gravity. We can calculate this value using the two equations below:

$$S_r = 1; S_r e = m G_s \Rightarrow e = m G_s$$

$$\rho_d = \frac{G_s \rho_w}{1 + e} \Rightarrow \rho_d = \frac{G_s \rho_w}{1 + m G_s}$$

so that

$$G_s = \frac{\rho_d}{\rho_w - m \rho_d}$$

Finally $\rho_s = G_s \rho_w$

$$\rho_s = \frac{\rho_d \rho_w}{\rho_w - m \rho_d}$$

$$\rho_s = \frac{(1640 \text{ kg m}^{-3})(1000 \text{ kg m}^{-3})}{1000 \text{ kg m}^{-3} - 0.25(1640 \text{ kg m}^{-3})} = \frac{(1640000 \text{ kg m}^{-3})(1 \text{ kg m}^{-3})}{590 \text{ kg m}^{-3}} = 2779.66 \text{ kg m}^{-3}$$

$$\rho_s = 2779.66 \text{ kg m}^{-3}$$

Example 32:

What is the saturated density of soil whose dry density is 1700 kg/m^3 ? Assume $\rho_s = 2710 \text{ kg/m}^3$

$$[\rho_{sat} = 2072.69 \text{ kg/m}^3]$$

Given:

$$\rho_d = 1700 \text{ kg/m}^3$$

$$\rho_s = G_s \rho_w = 2710 \text{ kg/m}^3 \Rightarrow G_s = 2.71$$

$$S_r = 1 \text{ (Complete saturation)}$$

SOLUTION:

$$\rho_{sat} = \rho_d + n\rho_w$$

$$\rho_{sat} = 1700 \text{ kg/m}^3 + \frac{e\rho_w}{1+e}$$

Now, since e is not known, we then express it in terms of the other known or given parameters, as the dry density is known. Therefore from

$$\rho_d = \frac{G_s\rho_w}{1+e} \Rightarrow e = \frac{G_s\rho_w - \rho_d}{\rho_d} = \frac{G_s\rho_w}{\rho_d} - 1$$

Therefore

$$\rho_{sat} = 1700 \text{ kgm}^{-3} + \frac{e\rho_w}{1+e}$$

Consider the second term of ρ_{sat} above, which is

$$\begin{aligned} \frac{e\rho_w}{1+e} &= \left(\frac{G_s\rho_w - \rho_d}{\rho_d} \right) \left(\frac{\rho_w\rho_d}{G_s\rho_w} \right) = \frac{G_s\rho_w - \rho_d}{G_s} \\ \Rightarrow \frac{e\rho_w}{1+e} &= \frac{2710\text{kgm}^{-3} - 1700 \text{ kgm}^{-3}}{2.71} = \frac{1010\text{kgm}^{-3}}{2.71} = 372.69\text{kgm}^{-3} \end{aligned}$$

Hence

$$\frac{e\rho_w}{1+e} = 372.69\text{kgm}^{-3}$$

Therefore

$$\begin{aligned} \Rightarrow \rho_{sat} &= 1700 \text{ kgm}^{-3} + \frac{e\rho_w}{1+e} = 1700 \text{ kgm}^{-3} + 372.69\text{kgm}^{-3} \\ &= 2072.69\text{kgm}^{-3} \\ \rho_{sat} &= \mathbf{2072.69\text{kgm}^{-3}} \end{aligned}$$

Example 33:

The volume of water in a sample of moist soil is $0.056m^3$. The volume of solids V_s is $0.28m^3$. Given that the density of soil solids ρ_s is $2590kgm^{-3}$, find the water content of the sample. [$m = 7.72\%$]

Given:

$$V_w = 0.056m^3$$

$$V_s = 0.28m^3$$

$$\rho_s = 2590kgm^{-3} \Rightarrow G_s = 2.59$$

SOLUTION:

$$m = \frac{M_w}{M_s}$$

$$M_s = V_s G_s \rho_w; \quad M_w = V_w \rho_w \Rightarrow m = \frac{V_w \rho_w}{V_s G_s \rho_w}$$

$$m = \frac{V_w}{V_s G_s}$$

$$m = \frac{0.056m^3}{(0.28m^3)(2.59)} = 0.0772 = 7.72\%$$

$$m = 7.72\%$$

Example 34:

A soil sample is contained in a tube. The tube has a weight of 1.15kg. The weight of soil and the sampler tube is 2.07kg. After extruding the sample from the tube and oven drying, it was found to have a dry weight of 0.27kg. Determine:

- (i) The moisture content of the sample [$m = 240.74\%$]

(ii) If the soil's theoretical saturation moisture content is 262.3%, what is the percentage degree of saturation of the sample as per the value of the moisture content obtained in (i) above? [$S_r = 91.78\%$]

(iii) A moist sample specimen of soil has a volume of 0.0011 cubic metres and weighs 1.20kg when in a moist state and it weighed 1.0kg after oven-drying. Determine both its moist and dry unit weights [$\gamma = 10910N/m^3$ or $10.91kN/m^3$; $\gamma_d = 9091N/m^3$ or $9.091kN/m^3$]

Given:

Weight of sampler, $W_{TS} = 1.15kg$

Weight of soil and sampler, $W_{STS} = 2.07kg$

$M_s = 0.27kg$

$M = 2.07kg - 1.15kg = 0.92kg$

SOLUTION:

(i) **Moisture content:**

$$m = \frac{M_w}{M_s}$$

$$m = \frac{0.92kg - 0.27kg}{0.27kg} = \frac{0.65}{0.27} = 2.4074 = 240.74\%$$

$$m = 240.74\%$$

(ii) **Percentage degree of saturation of the sample when $m = 240.74\%$**

To solve this problem, we use our usual mathematical trick, that is:

if $S_r = 100\%$ when $m = 262.30\%$;

then; what is S_r ? when $m = 240.74\%$?

$$S_r = \frac{100\% \times 240.74\%}{262.30\%} = 91.78\%$$

$$S_r = 91.78\%$$

(iii) **Given information:**

$$V = 0.0011m^3$$

$$M = 1.20kg$$

$$M_s = 1.0kg$$

Moist unit weight:

$$\gamma = \frac{Mg}{V}$$

$$\gamma = \frac{1.20kg \times 10ms^{-2}}{0.0011m^3} = \frac{12.0kgms^{-2}}{0.0011m^3} = 10909Nm^{-3} = 10.91kNm^{-3}$$

$$\gamma = 10.91kNm^{-3}$$

Dry unit weight:

$$\gamma_d = \frac{M_s g}{V}$$

$$\gamma_d = \frac{1.0kg \times 10ms^{-2}}{0.0011m^3} = \frac{10.0kgms^{-2}}{0.0011m^3} = 9090.91Nm^{-3} = 9.091kNm^{-3}$$

$$\gamma_d = 9.091kNm^{-3}$$

Example 35:

A **saturated** soil sample weighs 184g and it weighed 126g after it was oven dried. If the volume of the sample is 105cc, calculate:

(a) the water content [$m_{sat} = 46.03\%$]

(b) The saturated unit weight [$\gamma_{sat} = 17.52kN/m^3$]

(c) The buoyant unit weight [$\gamma' = 7.52kN/m^3$]

[Hint: $\gamma' = \gamma_{sat} - \gamma_w$]

(d) The dry unit weight of the sample [$\gamma_d = 12.0kN/m^3$]

Given:

$$M = 184.0g = 0.184kg$$

$$M_s = 126g = 0.126kg$$

$$V = 105cm^3 = 1.05 \times 10^{-4}m^3$$

(a) **Moisture content**

$$m_{sat} = \frac{M_w}{M_s}$$

$$m = \frac{0.184kg - 0.126kg}{0.126kg} = \frac{0.058}{0.126} = 0.4603 = 46.03\%$$

$$m_{sat} = 46.03\%$$

(b) **Saturated unit weight**

$$\gamma_{sat} = \frac{Mg}{V}$$

$$\gamma_{sat} = \frac{0.184kg \times 10ms^{-2}}{1.05 \times 10^{-4}m^3} = 17523.8Nm^{-3} = 17.52kNm^{-3}$$

$$\gamma_{sat} = 17.52kNm^{-3}$$

(c) The buoyant unit weight, γ'

The buoyant unit weight is simply the difference between the saturated unit weight of soil and the unit weight of water. Therefore

$$\gamma' = \gamma_{sat} - \gamma_w$$

$$\gamma' = 17.52kNm^{-3} - 10.00kNm^{-3} = 7.52kNm^{-3}$$

$$\gamma' = 7.52kNm^{-3}$$

(d) The dry unit weight of the sample

$$\gamma_d = \frac{\gamma_{sat}}{1 + m_{sat}} = \frac{17.52kNm^{-3}}{1 + 0.4603} = 12.0kNm^{-3}$$

$$\gamma_d = 12.0kNm^{-3}$$

Example 36:

A sample of saturated clay was placed in a container and weighed. The weight was 6N. The clay and its container was placed in an oven and dried for 24 hours at 105°C. The weight was reduced to 5N. The weight of the container is 1N. If $G_s = 2.7$, determine:

(a). Water content.

(b). Void ratio.

(c). Saturated/ moist unit weight.

(d). Dry unit weight.

(e). Effective unit weight (Submerged unit weight).

$$[m = 25\%; e = 0.675; \gamma_{sat} = 20.15 \text{ kN/m}^3; \gamma_d = 16.12 \text{ kN/m}^3, \\ \gamma' = 10.15 \text{ kN/m}^3]$$

Given:

$$W_1 = 6N$$

$$W_2 = 5N$$

$$W_c = 1N$$

$$W = 5N \quad (W = W_1 - W_c = 6 - 1 = 5N)$$

$$W_s = 4N \quad (W_s = W_2 - W_c = 5 - 1 = 4N)$$

$$G_s = 2.70$$

$$S_r = 1; \text{(soil fully saturated)}$$

SOLUTION:

(a) Moisture content:

$$m = \frac{W_w}{W_s}$$

$$m = \frac{5N - 4N}{4N} = 0.25 = 25.0\%$$

$$m = 25\%$$

(b) Void ratio:

$$S_r = 1 \Rightarrow e = mG_s$$

$$e = 0.25 \times 2.70 = 0.675$$

$$\mathbf{e = 0.675}$$

(c) Saturated/ moist unit weight:

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e} =$$

$$\gamma_{sat} = \frac{(2.70 + 0.675)(10kNm^{-3})}{1 + 0.675} = \frac{33.75kNm^{-3}}{1.675} = 20.15kNm^{-3}$$

$$\mathbf{\gamma_{sat} = 20.15kNm^{-3}}$$

(d) Dry unit weight:

$$\gamma_d = \frac{G_s\gamma_w}{1 + e}$$

$$\gamma_d = \frac{2.70 \times 10kNm^{-3}}{1 + 0.675} = \frac{27.0kNm^{-3}}{1.675} = 16.12kNm^{-3}$$

$$\mathbf{\gamma_d = 16.12kNm^{-3}}$$

Alternatively:

Since the saturated unit weight is known, we can use the saturated unit weight to determine the dry unit weight as follows:

$$\gamma_{sat} = \gamma_d + n\gamma_w \Rightarrow \gamma_d = \gamma_{sat} - n\gamma_w = \gamma_{sat} - \frac{e\gamma_w}{1 + e}$$

$$\gamma_d = \gamma_{sat} - \frac{e\gamma_w}{1 + e}$$

From above, $\gamma_{sat} = 20.15kNm^{-3}$

Therefore we need to find the value of $e\gamma_w/(1 + e)$ and then subtract it from γ_{sat} to get the value of γ_d , that is:

$$\frac{e\gamma_w}{1+e} = \frac{0.675 \times 10.0 \text{ kNm}^{-3}}{1+0.675} = \frac{6.75 \text{ kNm}^{-3}}{1.675} = 4.02985 \text{ kNm}^{-3}$$

$$\frac{e\gamma_w}{1+e} = 4.02985 \text{ kNm}^{-3}$$

Hence

$$\gamma_d = \gamma_{sat} - \frac{e\gamma_w}{1+e} = 20.15 \text{ kNm}^{-3} - 4.02985 \text{ kNm}^{-3} = 16.12 \text{ kNm}^{-3}$$

$$\gamma_d = 16.12 \text{ kNm}^{-3}$$

(e) Effective unit weight (Submerged unit weight).

$$\gamma' = \gamma_{sat} - \gamma_w$$

$$\gamma' = 20.15 \text{ kNm}^{-3} - 10.0 \text{ kNm}^{-3} = 10.15 \text{ kNm}^{-3}$$

$$\gamma' = 10.15 \text{ kNm}^{-3}$$

Example 37:

Soil has been compacted in an embankment at a bulk unit weight of 2.15 t/m^3 and water content of 12%, the solid particles of soil having specific gravity of 2.65. Calculate the dry unit weight, degree of saturation, and air content.

$$\gamma_d = 19.20 \text{ kNm}^{-3}; S_r = 83.64\%; N_a = 4.51\%$$

Given:

$$\gamma = 2.15 \text{ t/m}^3 = 2150 \text{ kg/m}^3 \times 10 \text{ m/s}^2 = 21.5 \text{ kN/m}^3$$

$$m = 12\%$$

$$G_s = 2.65$$

SOLUTION:

Dry unit weight:

$$\gamma_d = \frac{\gamma}{1 + m}$$

$$\gamma_d = \frac{21.5 \text{ kNm}^{-3}}{1 + 0.12} = \frac{21.5 \text{ kNm}^{-3}}{1.12} = 19.20 \text{ kNm}^{-3}$$

$$\gamma_d = 19.20 \text{ kNm}^{-3}$$

Degree of saturation:

$$S_r = \frac{mG_s}{e}; \quad e = \left(\frac{G_s \gamma_w}{\gamma_d} \right) - 1 = \frac{G_s \gamma_w - \gamma_d}{\gamma_d}$$

$$\Rightarrow S_r = mG_s \left(\frac{\gamma_d}{G_s \gamma_w - \gamma_d} \right) = \frac{mG_s \gamma_d}{G_s \gamma_w - \gamma_d}$$

$$S_r = \frac{0.12 \times 2.65 \times 19.20 \text{ kNm}^{-3}}{2.65 \times 10.0 \text{ kNm}^{-3} - 19.20 \text{ kNm}^{-3}} = \frac{6.1056}{7.30} = 0.83638 = 83.64\%$$

$$S_r = 83.64\%$$

Air content:

$$N_a = n(1 - S_r); \quad n = \frac{e}{1 + e}; \quad e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{G_s \gamma_w - \gamma_d}{\gamma_d}$$

$$n = \frac{e}{1 + e} = \frac{\left(\frac{G_s \gamma_w - \gamma_d}{\gamma_d} \right)}{\frac{G_s \gamma_w}{\gamma_d}} = \frac{G_s \gamma_w - \gamma_d}{G_s \gamma_w}$$

$$\Rightarrow N_a = \left(\frac{G_s \gamma_w - \gamma_d}{G_s \gamma_w} \right) (1 - S_r)$$

$$\begin{aligned}
 N_a &= \left(\frac{2.65 \times 10.0 \text{ kNm}^{-3} - 19.20 \text{ kNm}^{-3}}{2.65 \times 10.0 \text{ kNm}^{-3}} \right) (1 - 0.083638) \\
 &= \left(\frac{7.30 \text{ kNm}^{-3}}{26.50 \text{ kNm}^{-3}} \right) (0.16362) = 0.0451 = 4.51\%
 \end{aligned}$$

$$N_a = \mathbf{4.51\%}$$

FURTHER WORKED EXAMPLES (GUIDED EXAMPLES)

Example 38:

In its condition a soil sample has a mass of 2290 g and a volume of $1.15 \times 10^{-3} m^3$. After being completely dried in an oven, the mass of the sample is found to be 2035g. If $G_s = 2.68$, determine the bulk density, unit weight, water content, void ratio, porosity, degree of saturation and air content.

Given:

$$M = 2290g$$

$$M_s = 2035g$$

$$G_s = 2.68$$

$$V = 1.15 \times 10^{-3} m^3$$

Solution:

- Bulk density, $\rho = 1991 kg/m^3$
- Unit weight $\gamma = 19.91 kN/m^3$
- Water content, $m = 12.5\%$
- Void ratio $e = 0.514$
- Porosity, $n = 0.34$
- Degree of saturation, $S = 65.20\%$
- Air content, $N_a = n(1 - S) = 0.34(1 - 0.652) = 0.12$

Important hints:

$$\rho = M/V$$

$$m = (M_w/M_s) \times 100\%$$

$$\gamma = \rho g$$

$$e = (G_s \rho_w / \rho_d) - 1$$

$$n = e / (1 + e)$$

$$S_r = m G_s / e$$

$$N_a = n (1 - S_r)$$

Example 39:

A moist soil has these values:

- $V = 7.08 \times 10^{-3} m^3$
- $M = 13.95 kg$
- $m = 9.8\%$
- $G_s = 2.66.$

Determine the bulk density, the void ratio, the porosity, degree of saturation, the air content, the volume occupied by water as well as the volume of solids

Solution:

- $\rho = 1970.3 kg/m^3$
- $\rho_d = 1794.4 kg/m^3$
- $e = 0.48$
- $n = 0.32$
- $S = 54.3\%$
- $N_a = 0.15$

- $V_w = 0.00125m^3$
- $V_s = 0.00583m^3$

Important hints:

$$\rho = M/V$$

$$\rho_d = \rho/(1 + m)$$

$$e = (G_s \rho_w / \rho_d) - 1$$

$$S_r = m G_s / e$$

$$N_a = n (1 - S)$$

$$V_w = M_w / \rho_w$$

$$M_w = M - M_s$$

$$M_s = M / (1 + m)$$

$$V_w = [M - (\frac{M}{1 + m})] / \rho_w$$

$$V_s = M_s / G_s \rho_w$$

Example 40:

In the natural state, a moist soil has a volume of $0.0093m^3$ and weighs 177.6 N. The oven dry weight of the soil is 153.6 N. If $G_s = 2.71$, calculate the moisture content, moist unit weight, dry unit weight, void ratio, porosity and degree of saturation.

Given:

$$V = 0.0093m^3$$

$$W = 177.6N$$

$$W_s = 153.6N$$

$$G_s = 2.71$$

Solution:

- $m = 15.6\%$
- $\gamma = 19.1kN/m^3$
- $\gamma_d = 16.52kN/m^3$
- $e = 0.60$
- $n = 0.375$
- $S = 70.46$

Important hints:

$$m = (W_w/W_s) \times 100\%$$

$$\gamma = W/V$$

$$\gamma_d = W_s/V$$

$$e = (G_s \gamma_w / \gamma_d) - 1$$

$$n = e / (1 + e)$$

$$S_r = m G_s / e$$

Example 41:

A soil specimen has a volume of $0.05 m^3$ and a mass of 87.5kg. If the water content is 15% and specific gravity is 2.68. Determine the void ratio, porosity, dry unit weight, saturated unit weight and the degree of saturation.

Given:

$$V = 0.05m^3$$

$$W = 87.5kg$$

$$m = 15\%$$

$$G_s = 2.68$$

Solution:

- $e = 0.77$
- $n = 0.43$
- $\gamma = 15.14kN/m^3$
- $\gamma_{sat} = 19.49kN/m^3$
- $S_r = 52.20\%$

Important hints:

$$\rho = M/V$$

$$\rho_d = \rho/(1 + m)$$

$$e = (G_s \rho_w / \rho_d) - 1$$

$$n = e/(1 + e)$$

$$\gamma_d = G_s \gamma_w / (1 + e) \text{ or simply } \gamma_d = \rho_d g$$

$$\gamma_{sat} = [(G_s + e) \gamma_w] / (1 + e)$$

$$S_r = m G_s / e$$

Example 42:

A soil specimen has a void ratio of 0.7. Given that $G_s = 2.72$, calculate the dry unit weight, bulk unit weight and the water content at the degree of saturation of 75%.

Given:

$$e = 0.7$$

$$G_s = 2.72$$

$$S_r = 75\%$$

Solution:

- $\gamma_d = 16 \text{ kN/m}^3$
- $\gamma = 20.11 \text{ kN/m}^3$
- $m_{s=75\%} = 19.30\%$

Important hints:

$$\gamma_d = G_s \gamma_w / (1 + e)$$

$$\gamma_{sat} = (G_s + e) \gamma_w / (1 + e)$$

$$\gamma_{sat=75\%} = (G_s + Se) \gamma_w / (1 + e)$$

$$\gamma_{sat=75\%} = (G_s + 0.75e) \gamma_w / (1 + e)$$

$$m_{s=75\%} = \left(\frac{\gamma_{sat=75\%}}{\gamma_d} \right) - 1$$

Example 43:

The moisture content of saturated clay is 160.0 %. If the specific gravity of the soil is 2.40, what are the wet and dry densities of the saturated clay?

Given:

$$m = 160\%$$

$$G_s = 2.40$$

$$S_r = 1$$

Solution:

- $\rho = 1289 \text{ kg/m}^3$
- $\rho_d = 496 \text{ kg/m}^3$

Important hints:

Here $e = mG_s$ and therefore

$$\rho_{sat} = \frac{(G_s + e)\rho_w}{1 + e} = \frac{(G_s + mG_s)\rho_w}{1 + mG_s}$$

$$\rho_d = \frac{G_s\rho_w}{1 + e} = \frac{G_s\rho_w}{1 + mG_s}$$

Example 44: (HIGHLY RECOMMENDED FOR REVISION)

A saturated sample of clay has a volume of $0.224 \times 10^{-4} \text{ m}^3$ and weighs 0.0367 kg. After oven drying, the volume is found to be $0.140 \times 10^{-4} \text{ m}^3$. The weight of dry soil is 0.0232 kg. For the soil in its natural state, find:

(a) Water content

(b) Specific gravity

(c) Void ratio

(d) Saturated density

(e) Dry density

Given

$$V = 0.224 \times 10^{-4} m^3$$

$$V_s = 0.140 \times 10^{-4} m^3$$

$$M = 0.0367 kg$$

$$M_s = 0.0232 kg$$

$$S_r = 1$$

Solution:

- $m = 58.18\%$,
- $G_s = 2.684$
- $e = 1.512$
- $\rho_{sat} = 1638 kg/m^3$
- $\rho_d = 1036 kg/m^3$

Important hints

$$m = M_s / M_w$$

$$M_w = M - M_s$$

$$\rho_d = M_s / V$$

$$\rho_{sat} = M / V$$

Now find n first using the relationship

$$\rho_{sat} = \rho_d + n\rho_w \text{ Then;}$$

$$e = n/1 - n$$

Note that:

$$\rho_{sat} = \frac{(G_s + e)\rho_w}{1 + e} = \frac{G_s\rho_w}{1 + e} + \frac{+e\rho_w}{1 + e} = \rho_d + n\rho_w$$

Finally, calculate G_s either from ρ_d , ρ_{sat} or from the popular relationship $G_s = S_r e / m$

Example 45:

A soil specimen with a volume of $0.60 \times 10^{-4} \text{ m}^3$ weighs 0.105 kg. Its dry weight is 0.0802 kg, and G_s is 2.65. Compute the following:

- (a) Water content
- (b) Void ratio and porosity
- (c) Degree of saturation

Given:

- $V = 0.60 \times 10^{-4} \text{ m}^3$
- $M = 0.105 \text{ kg}$
- $M_s = 0.0802 \text{ kg}$
- $G_s = 2.65$

Solution:

$$m = 31\%$$

$$e = 0.982$$

$$n = 0.495$$

$$S_r = 84\%$$

Important hints:

$$m = M_s/M_w$$

$$M_w = M - M_s$$

$$\rho_d = M_s/V$$

Therefore compute e from the relationship

$$\rho_d = \frac{M_s}{V} = \frac{G_s \rho_w}{1 + e}$$

Finally;

$$S_r = mG_s/e$$

Example 46:

A soil has a bulk density of 1810 kg/m^3 . The moisture content was found to be 16.1 %. Assume $G_s = 2.7$, find:

(a) Void ratio

(b) Dry density

(c) Degree of saturation

Given:

$$\rho = 1810 \text{ kg/m}^3$$

$$m = 16.10\%$$

$$G_s = 2.70$$

Solution:

- $e = 0.731$
- $\rho_d = 1559 \text{ kg/m}^3$
- $S_r = 59.47\%$

Important hints:

Since $\rho = (1 + m)G_s\rho_w/(1 + e)$ then

$$\rho_d = \rho/(1 + m);$$

$$e = (G_s\rho_w/\rho_d) - 1$$

$$S_r = mG_s/e$$

Example 47:

A saturated soil has $m = 25\%$ and $e = 0.6725$. Find:

(a) Specific gravity

(b) Saturated density

Given:

$$m = 25\%$$

$$e = 0.6725$$

$S_r = 1$; since the soil is *fully saturated*

Solution:

- $G_s = 2.69$
- $\rho_{sat} = 2010.50 \text{ kg/m}^3$

Important hints:

$$G_s = S_r e / m \text{ and hence}$$

$$G_s = e / m; \text{ since } S_r = 1$$

$$\rho_{sat} = (G_s + e) \rho_w / (1 + e)$$

Example 48:

A sand has a porosity of 45% and $G_s = 2.65$. Find:

(b) e

(c) ρ_d and ρ_{sat}

(d) ρ_{wet} if $S_r = 30\%$

Given:

$$n = 45\%$$

$$G_s = 2.65$$

Solution:

- $e = 0.818$
- $\rho_d = 1457 \text{ kg/m}^3$
- $\rho_{sat} = 1907 \text{ kg/m}^3$
- $\rho_{wet} = \rho = 1592.62 \text{ kg/m}^3$

Important hints:

$$e = n/(1 - n)$$

$$\rho_d = \frac{G_s \rho_w}{1 + e}$$

$$\rho_{sat} = \rho_d + n \rho_w$$

$$\rho_{wet} = \rho = \frac{(G_s + e S_r) \rho_w}{1 + e} = \rho_d + 0.30 n \rho_w; \quad S_r = 30\%$$

Example 49:

A 1 m^3 volume of dry soil has a mass of 1.7 tons ($1 \text{ ton} = 1000 \text{ kg}$). The water content is 20%, and $G_s = 2.70$. Find:

(a) e and n

(b) ρ in kg/m^3

Given:

$$V = 1 \text{ m}^3$$

$$M = 1700 \text{ kg}$$

$$m = 20\%$$

$$G_s = 2.70$$

Solution:

- $e = 0.587$
- $n = 0.37$
- $\rho_d = 1700 \text{ kg/m}^3$
- $\rho = 2040 \text{ kg/m}^3$

Important hints:

$$\rho_d = M_s/V$$

$$\rho_d = G_s \rho_w / (1 + e) = M_s/V$$

$$e = (G_s \rho_w / \rho_d) - 1$$

$$n = e / (1 + e)$$

$$\rho = (M_s + M_w) / V$$

$$M_w = m M_s$$

Example 50:

A $5.80 \times 10^{-4} \text{ m}^3$ volume of soil weighs 1.1 kg and its dry weight is 1.01 kg. If $G_s = 2.67$; find:

(a) e, n, m

(b) ρ and ρ_d in kg/m^3

Given:

$$V = 5.80 \times 10^{-4} m^3$$

$$M = 1.10 kg$$

$$M_s = 1.01 kg$$

$$G_s = 2.67$$

Solution:

- $e = 0.534$
- $n = 0.348$
- $m = 8.91\%$
- $\rho_d = 1741 kg/m^3$
- $\rho = 1897 kg/m^3$

Important hints:

$$m = M_s/M_w; M_w = M - M_s$$

$$\rho_d = M_s/V$$

$$e = (G_s \rho_w / \rho_d) - 1$$

$$n = e / (1 + e)$$

$$\rho = \rho_d (1 + m)$$

Example 51:

A water content test was made on a sample of silty clay. The weight of the wet soil plus container was 17.53 g, and the weight of the dry soil plus container was

14.84 g. Weight of the empty container was 7.84 g. Calculate the water content of the sample.

Solution:

MOISTURE CONTENT DETERMINATION EXPERIMENT	
Weight of container + wet soil (g)	17.53 (A)
Weight of container + dry soil (g)	14.84 (B)
Weight of empty container (g)	7.84 (C)
<i>Weight of wet soil (g) = A - C</i>	9.69 (D)
<i>Weight of dry soil (g) = B - C</i>	7.0 (E)
<i>Weight of water (g) = D - E</i>	2.69 (F)
<i>Moisture content, $m = [(D - E)/(B - C)] \times 100\%$</i> $m = \left(\frac{E}{F} \right) \times 100$	38.43%

Example 52:

A sample of fully saturated clay weighs 1.35 kg in its natural state and 0.975 kg after drying. If the specific gravity of the solid is 2.7, what is the natural water content of the soil? Compute also the void ratio, the porosity, the wet density and the dry density?

Given:

$$M = 1.35 \text{ kg}$$

$$M_s = 0.975 \text{ kg}$$

$$G_s = 2.70$$

$S_r = 1$; since the soil is *fully saturated*

Solution:

- $m = 38.50\%$
- $e = 1.04$
- $n = 0.51$
- $\rho_{sat} = 1833 \text{ kg/m}^3$
- $\rho_d = 1323.50 \text{ kg/m}^3$

Important hints:

$$m = M_s/M_w; M_w = M - M_s$$

$$e = mG_s/S_r$$

$$n = e/(1 + e)$$

$$\rho = (G_s + e)\rho_w/(1 + e)$$

$$\rho_d = \rho/(1 + m)$$

Example 53:

The dry density of compacted sand is 1820 kg/m^3 and density of the solids ($G_s \times \rho_w$) is 2670 kg/m^3 . What are the density and water content of the soil when saturated?

Given:

$$\rho_d = 1820 \text{ kg/m}^3$$

$$G_s \rho_w = 2670 \text{ kg/m}^3$$

and $S_r = 1$ at full saturation

Solution:

- $m = 17.50\%$
- $\rho = 2138 \text{ kg/m}^3$

Important hints:

Using the relationship $\rho_d = G_s \rho_w / (1 + e)$ where $e = m G_s$ at saturation, then $\rho_d = G_s \rho_w / (1 + m G_s)$ so that

$$m = (G_s \rho_w - \rho_d) / G_s \rho_d; \text{ then}$$

$$\rho = \rho_d (1 + m) \text{ or}$$

$$\rho = (G_s + e) \rho_w / (1 + e)$$

$$\rho_{sat} = (G_s + m G_s) \rho_w / (1 + m G_s)$$

Example 54:

A dry sand sample has a density of 1680 kg/m^3 . Determine its density when the degree of saturation is 75%. The density of solids for quartz is 2650 kg/m^3

Given:

$$\rho_d = 1680 \text{ kg/m}^3$$

$$\rho_s = G_s \rho_w = 2650 \text{ kg/m}^3 ; \text{ implying that}$$

$$G_s = 2.65$$

$$S_r = 75\% \text{ or } 0.75$$

Solution:

- $\rho = 1955 \text{ kg/m}^3$

Important hints:

From $\rho_d = G_s \rho_w / (1 + e)$ then

$e = (G_s \rho_w - \rho_d) / \rho_d$; so that, for 75% partial saturation

$$\rho_{s=75\%} = \rho = (G_s + 0.75e) \rho_w / (1 + e)$$

Example 55:

The dry density of a soil is 1650 kg/m^3 and the solids have a density of 2680 kg/m^3 . Find the following when the soil is saturated.

a) Water content,

(b) Void ratio

(c) Total density when the soil is saturated.

Given:

$$\rho_d = 1650 \text{ kg/m}^3$$

$$\rho_s = G_s \rho_w = 2680 \text{ kg/m}^3 ; \text{ implying that}$$

$$G_s = 2.68$$

$$S_r = 1 \text{ since the soil is saturated}$$

Solution:

- $e = 0.62$
- $m = 23.30\%$
- $\rho_{sat} = 2034 \text{ kg/m}^3$

Important hints:

From $\rho_d = G_s \rho_w / (1 + e)$ then

$e = (G_s \rho_w / \rho_d) - 1$; so that at *full saturation*

$$m = e / G_s$$

$$m = (G_s \rho_w / \rho_d) - 1 \times G_s \text{ and}$$

$$\rho = \rho_d (1 + m); \text{ or simply}$$

$$\rho_{sat} = (G_s + e) \rho_w / (1 + e)$$

Example 56:

A natural deposit of soil was found to have a water content of 20% and to be 90% saturated. If $G_s = 2.7$, what is the void ratio of this soil?

Given:

$$m = 20\%$$

$$G_s = 2.70$$

$$S_r = 90\% \text{ or } 0.90$$

Solution:

- $e = 0.60$

Important hints:

$$S_r = 0.90$$

$$e = mG_s/S_r$$

Example 57:

The void ratio of clay soil is 0.5 and the degree of saturation is 70%. Assuming the density of the solids is 2750 kg/m^3 ; compute

(a) the water content and

(b) dry and wet densities.

Given:

$$e = 0.50$$

$$\rho_s = 2750 \text{ kg/m}^3; \text{ implying that } G_s = 2.750$$

$$S_r = 70\% \text{ or } 0.70$$

Solution:

- $m = 12.73\%$
- $\rho_d = 1833.33 \text{ kg/m}^3$
- $\rho = 2067 \text{ kg/m}^3$

Important hints:

$$m = S_r e / G_s$$

$$\rho_d = \rho_s / (1 + e)$$

$$\rho = \rho_d (1 + m) \text{ or simply}$$

$$\rho = (G_s + e S_r) / (1 + e); \text{ where } S_r = 0.70$$

Example 58:

A soil has a bulk density of 1910 kg/m^3 and a water content of 9.5%. The value of G_s is 2.70.

(a) Calculate the void ratio and degree of saturation of the soil.

(b) What would be the values of density and water content if the soil were fully saturated at the same void ratio?

(a) Given:

$$\rho = 1910 \text{ kg/m}^3$$

$$m = 9.50\%$$

$$G_s = 2.70$$

Solution:

- $e = 0.55$
- $S_r = 46.81\%$

Important hints:

$$e = (G_s \rho_w / \rho_d) - 1; \rho_d = \rho / (1 + m)$$

$$S_r = m G_s / e$$

(b) Given:

$$e = 0.55$$

$$S_r = 1$$

$$\rho_{sat} = ?$$

$$m = ?$$

$$\rho_{sat} = \frac{(G_s + e S_r) \rho_w}{1 + e} = \frac{(G_s + e) \rho_w}{1 + e}$$

$$m = \frac{S_r e}{G_s} = \frac{e}{G_s}$$

- $\rho_{sat} = 2097 \text{ kg/m}^3$
- $m = 20.37\%$

Example 59:

Calculate the dry unit weight, the saturated unit weight and the buoyant unit weight of a soil having a void ratio of 0.70 and a value of G_s of 2.72.

Calculate also the unit weight and water content at a degree of saturation of 75%.

Given:

$$e = 0.70$$

$$G_s = 2.70$$

$$S_{r1} = 1$$

$$S_{r2} = 0.75$$

Solution:

- $\gamma_d = 15.88 \text{ kN/m}^3$
- $\gamma_{sat} = 20 \text{ kN/m}^3$
- $\gamma_{bouyant} = \gamma' = 10 \text{ kN/m}^3$

When $S_r = 0.75$

- $\gamma = 18.97 \text{ kN/m}^3$
- $m = 19.44\%$;

(Note that $g = 10 \text{ m/s}^2$)

Important hints:

$$\gamma_d = G_s \gamma_w / (1 + e)$$

$$\gamma_{sat} = G_s \gamma_w (1 + e) / (1 + e); S_r = 1$$

$$\gamma_{bouyant} = \gamma' = \gamma_{sat} - \gamma_w$$

For $S_r = 0.75$

$$\gamma = (G_s + e S_r) \gamma_w / (1 + e)$$

$$m = S_r e / G_s; \text{ where } S_r = 0.75$$

Example 60:

A sample of saturated clay weighed 1526 gm in its natural state, and 1053 gm after drying. Determine the natural water content. If the specific gravity of the solid constituents was 2.70, what is the void ratio, the porosity and the unit weight of the soil sample?

Given:

$$W = 1526g = 1.526kg$$

$$W_s = 1053g = 1.053kg$$

$$G_s = 2.70$$

$$S_r = 1$$

$$m = ?$$

Solution:

- $m = 45\%$
- $e = 1.213$
- $n = 0.55$
- $\gamma_{sat} = 17.68kN/m^3$

Important hints:

$$m = (W_w/W_s) \times 100\%; W_w = W - W_s$$

$$e = mG_s; S_r = 1$$

$$n = e/(1 + e)$$

$$\gamma_{sat} = (G_s + e)\gamma_w/(1 + e)$$

Example 61:

A sandy soil is composed of solid constituents having a specific gravity of 2.60. The void ratio is 0.572. Compute the unit weight of the sand when dry and when saturated, and compare with the effective unit weight when submerged.

Given:

$$G_s = 2.60$$

$$e = 0.572$$

Solution:

- $\gamma_d = 16.54 \text{ kN/m}^3$
- $\gamma_{sat} = 20.18 \text{ kN/m}^3$
- $\gamma' = 10.18 \text{ kN/m}^3$

Important hints:

$$\gamma_d = G_s\gamma_w/(1 + e)$$

$$\gamma_{sat} = (G_s + e)\gamma_w/(1 + e)$$

$$\gamma' = \gamma_{sat} - \gamma_w$$

Note: Unless otherwise stated, some of the problems below should use the value of G_s as 2.7

Example 62:

A soil was found to have the following properties: $G_s = 2.74$; $e = 0.59$ and $m = 14\%$.

Find:

- a. Degree of Saturation
- b. Porosity
- c. Dry density

Given:

$$G_s = 2.74$$

$$e = 0.59$$

$$m = 14\%.$$

Solution:

- $S_r = 65\%$
- $n = 0.371$
- $\rho_d = 1723.3 \text{ kg/m}^3$

Important hints:

$$S_r = mG_s/e$$

$$n = e/(1 + e)$$

$$\rho_d = G_s\rho_w/(1 + e)$$

Example 63:

A wet soil was found to have a volume of 22.3 cm^3 and mass of 29.7g. The dry mass of the soil was found to be 23 g.

Find the:

- a. void ratio
- b. Water content
- c. Porosity
- d. Degree of saturation

Given:

$$V = 22.3 \text{ cm}^3 = 22.3 \times 10^{-6} \text{ m}^3$$

$$M = 29.7 \text{ g} = 29 \times 10^{-3} \text{ kg}$$

$$M_s = 23.0 \text{ g} = 23 \times 10^{-3} \text{ kg}$$

$$G_s = 2.70$$

Solution:

- $m = 29.13\%$
- $e = 1.62$
- $n = 0.62$
- $S_r = 48.60\%$

Important hints:

$$W_w = W - W_s$$

$$m = (W_w/W_s) \times 100\%;$$

$$\rho_d = M_s/V$$

$$\rho_d = G_s \rho_w / (1 + e)$$

$$e = \frac{G_s \rho_w}{\rho_d} - 1$$

$$n = e / (1 + e)$$

$$S_r = m G_s / e$$

Example 64:

A wet soil sample weighs 250g and 160g when dry. What is the volume of water in the soil, in cc? [$V_w = 90 \text{ cm}^3$]

Example 65:

A wet sample of soil weighs 4.20kg. The same sample weighs 3.65kg when dry. Calculate its water content of the sample. [$m = 15.07\%$]

Example 66:

A moist soil sample is weighed in a metal container. The sample and container weigh 126.3g. The empty container weighs 13.6g. After oven drying, the sample

and container weigh 104.2g. Calculate the water content of the sample.
[$m = 8.16\%$]

Example 67:

A dry soil sample weighs 269g. An amount of water of 27.9g is added to the soil. What is the resulting water content of the mixture? [$m = 10.37\%$]

Example 68:

A moist soil sample has a weight of 10.2kg and its volume is 0.008 cubic metres. If its moisture content was found to be 21.30%, calculate both its moist and dry unit weights [$\gamma = 12750\text{N}/\text{m}^3 = 12.75\text{kN}/\text{m}^3$, $\gamma_d = 10.51\text{kN}/\text{m}^3$]

Hints:

$$\gamma = W/V$$

$$\gamma_d = \gamma/(1 + m)$$

Example 69

A soil sample weighs 126g in its natural condition, has a volume of 62cc and its water content is 8.62g. What is its dry unit weight? [$\gamma_d = 18.71\text{kN}/\text{m}^3$]

Hints:

$$\gamma = W/V$$

$$\gamma_d = \gamma/(1 + m)$$

Example 70:

A soil weighs 16.29kg when moist and its dry weight is 13.10kg. If its volume is 0.0075 cubic metres:

(a) Calculate its void ratio and the porosity, assuming the soil's specific gravity is 2.68.

(b) What is the saturated water content of the sample?

[$e = 0.534$; $n = 0.348$; $m_{sat} = 20\%$]

Given:

$$W = 16.29kg$$

$$W_s = 13.10kg$$

$$V = 0.0075m^3$$

$$G_s = 2.68$$

Hints:

$$\rho = M/V$$

$$\rho_d = M_s/V \text{ or } \rho_d = \rho/(1 + m)$$

$$e = (G_s \rho_w / \rho_d) - 1$$

$$n = e/(1 + e)$$

$$m_{sat} = e/G_s$$

Example 71:

A soil sample has a volume of 953.24cm^3 and its moist weight is 1.87kg . The water content determined by oven drying was found to be 22.5% . If its specific gravity is 2.68 determine its

- (i) void ratio
- (ii) porosity
- (iii) degree of saturation
- (iv) saturated water content

$$[e = 0.6735; n = 0.4025; S_r = 89.53\%; m_{sat} = 25.13\%]$$

Given:

$$V = 953.24 \times 10^{-6} \text{m}^3$$

$$W = 1.87 \text{kg}$$

$$m = 22.5\%$$

$$G_s = 2.68$$

Hints:

$$\rho = M/V$$

$$\rho_d = \rho/(1 + m)$$

$$e = (G_s \rho_w / \rho_d) - 1$$

$$n = e/(1 + e)$$

$$m_{sat} = e/G_s$$

$$S_r = mG_s/e; \text{ where } m = 22.5\%$$

Example 72:

A sample of soil has a volume of $0.016m^3$ and a weight of $32.4kg$ when wet and $28.8kg$ when dry. If the specific gravity of the soil is 2.68 , calculate:

- (i) The moist unit weight
- (ii) The water content
- (iii) The dry unit weight
- (iv) The degree of saturation of the sample

$[\gamma = 20.25kN/m^3; m = 12.5\%; \gamma_d = 18.0kN/m^3; S_r = 68.52\%]$

Given:

$$M = 32.4kg$$

$$M_s = 28.8kg$$

$$V = 0.016m^3$$

$$G_s = 2.68$$

Hints:

$$\gamma = W/V = Mg/V$$

$$m = \left(\frac{M - M_s}{M_s} \right) \times 100\%$$

$$\gamma_d = \frac{\gamma}{1 + m}$$

$$e = (G_s \gamma_w / \gamma_d) - 1$$

$$S_r = mG_s/e$$

Example 73:

A **block** of a wet soil was found to have the following measurements: height 26cm, length 22cm, width 19.70cm. The block weighs 19.50kg and it was found to weigh 15.30kg when dry. If the soil's specific gravity is 2.70, find

- (i) void ratio
- (ii) porosity
- (iii) saturated water content
- (iv) %/degree of saturation

[$e = 0.989$; $n = 0.497$; $m_{sat} = 36.61\%$; $S_r = 75\%$]

Given:

Height, $h = 26cm$;

Length, $l = 22cm$

Width, $w = 19.70cm$

$$V = h \times l \times w = hlw$$

$$M = 19.50kg$$

$$M_s = 15.30kg$$

$$G_s = 2.70$$

Hints:

$$\rho_d = M_s/V$$

$$e = (G_s \rho_w / \rho_d) - 1$$

$$n = e/(1 + e)$$

$$m_{sat} = e/G_s$$

$$m = \left(\frac{M - M_s}{M_s} \right) \times 100\%$$

$$S_r = mG_s/e$$

Example 74:

An undisturbed sample of clayey soil is found to have a wet weight of 285 N, a dry weight of 250N, and a total volume of $14 \times 10^{-3} m^3$. If the specific gravity of soil solids is 2.70, determine the water content, void ratio, and the degree of saturation.

$$[m = 14\%; e = 0.512; S_r = 73.83\%]$$

Given:

$$W = 285N$$

$$W_s = 250N$$

$$V = 14 \times 10^{-3} m^3$$

$$G_s = 2.70$$

Hints:

$$\gamma = W/V$$

$$\gamma_d = W_s/V$$

$$m = \left(\frac{\gamma}{\gamma_d} \right) - 1$$

$$e = (G_s \gamma_w / \gamma_d) - 1$$

$$S_r = m G_s / e$$

Example 75:

The total volume of a soil specimen is 80,000 mm³, weighs 150 grams and its dry weight and the density of the soil solids are 130g and 2680 Kg/m³, respectively, Find the following:

a) The water content.

b) Void Ratio.

c) Porosity.

d) Degree of saturation.

e) Saturated unit weight.

f) Dry unit weight.

Hints:

$$\rho = M/V$$

$$\rho_d = M_s/V$$

$$m = \left(\frac{\rho}{\rho_d} \right) - 1$$

$$e = (G_s \rho_w / \rho_d) - 1$$

$$n = e / (1 + e)$$

$$S_r = m G_s / e$$

$$\gamma_{sat} = (G_s + e) \gamma_w / (1 + e)$$

$$\gamma_d = \rho_d g$$

REFERENCES:

<http://site.iugaza.edu.ps/hbaker/files/2018/02/Solved-Problems-in-Soil-Mechanics.pdf>

http://www2.hkedcity.net/citizen_files/aa/az/g55401/public_html/Revision/IndexPropertiesProblems.html

http://www.uotechnology.edu.iq/dep-building/LECTURE/SHARING/third_class/soil-mech/Soil%20Mechanics-First%20course%20.pdf

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