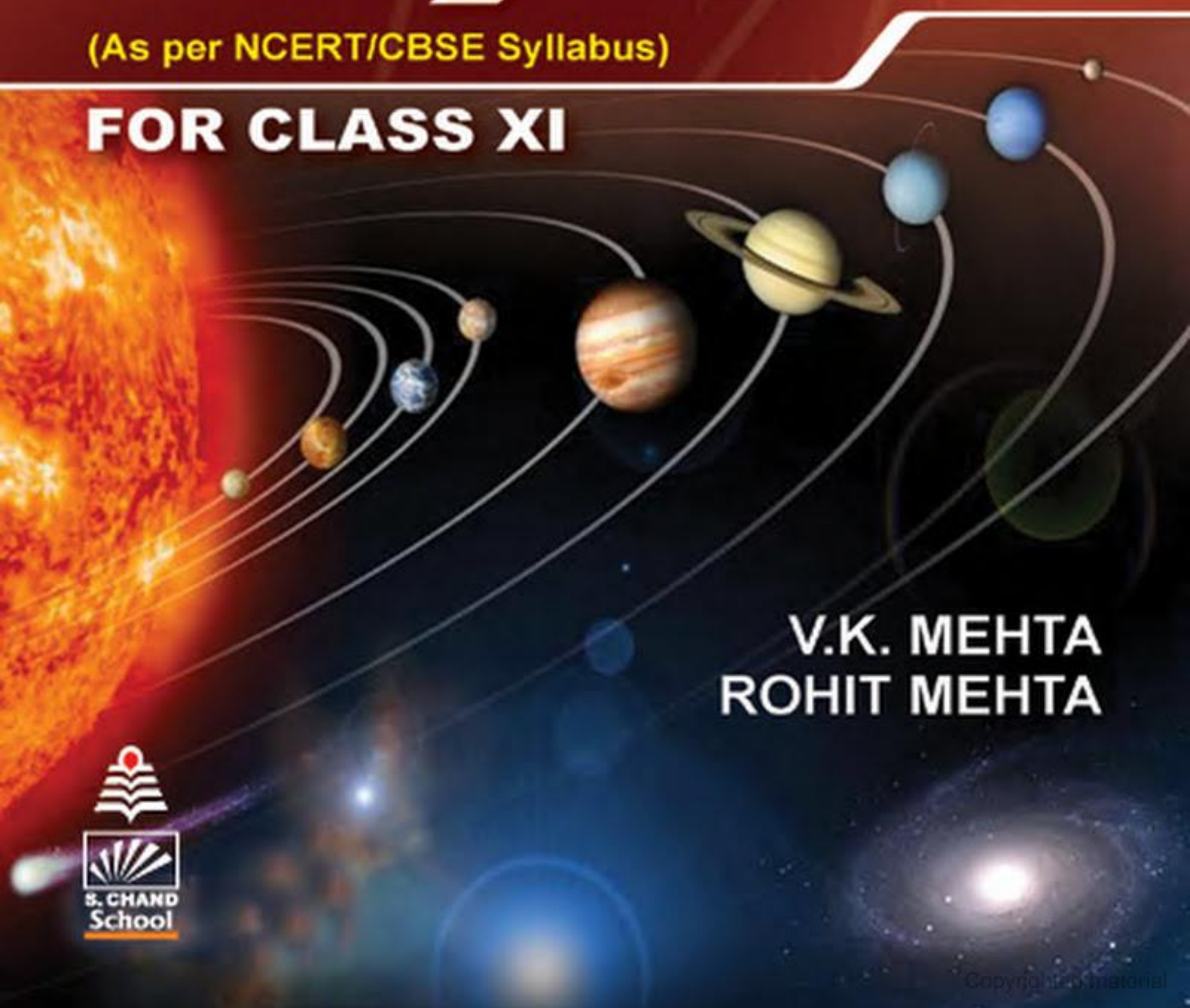
Revised Edition

S. Chand's Principles of Physics

(As per NCERT/CBSE Syllabus)

FOR CLASS XI



V.K. MEHTA
ROHIT MEHTA

S. CHAND School

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S. Chand's Principles of Physics

For

CLASS XI

Senior Secondary Certificate Examinations of CBSE and Other State Boards of School Education

V.K. MEHTA
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Introduction

GENERAL

In the present century, we find that science is held in high esteem. Although the word "science" has its origin in a Latin verb meaning "to know", science has come to mean not merely knowledge but a body of knowledge of natural world in an organised and rational way. The scientific developments have exerted influence on every phase of human activity. The intelligent man recognises that the material progress of technology is a by-product of the discoveries of science. In all fairness, honest credit is being given to science for its contribution to knowledge, comfort, efficiency and to the general well-being of mankind.

Physics is the fundamental science. *The main goal of physics is to give correct and precise description of the material universe.* The study of physics is an adventure. It is challenging, sometimes frustrating, occasionally painful and often richly rewarding and satisfying. Although physics has earned the reputation as a difficult subject, yet it is inherently simple. It is because there are only a few fundamental laws that you need to lodge in your memory bank. If you really understand these laws, you can readily use them to deduce how nature behaves in a variety of situations. As you read text and work physics problems, ask yourself how each problem you approach is really similar to other problems and to the text examples. And you will find that similarity because most of the problems and examples really involve only a few underlying laws. So physics is simple provided if you understand its fundamental laws and have the experience in reasoning out how these laws apply to the world about you. In this chapter, we shall deal with some fundamental aspects of physics.

1.1. PHYSICS

The branch of science which deals with correct and precise description of the material universe is known as **physics**.

The word physics comes from a Greek word meaning "knowledge of nature". Physics attempts to describe the fundamental nature of the universe and how it works. For example, physics explains why the sky is blue, why rainbows have colours, why steel ships float, what atoms and nuclei are made of etc. To all these and many more questions, physics explains using as few laws as possible, revealing their underlying simplicity and beauty. The amount of knowledge available on physics is so vast that as a matter of convenience, it has been divided into two classes :

(*i*) Classical (old) physics (*ii*) Modern physics

(*i*) Classical physics. *The physics prior to 1890 is known as* **classical physics.**

By 1890, the various laws governing the material universe were discovered and it appeared that knowledge of this subject was complete. Newton's laws of motion and gravitation had provided

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a solid foundation on which science of mechanics had been raised to impressive heights. The laws of thermodynamics and kinetic theory of matter gave a satisfactory account of the entire subject of heat. Maxwell developed the theory of electromagnetic radiation and explained all electrical, magnetic and optical phenomena. It appeared that the laws and theories of classical physics would hold the field unchallenged. However, startling discoveries after 1890 gave a death blow to the so-called classical physics.

(*ii*) Modern physics. *The physics after 1890 is known as* **modern physics.**

After 1890, there had been enormous advances in physics which marked the beginning of revolutionary concepts. The thrilling discoveries posed new problems unknown to classical physics. In 1895, Roentgen discovered **X-rays** and a few weeks later, Henry Becquerel announced the discovery of **radioactivity**. In the year 1897, J.J. Thomson discovered **electron**, a fundamental constituent of all matter. In 1901, Max Planck developed **quantum theory** according to which energy is not radiated continuously but in discrete packets of energy called quanta. These *quanta* eventually came to be known as *photons*. The **special theory of relativity** proposed by Albert Einstein in 1905 gave new dimensions to atomic and nuclear physics. In short, the rapid startling discoveries after 1890 gave a new shape to the entire concept of physics. The study of these concepts has been placed under the head of modern physics.

The reader may wonder about the difference between classical physics and modern physics. The answer is that classical physics does not work for microscopic particles (*e.g.*, atoms) or for objects travelling at very high speed. In fact, these two aspects gave birth to what is now called *modern physics. Indeed modem physics may be considered as mere 'fine-tuning" of classical physics.* Except for the microscopic particles and for motion at speeds near the speed of light, classical physics correctly and precisely describes the behaviour of the physical world. Although modern physics has made many important contributions to technology, great bulk of technical knowledge and skill is still based squarely on classical physics. It is through the applications of classical physics that we have been able to exploit natural resources successfully. *In this book, we shall discuss classical physics only.*

1.2. SOME SCIENTIFIC NOTIONS

Some important scientific notions are: (*i*) scientific law (*ii*) theory (*iii*) model (*iv*) scientific method.

(*i*) Scientific law. *The general statement of happenings in nature after repeated observations and experiments is known as* **scientific law**.

A scientific law is a concise statement how nature behaves. It is actually the *essence* of a large number of observations and experiments. For example, *the law of gravitation* states that the two objects attract each other. Similarly, "unlike charges attract each other and like charges repel" is also a law and is based on observations and experiments. A law has no theoretical or other basis. In other words, we cannot ask "why like charges repel and not attract? Why two bodies attract each other and not repel?" The only answer to such questions is that this is how nature behaves. And the matter ends there. No "buts" and "ifs".

Physical laws are usually expressed as mathematical equations which are then used to make predictions about other phenomena. For example, the law of gravitation states that the two objects attract each other with a force which is proportional to the mass of each object and inversely proportional to the square of the distance between them. It is written in the mathematical form as $F = G\, m_1 m_2 / r^2$. Because this expresses the law in the form of an equation, it is far preferable to the qualitative statement. *It may be noted that a scientific law has only experimental proof.*

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(*ii*) Theory. *The speculation on the basis of determined laws is known as* **theory**.

When several laws are determined, then one may make a guess about the way the universe must be constituted in order that such things happen ; this guess is called *theory*. It may be noted that there is a basic difference between a theory and law. *Theory is given by man and hence may be wrong but law is made by nature and is always right.*

Theories are never derived from observations — they are *created* to explain observations. They are inspirations that come from the minds of intelligent people. For example, the idea that matter is made up of atoms (atomic theory) was certainly not arrived at because someone observed atoms. Rather the idea was concieved by a creative mind. The theory of relativity and the electromagnetic theory were likewise the result of inspiration. Nevertheless, the history of physics tells us that theories come and go ; that long-held theories are replaced by new ones. However, a new theory is accepted only if it explains a *greater range* of phenomena than does the older one.

(*iii*) Model. *A* **model** *is a visual picture of the phenomenon under study.*

When scientists are trying to understand a particular phenomenon, they often make use of a model. The purpose of a model is to give us a visual picture — something to hold on — when we cannot see what actually is happening. One example is the wave model of light. We cannot see waves of light as we can see water waves. But it is valuable to think of light as if it were made up of waves because experiments on light show that it behaves in many respects as water waves do.

(*iv*) Scientific method. *To understand any natural process by a combination of logical reasoning and controlled experimentation is called the* **scientific method**.

Prior to about 1600, questions of truth and falsehood were most often determined by political or religious dictates. But great scientists like Isaac Newton introduced scientific methods to the world. The two main tools of scientific method are logic and experimentation. The result is that the experimental results are *reproducible*. That is, the same set of circumstances will always produce the same observed results in the same experiment, no matter who the observer is.

Notes. (*i*) Scientific laws are different from political laws in that the latter tell us how we must behave. Scientific laws do not say how nature *must* behave but rather describe how nature *does* behave.

(*ii*) To be called a law, a statement must be found experimentally valid over a wide range of observed phenomena. For less general statements, the term "principle" is often used (such as Archimede's principle). Where to draw the line between laws and principles is, of course, arbitrary and there is not always complete consistency.

1.3. SCOPE AND EXCITEMENT OF PHYSICS

The scope of physics is very wide. It covers an immense range of natural phenomena. Distances extend from incredibly small dimensions of subatomic particles to thousands of million metres that separate galaxies of the universe. For example, the radius of atomic nucleus is about 10^{-14} m while the radius of the universe is about 10^{25} m. The time intervals encountered in the physical world vary over an extremely wide range. For instance, the time taken by light to cross distance of nuclear size is about 10^{-22} s while the life of sun is about 10^{18} s. Similarly, the mass of an electron is approximately 10^{-30} kg whereas the mass of universe is about 10^{55} kg. All these wide range of measurements are in the domain of physics.

The subject of physics has wide applications. It includes medical physics, computers, meteorology, material science, geophysics, engineering communication, environmental physics *etc*. Physics is a very exciting subject. It provides answers to exciting questions such as :

(*i*) Why is the sky blue? (*ii*) Why is sunset red?

(*iii*) Why is glass transparent? (*iv*) Why are atoms held together?

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These are only a few exciting questions. In fact, there are a very large number of such questions to which physics provides answers.

1.4. FIELDS OF PHYSICS

Physics is the *physical science which deals with matter and energy and their transformations. Knowledge of physics will help you understand both natural phenomena and the technologies that increasingly prevade our lives. There are numerous problems that physics faces. Yet we can describe and understand any problem with the help of the following five fields of physics :

(*i*) **Classical mechanics.** *It deals with objects from molecules to galaxies that are moving at speeds small compared with the speed of light.* It deals with such ideas as inertia, motion, force and energy. Mechanics also includes the properties and laws of both solids and fluids. You use the laws of classical mechanics (also known as Newtonian mechanics) when you drive a car, ride a skateboard, build a skyscrapper *etc.* Indeed many physics problems can be solved through the application of laws of classical mechanics. You will see that classical mechanics is but an approximation to a more comprehensive set of physical laws that includes Einstein's theory of relativity and the theory of quantum mechanics.

(*ii*) **Thermodynamics.** *It deals with the study of heat and its interaction with matter.* It includes effects of temperature on the properties of materials, heat flow and transformations involving heat and work.

(*iii*) **Electromagnetism.** *It deals with electricity, magnetism and electromagnetic radiation.* Today electromagnetic technology dominates our civilisation as evidenced by radio, television, taperecorders, microwave ovens and computers.

(*iv*) **Relativity.** In 1905, Albert Einstein gave the theory of relativity. This theory has changed our notions of space and time and of mass and energy. Space and time are seen to be intimately connected with time being the fourth dimension in addition to space's three dimensions. Although theory of relativity provides a more correct description of physical reality than does classical mechanics, the two theories differ significantly only at speeds approaching that of light. *In our everyday existence, we need not worry about relativity.*

(*v*) **Quantum mechanics.** *It deals with the behaviour of objects on the extremely small scale of atomic dimensions.* Indeed classical physics fails to explain the behaviour of such microscopic particles. The quantum mechanics touches every aspect of modem physics and most of classical physics. Indeed one requires an understanding and the application of quantum mechanics for a satisfactory explanation of most of the phenomena.

1.5. THE SIMPLICITY OF PHYSICS

There is a common notion that physics is incomprehensible to the people of average ability. If you think that, you are missing the point because it is so fundamental, physics is inherently simple. There are only a few basic laws to learn ; if you really understand those laws, you can readily apply them in a wide variety of situations. *The laws and theories of physics describe the working of the universe at the most basic level.* For that reason, physics is extremely powerful ; the same laws describe the behaviour of molecules, aeroplanes and galaxies. Scientists believe that it would be possible to describe the operation of a living cell or organism using only the fundamental laws of physics. So physics is simple — challenging too — but with an underlying simplicity that reflects the scope and power of this fundamental science.

* A physical science is one which primarily deals with non-living things. On the other hand, biological science deals with living things.

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1.6. PHYSICS IS BASIC SCIENCE

Physics is the basic science. It is about the nature of basic things such as motion, force, energy, matter, heat, sound, light and the inside of atoms. An understanding of science begins with an understanding of physics. There is physics in everything you see, hear, smell, taste and touch.

(*i*) **Physics and Mathematics.** The laws of physics are usually expressed as *mathematical equations which are then used to make predictions about other phenomena and to test the range of validity of the laws. In order to understand physics on any level beyond qualitative description, you require a considerable knowledge of mathematics. The mathematical description of the laws and theories of physics gives new insight to the subject. Indeed, the primary goal of mathematics is to aid physics.

(*ii*) **Physics and Chemistry.** Chemistry is about how matter is put together, how atoms combine to form molecules and how molecules combine to make up the many kinds of matter around us. In order to explain these aspects, we have to apply the laws and theories of physics. So underneath chemistry is physics.

(*iii*) **Physics and Biology.** Biology is more complex and involves matter that is alive. So underneath biology is chemistry and underneath chemistry is physics. That is why physics is the most basic science.

(*iv*) **Physics and other sciences.** Since physics is a fundamental science, its laws and theories are applied to various physical sciences. The knowledge of physics is required in many fields and the list below shows some of these :

(*a*) Medical physics (*b*) Meterology (*c*) Geophysics (*d*) Environmental physics
(*e*) Communication (*f*) Material science.

1.7. PHYSICS AND TECHNOLOGY

Physics and technology are different from each other. *Physics is the knowledge of material universe for its own sake, whereas technology transfers the knowledge of physics into practical shape for the general well-being of human race.* In other words, physicists provide the knowledge of material universe, whereas the engineers provide the tools, techniques and procedures for putting this knowledge to practical use. For example, physics has provided the knowledge of semiconductors but technology made of them transistor radios. Many technologies have been developed on the basis of the knowledge of physics and a few of them are mentioned below by way of illustration :

(*a*) The discovery of Faraday's laws of electromagnetic induction enabled the engineers to develop electric generator, electric motor, transformer and a host of other equipment.

(*b*) The discovery of laws of thermodynamics led to the development of many types of heat engines.

(*c*) The discovery of electromagnetic radiation, vacuum tubes and semiconductors led to wireless communication *e.g.* radio and television *etc.*

(*d*) The discovery of nuclear fission led to the development of nuclear power plants which are generating huge amount of electric power all over the world.

The above short list indicates how important is the knowledge of physics in developing new technologies. *Indeed, technology may be regarded as applied physics.*

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* When laws of physics are expressed in mathematical form, they do not have the double meanings that so often confuse the discussion of the ideas expressed in common language. When laws are expressed mathematically, they are easier to verify or disprove by experiments.

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1.8. PHYSICS AND SOCIETY

The advancement of technology as a result of discoveries of knowledge of physics has made a significant impact on human lives. We travel faster than ever before in aeroplanes. We enjoy colour television. We use electricity to operate a host of electrical appliances. We can solve complex problems at amazing speed with the help of computers. We send satellites into orbit to learn about the conditions in outer space. The development of radio and television has led to instant communication. Indeed, the uses of physics have exerted considerable influence on every phase of human activity.

We are all familiar with the abuses of applied physics or technology. Many people blame technology itself for widespread pollution, resource depletion and even social decay in general. It is much wiser to combat the misuse of technology with knowledge than with ignorance. Wise applications of physics can lead to a better world.

1.9. SOME DISCOVERIES AND INVENTIONS

The discoveries and inventions in the field of physics were made by brilliant, logical and creative minds. The table below gives some discoveries and inventions in physics.

Some Inventions and Discoveries

S.No.	Invention or Discovery	Inventor or Discoverer	Year
1.	Compound microscope	Janssen	1590
2.	Refracting telescope	Lippershey	1608
3.	Laws of refraction	Snell	1621
4.	Spectrum (light)	Newton	1666
5.	Reflecting telescope	Newton	1669
6.	Capacitor	Kleist	1745
7.	Gold-leaf electroscope	Bennet	1787
8.	Simple cell	Volta	1800
9.	Double-slit interference	Young	1801
10.	Diffraction	Fraunhoffer	1819
11.	Magnetic effect	Oersted	1819
12.	Electric motor	Faraday	1821
13.	Ohm's law	Ohm	1826
14.	Transformer	Faraday	1831
15.	Dynamo	Faraday	1831
16.	Laws of Electrolysis	Faraday	1834
17.	Semiconductors	Rosenschold	1834
18.	Cathode rays	(several)	1859
19.	Electric lamp	Edison	1879
20.	Electric iron	Seeley	1882
21.	Radioactivity	Becquerel	1896
22.	Nuclear reactor	Fermi	1942
23.	Transistor	Shockley, Bardeen and Brattain	1947
24.	Integrated circuits	Kilby	1958
25.	Optical fibre	Kao, Newnes and Beales	1966

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1.10. BASIC FORCES IN NATURE

There are four basic forces that operate in nature. These are (*i*) the gravitational force (*ii*) the electromagnetic force (*iii*) the strong nuclear force and (*iv*) the weak nuclear force. For detailed discussion, the reader may refer to Art. 10.8.

1.11. TOWARDS UNIFICATION OF FORCES

In physics, we attempt to explain different physical phenomena in terms of a few concepts and laws. This thrust of unification in physics has greatly simplified its study. We give here a few examples to illustrate this important point.

(*i*) Newton unified terrestrial and celestial domains under a common law of gravitation.

(*ii*) The experimental discoveries of Oersted and Faraday showed that electric and magnetic phenomena are in general inseparable.

(*iii*) Maxwell's equations form the basis of all electromagnetic phenomena.

(*iv*) Maxwell unified electromagnetism and optics with the discovery that light is an electromagnetic wave.

1.12. CONSERVATION LAWS

In physics, there are some physical quantities which remain the same before and after an event or interaction. Such quantities are called **conserved quantities.* For example, electric charge is a conserved quantity. In every event, the principle of *conservation of charge* has always been found to apply. Thus when we produce a free negative charge by tearing an electron loose from an atom, an equal positive charge is left behind, thereby causing no change in total charge of the system. The laws relating to conserved quantities are called conservation laws.

Those laws of nature that state that some physical quantity is the same before and after an event or interaction are called **conservation laws.**

Some important physical quantities that are conserved in an event or interaction are :

(*i*) Electric charge _____ **Law of conservation of electric charge**

(*ii*) Energy _____ **Law of conservation of energy**

(*iii*) **Momentum _____ **Law of conservation of momentum**

We do not know the reason for it. But scientists have performed a very large number of experiments over the years to test the conserved nature of the above physical quantities. In all the experiments, the conserved nature of the above physical quantities is proved beyond any doubt. Nevertheless, conservation laws reflect one of the most basic ways of describing nature. Because conservation laws allow us to consider quantities, such as energy, that do not change during an event, we do not need to know the details of the interaction. We simply deal with the value of the conserved quantity before and after the interaction or event. Moreover, the fact that some physical quantities are conserved and some are not, tells us something about nature itself. Note that conservation laws are applicable to only isolated systems. The reader will find detailed discussion on these conservation laws in the text at apporopriate places.

1.13. SYSTEM, ENVIRONMENT AND ISOLATED SYSTEM

Since conservation laws are valid in isolated systems, it is desirable to know, what is an isolated system?

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* A physical quantity that is the same before and after an event or interaction is called a *conserved quantity.*

** Linear momentum as well as angular momentum.

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(*i*) System and Environment. *A* **system** *is some part of the universe in which we have a special interest.* All the rest of the universe outside the system is called **environment**.

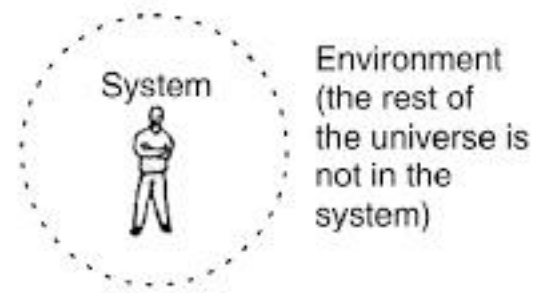


Fig. 1.1

The system can be any part of the universe we choose—a human body, the whole earth, a single atom *etc*. Fig. 1.1 shows the dashed line around a person. The part of the universe within the dashed line is the system. All the rest of the universe outside our chosen system is the environment. The dashed line can be an actual boundary between the system and its environment, or it can be an imaginary boundary that we use to help identify the system.

(*ii*) Isolated system. *The system is said to be* **isolated** *if nothing (energy, matter, light) crosses the boundary of the system to or from the environment and if the net sum of all the external forces on the system is zero.*

Within an isolated system, the amount of conserved quantity (*e.g.* electric charge, energy, momentum *etc*.) will not change during an event or interaction. Often we can consider a system approximately isolated by simply choosing its boundaries in an appropriate manner. Then we can apply conservation laws to learn more about elements within the system. For instance, when two cars collide, we can choose the system to be the region of space that includes both cars. The interaction of each car with the other is much greater than any other force affecting the cars. As a result, we consider the system as being approximately isolated. Therefore, we can use conservation laws to learn more about the motions of the cars during their collision.

CONCEPTUAL QUESTIONS

Q.1. Which are the two main tools of physics?

Ans. The two main tools of physics are logic and experimentation.

Q.2. What is the drawback of classical physics?

Ans. Some aspects of classical physics do not work for the tiny world of the atoms or objects travelling at very high speed.

Q.3. What is the importance of mathematics for physics?

Ans. The laws of physics are generalisations from observations and experiments. When these laws are expressed in mathematical forms, they are unambiguous. They do not have double meanings that so often confuse the discussion of ideas expressed in common language. When findings of nature are expressed mathematically, they are easier to verify or disprove by experiments.

Q.4. What is the difference between a theory and a model?

Ans. Usually, a model is fairly simple and provides a structural similarity to the phenomenon being studied. However, a theory is broader, more detailed, and attempts to solve a set of problems often with mathematical precision.

Q.5. The theories of science are not fixed but undergo change. Discuss.

Ans. The history of science shows that theories come and go ; that long-held theories are replaced by new ones. For example, during the last hundred years, the theory of atom has been repeatedly refined as new evidence on atomic behaviour has been gathered. A new theory is accepted only if it explains a *greater range* of phenomena than does the older one.

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Q.6. What are the laws of physics?

Ans. Among all the sciences, physics is the most basic science. *It is a quantitative science*. Its goal is to describe all phenomena in the physical world in terms of a few relationships between measured properties of matter and energy. These fundamental relationships are called *laws* of physics. They are statements of great generality derived from a wide range of phenomena.

Q.7. Why is physics inherently simple?

Ans. Physics is inherently simple. It is because there are only a few basic laws to learn ; if you really understand these *laws*, you can readily apply them in a wide variety of situations.

Q.8. Explain the terms (*i*) fact (*ii*) hypothesis.

Ans. (*i*) A *fact* is a phenomenon about which competent observers who have made a series of observations are in agreement.

(*ii*) A *hypothesis* is an educated guess; a reasonable explanation of an observation or experimental result that is not fully accepted as factual until tested over and over again by experiments.

VERY SHORT ANSWER QUESTIONS

Q.1. What is physics?

Ans. Physics is a branch of science which deals with the study of nature and natural phenomena.

Q.2. What is a scientific law?

Ans. The general statement of happenings in nature after repeated observations and experiments is known as a scientific law.

Q.3. What is theory?

Ans. The speculation on the basis of determined scientific laws is known as theory.

Q.4. What is a model?

Ans. A model is a visual picture of the phenomenon under study.

Q.5. What is the basic difference between science and technology?

Ans. Science is the study of laws of nature. Technology is the application of these laws for the general well-being of human race.

Q.6. What are the five main branches of physics?

Ans. The five main branches of physics are : Classical mechanics, Thermodynamics, Electromagnetism, Theory of relativity and Quantum mechanics.

Q.7. What is a scientific method?

Ans. To understand any natural process by a combination of logical reasoning and controlled experimentation is called the scientific method.

Q.8. What is classical physics?

Ans. The physics prior to 1890 is known as classical physics.

Q.9. What is modern physics?

Ans. The physics after 1890 is known as modern physics.

Q.10. What are conservation laws?

Ans. Those laws of nature which state that some physical quantity is the same before and after an event or interaction are called conservation laws.

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SHORT ANSWER QUESTIONS

Q.1. What is the difference between classical physics and modern physics?

Ans. The classical physics does not work for microscopic particles (*e.g.,* atoms) or for objects travelling at very high speed (near the speed of light). These two aspects can only be explained on the basis of modern physics. Indeed modern physics may be considered as mere "*fine-tuning*" of classical physics. Although modern physics has made many important contributions to technology, great bulk of technical knowledge and skill is still based squarely on classical physics.

Q.2. Why do we express laws of physics in mathematical form?

Ans. The laws of physics are generally expressed as mathematical equations which are then used to make predictions about other phenomena. When a physics law is expressed in mathematical form, it is far preferable to the qualitative statement. Moreover, mathematical form permits us to verify the various laws experimentally.

Q.3. Name two advancements made in technology on the basis of physics.

Ans. (*i*) The discovery of Faraday's laws of electromagnetic induction has enabled the engineers to develop electric generator, electric motor, transformer *etc.*

(*ii*) The discovery of nuclear fission led to the development of nuclear power plants which are generating huge amount of electric power all over the world.

Q.4. Why is physics called science of measurement?

Ans. Physics is an experimental science. In physics, we are to deal with a large number of physical quantities like length, mass, time, velocity, pressure *etc.* These quantities can give clear understanding only if we can measure them and express our conclusions into these measurements. For this reason, physics is sometimes called the science of measurement.

Q.5. Discuss relation of physics to chemistry.

Ans. The knowledge of physics has enabled us to study the structure of atom, radioactivity, X-ray, diffraction *etc.* These concepts of physics find wide applications in chemistry *e.g.*, determining the nature of valency and chemical bonding, to detect traces of substances in a given sample *etc.*

Q.6. Discuss relation of physics to biology.

Ans. Biology involves matter that is alive. The knowledge of physics has provided us a powerful tool to study the biological sciences. For example, electron microscope has made it possible to see even the structure of a cell. Similarly, optical microscope is widely used in the study of biology.

Q.7. What do you understand by system, environment and isolated system?

Ans. A *system* is some part of the universe in which we have a special interest. All the rest of the universe outside the system is the *environment*. The system is said to be *isolated* if nothing (energy, matter, light) crosses the boundary of the system to or from the environment and if the net sum of all external forces on the system is zero. Note that conservation laws are valid only in an isolated system.

LONG ANSWER QUESTIONS

Q.1. State the laws of conservation of charge, energy and linear momentum. **[Refer to Art. 1.12]**

Q.2. How is physics related to society? **[Refer to Art. 1.8]**

Q.3. Discuss the basic forces in nature. **[Refer to Art. 1.10]**

Q.4. Give a short note on physics and technology. **[Refer to Art. 1.7]**

Q.5. Give three examples where physics has been used in technology. **[Refer to Art. 1.7]**

Q.6. Explain briefly the scope and excitement of physics. **[Refer to Art. 1.3]**

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MCQ FOR COMPETITIVE EXAMINATIONS

1. The physicist who discovered the law of gravitation was
   (*a*) Newton (*b*) Faraday
   (*c*) Chadwick (*d*) Curie
2. The theory of relativity was proposed by
   (*a*) Newton (*b*) Bohr
   (*c*) Einstein (*d*) None of the above
3. Of all the sciences, the most basic science is
   (*a*) chemistry (*b*) biology
   (*c*) physics (*d*) none of the above
4. Which of the following activities involve the utmost expression of passion, talent and intelligence?
   (*a*) literature (*b*) science
   (*c*) music (*d*) all the above
5. The classical physics is applicable to
   (*a*) microscopic world
   (*b*) macroscopic world
   (*c*) both microscopic and macroscopic world
   (*d*) cannot say
6. Electron was discovered by
   (*a*) Einstein (*b*) Faraday
   (*c*) Bohr (*d*) J.J. Thomson
7. Who is the greatest physicist of the twentieth century?
   (*a*) Chadwick (*b*) J.J. Thomson
   (*c*) Bohr (*d*) Einstein
8. J.J. Thomson belonged to
   (*a*) England (*b*) America
   (*c*) France (*d*) India
9. Physics is
   (*a*) experimental science
   (*b*) quantitative science
   (*c*) the most basic science
   (*d*) all the above
10. The laws of electromagnetic induction were discovered by
    (*a*) Faraday (*b*) Newton
    (*c*) Bohr (*d*) J.J. Thomson

ANSWERS TO MCQ FOR COMPETITIVE EXAMINATIONS

1. (*a*)	**2.** (*c*)	**3.** (*c*)	**4.** (*d*)	**5.** (*b*)
6. (*d*)	**7.** (*d*)	**8.** (*a*)	**9.** (*d*)	**10.** (*a*)

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2

Systems of Units

INTRODUCTION

Physics is an experimental science. To understand physics or any experimental science, we must be able to connect our theoretical description of nature with our experimental observations of nature. This connection is made through quantitative measurements. In physics, we are to deal with a large number of physical quantities like length, mass, time, velocity, force, pressure *etc*. These quantities can give clear understanding only if we can measure them and express our conclusions into these measurements. For this reason, physics is sometimes called the science of measurement. There were several systems of units in use over the years. For example, the unit of length was foot in British system, metre in metric system and so on. *But today, all scientists and engineers have agreed to state their observations of nature in terms of single universal system of units called Systeme International (French for International system) which is abbreviated as SI.* In this chapter, we shall discuss the various aspects of systems of units.

2.1. MEASUREMENT

For the measurement of a physical quantity, we consider a constant quantity as a standard and then find the number which expresses how many times the standard quantity is contained in the physical quantity. This standard quantity is called the **unit.**

Measurement *is thus the comparison of unknown physical quantity with a known fixed unit quantity.*

For example, if the length of a rod is 3 metres, it means that the unit of length is the *metre* and this unit is contained 3 times in the length of that rod. Similarly, if mass of a ball is 2 kilogrammes, it means that unit of mass is *kilogramme* and this unit is contained 2 times in the mass of the ball. Hence to express the magnitude of a physical quantity, we should know two things *viz.*

(*i*) The *unit* in which the quantity is measured.

(*ii*) The *numerical value* — which expresses how many times the above mentioned unit is contained in the given physical quantity.

$\therefore$ *Magnitude of physical quantity = Numerical value of physical quantity* $\times$ *size of its unit*

If n is the numerical value of a physical quantity and u is the size of the unit, then, magnitude Q of the physical quantity is

$$Q = n\,u$$

Thus when we say that the length of the rod is 3 m *i.e.* $Q = 3$ m, it means $n = 3$ and $u = 1$ m.

The size of unit is different in different systems of units. Suppose the magnitude of a physical quantity is Q. If n_1 is the numerical value of the physical quantity for a unit u_1 and n_2 for a unit u_2, then,

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$$Q = n_1 u_1 \quad \text{... for first system of units}$$

$$Q = n_2 u_2 \quad \text{... for second system of units}$$

$$\therefore \quad n_1 u_1 = n_2 u_2 \quad \ldots(i)$$

Two things are obvious from eq. (*i*) :

(*a*) The magnitude of a given physical quantity is always the same irrespective of the system of units adopted *i.e.*

$$n u = \text{Constant}$$

(*b*) If $u_1 > u_2$, then $n_1 < n_2$. Therefore, the larger the magnitude of the unit selected, the smaller is the numerical value of the physical quantity.

2.2. NEED FOR MEASUREMENT

In physics, we are to deal with a large number of physical quantities like length, mass, time, speed, acceleration, force, pressure *etc*. These quantities can give clear understanding if we can measure them. The need for measurement is clear from the following examples :

(*i*) $$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

We can find the speed if we can measure the distance travelled and the time taken to travel that distance.

(*ii*) $$\text{Acceleration} = \frac{\text{Force}}{\text{Mass}}$$

In order to find the acceleration produced in the body, we should know the magnitudes (*i.e.* measurement) of force acting on the body and mass of the body.

(*iii*) $$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

Again we can find pressure if we measure the force acting and the area over which the force acts.

2.3. UNIT

The quantity used as a standard of measurement is called the **unit**.

For example, when we say that the length of a rod is 3 metres, we compare the length of the rod with a standard quantity of length called *metre*. Hence metre is the unit of length. Several systems of units have been in use over the years. Today all scientists and engineers have agreed to use Systeme International (French for International system) which is abbreviated as SI. *In SI units, the standard of length is* **metre** *and that for time is the* **second** *and that for mass is* **kilogramme.**

2.4. FUNDAMENTAL UNITS AND DERIVED UNITS

Physical quantities can be divided into two categories : *base quantities* and *derived quantities*. The corresponding units for these quantities are called **fundamental units** (or **base units**) and **derived units.** Scientists in the interest of simplicity want the smallest number of base quantities possible to give full description of the physical world.

1. Fundamental units. In mechanics, the units of only three base quantities *viz. mass, length* and *time* need to be selected. All other physical quantities in mechanics (*e.g.* speed, acceleration, force, power, energy *etc.*) can be defined in terms of the units of these three base quantities. For example, speed is defined as the distance travelled divided by the time taken to travel that distance. Therefore, speed is related to distance (*i.e.* length) and time. The units selected for these base quantities (*i.e.* mass, length and time) are called fundamental units.

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The units selected for measuring mass, length and time are called **fundamental units.**

These three fundamental units are independent of each other. However, the units of other physical quantities encountered in mechanics can be expressed in terms of the units of these three fundamental units. This eliminates the need for assigning a separate unit for each physical quantity in mechanics.

Notes. (*i*) *The fundamental units are not definable in terms of other quantities. For this reason, they are called* **base** *or* **fundamental units.**

(*ii*) In order to cover all physical measurements in science and engineering, we require the units of seven base quantities *viz.* units of mass, length, time, temperature, current, amount of substance and luminous intensity. However, to express the units of physical quantities in mechanics, the units of mass, length and time are adequate.

2. Derived units. *The units of physical quantities which can be expressed in terms of fundamental units (mass, length and time) are called* **derived units.**

(*a*) The unit of velocity is a derived unit.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{\text{Length}}{\text{Time}}$$

Thus the unit of velocity can be expressed in the fundamental units of length and time.

(*b*) The unit of acceleration is a derived unit.

$$\text{Acceleration} = \frac{\text{Velocity}}{\text{Time}} = \frac{\text{Displacement}}{(\text{Time})^2} = \frac{\text{Length}}{(\text{Time})^2}$$

Thus the unit of acceleration can be expressed in the fundamental units of length and time.

(*c*) The unit of force is a derived unit.

$$\text{Force} = \text{Mass} \times \text{Acceleration} = \text{Mass} \times \frac{\text{Length}}{(\text{Time})^2}$$

Thus the unit of force can be expressed in the fundamental units of mass, length and time.

2.5. CHARACTERISTICS OF A STANDARD UNIT

We have seen that units of all physical quantities encountered in mechanics can be expressed in terms of fundamental units of mass, length and time. In SI units, the standard of length is *metre*, that for mass is *kilogramme* and that for time is *second*. The question arises how we should define these fundamental units. Obviously, the standard unit must be both accessible and invariable. Indeed, the fundamental unit selected must have the following features :

(*i*) It should be of convenient size.
(*ii*) It should be well defined.
(*iii*) It should be easily available so that as many laboratories as possible can duplicate it.
(*iv*) It should not change with time and place.
(*v*) It should not change with the change in physical conditions *e.g.* temperature, pressure *etc.*
(*vi*) It should be universally agreed upon so that results obtained in different countries are comparable.

If we define the length standard as the distance between one's nose and the index finger on an outstretched arm, we certainly have an accessible standard. But it will vary from person to person and cannot be accepted as the standard of length. This calls for precise and accurate definition of the standard quantity selected for fundamental units. The standard quantity must meet the above requirements. *There is international agreement on both the definitions of each of the seven standard quantities and the design of experiments used to measure them.* It is reminded that for mechanics, we only require standard units of length, mass and time.

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2.6. OLD SYSTEMS OF FUNDAMENTAL UNITS

Different units were assigned for mass, length and time in different countries. However, the following two systems of fundamental units need brief description.

(*i*) C.G.S. (centimetre, gramme, second) system.

(*ii*) M.K.S. (metre, kilogramme, second) system.

In C.G.S. system, the units of length, mass and time are centimetre (cm), gramme (g) and second (s) respectively. However, in M.K.S. system, the units of length, mass and time are metre (m), kilogramme (kg) and second (s) respectively. Note that M.K.S. and C.G.S. units are related to each other by multiples of ten. It is, therefore, easier to convert M.K.S. units into C.G.S. units and *vice-versa*. Although M.K.S. system held ground for a long time, it did not quite solve the problem of coherence. A system is said to be coherent when calculations involving one unit of A, one unit of B, one unit of C and so forth, yield one unit of X.

2.7. SI UNITS

The SI units (Systeme International) were formally introduced in 1960 and have been accepted by all the countries for scientific work. In this system, seven base quantities *viz mass, length, time, electric current, temperature, luminous intensity* and *quantity of matter* have been assigned proper and standard units. The units of all other physical quantities in science and engineering can be derived from the units of these seven fundamental quantities. The units assigned to these seven fundamental quantities are given below in the tabular form.

Base Units

Quantity	Symbol	Unit name	Unit symbol
Length	l, L	metre	m
Mass	m	kilogramme	kg
Time	t	second	s
Electric current	I	ampere	A
Temperature	T	degree kelvin	K (not °K)
Luminous intensity	I	candela	cd
Quantity of matter		mole	mol

In addition to these seven base units, two more supplementary units of plane angle and solid angle have been introduced in this system.

Supplementary Units

Plane angle	radian	rad
Solid angle	steradian	sr

Note. In mechanics, we use only the first three of the base units – metre, kilogramme and second – along with measures of angle.

2.8. DEFINITIONS OF SI BASE UNITS

The following are the official definitions of SI base units. Although not required for an understanding of SI, they illustrate the extraordinary precision associated with this modern system of units.

1. **Metre.** In the past, the metre has been defined in various ways.

(*i*) In 1889, the **metre** *was defined to be the distance between two fine lines engraved near the ends of a bar of platinum-iridium alloy when the bar was maintained at 0°C.* The original international prototype metre is still kept at the International Bureau of Weights and Measures in Paris.

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(*ii*) As new methods developed, the above definition of metre was not sufficiently accurate. In 1960, the **metre** *was redefined as 1,650,763.73 wavelengths of orange-red light emitted by the isotope krypton-86.*

(*iii*) Even the above definition became insufficient. In 1983, the metre was redefined in terms of speed of light in vacuum. The speed of light is now one of the most accurately determined quantities. Its value in vacuum has been measured at 299,79,2458 m/s. Therefore, the **metre** *is defined as the length of the path travelled by light in a vacuum during a time interval of 1/299,792,458 of a second.*

2. **Second.** The **second** *is the duration of 9192 631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atoms.*

3. **Kilogram.** The **kilogram** *is the mass equal to the mass of a standard platinum-iridium alloy cylinder (90% platinum and 10% iridium) kept at the International Bureau of Weights and Measures in Paris.* This cylinder is about 4 cm high and 4 cm in diameter. The standard of mass is least satisfactory. Unlike the standards of length and time which are based on measurement procedures that can be repeated by scientists anywhere, the unit of mass is still defined in terms of a particular cylinder kept in Paris.

4. **Ampere.** The **ampere** *is that constant current which, if maintained in two parallel straight conductors of infinite length and of negligible circular cross-section placed 1 metre apart in vacuum, would produce a force between them of* 2×10^{-7} *N per metre of length.*

5. **Kelvin.** The **kelvin** *is 1/273.16 of the thermodynamic temperature of the *triple point of water.*

6. **Candela.** The **candela** *is the luminous intensity in a given direction of a source which emits monochromatic radiation of frequency* 540×10^{12} *hertz and whose radiant intensity is 1/683 watt per steradian.*

7. **Mole.** The **mole** *is the amount of substance of a system that contains as many elementary particles as there are in 0.012 kg of carbon-12.*

SUPPLEMENTARY UNITS

8. **Radian.** The mathematics of circular motion is much simpler if we use the *radian* for angular measure. *We define the angle in radians* (θ) *to be the ratio of the arc length to the radius i.e.,*

$$\theta = \frac{l}{r} \text{ radians} \quad \text{[See Fig. 2.1]}$$

If $l = r$, $\theta = 1$ radian.

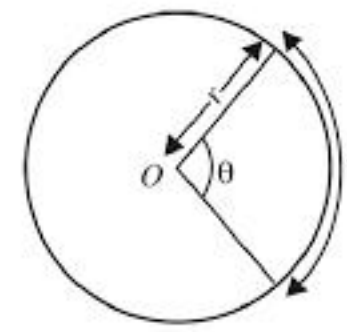


Fig. 2.1

One radian (*rad*) *is defined as the angle subtended by an arc whose length is equal to the radius.*

9. **Steradian.** *It is the measure of solid angle and is defined as the ratio of the subtended area to radius squared.* Referring to Fig. 2.2 (a part of sphere), we have, solid angle, $\omega = \frac{A}{r^2}$.

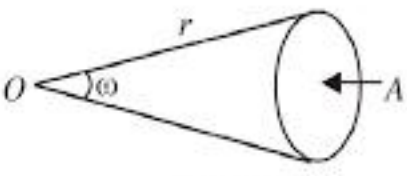


Fig. 2.2

If $A = 1$ m^2 and $r = 1$ m, then $\omega = 1$ steradian.

Hence **one steradian** *(sr) is the angle subtended at the centre of a sphere of radius 1 m by a part of its surface having an area of 1 m*^{2}.

* Pure water in an evacuated cell is cooled until ice begins to form. The resulting temperature where ice, water and water-vapour co-exist is called the triple point of water and is equal to 273.16 kelvin, by definition.

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Multiplication factor	Prefix	Symbol
1 000 000 000 000	10^{12} tera	T
1 000 000 000	10^{9} giga	G
1 000 000	10^{6} mega	M
1 000	10^{3} kilo	k
100	10^{2} hecto	h
10	10^{1} deca	da
0.1	10^{-1} deci	d
0.01	10^{-2} centi	c
0.001	10^{-3} milli	m
0.000 001	10^{-6} micro	μ
0.000 000 001	10^{-9} nano	n
0.000 000 000 001	10^{-12} pico	p

2.16. CONVERSIONS

It is sometimes necessary to convert SI units into C.G.S. or M.K.S. units and *vice-versa*. A few cases are discussed below by way of illustration.

(*i*) 1 km = 1000 m = 10^5 cm = 10^6 mm

(*ii*) 1 km^2 = 10^6 m^2 = 10^{10} cm^2 = 10^{12} mm^2 = 10^{18} μ m^2

(*iii*) 1 tonne/m^3 = 1000 kg/m^3 = 1 g/cm^3 = 1 kg/litre

(*iv*) 1 km/hr = $\frac{1000}{3600}$ m/s = 0.278 m/s = 27.8 cm/s

(*v*) 1 MW = 10^3 kW = 10^6 W = 10^9 mW (*vi*) 1 kWh = 3.6×10^6 J

CONCEPTUAL QUESTIONS

Q.1. In what three types of units can all physical measurements encountered in mechanics be expressed?

Ans. All physical measurements encountered in mechanics can be expressed in the units of mass, length and time.

Q.2. Why is metre now defined in terms of speed of light?

Ans. The speed of light is one of the most accurately determined quantities and its value in vacuum is 299,792,458 kms^{-1}.

Q.3. Why is the unit of mass least satisfactory?

Ans. Unlike the units of length and time, which are based on measurement procedures that can be repeated by scientists anywhere, the unit of mass is still defined in terms of a particular object — the international prototype kilogram kept at the International Bureau of Weights and Measures in France. Although kept under controlled conditions, it could conceivably change in value. Further, comparison with such a standard is less convenient than a procedural standard that can be checked in a laboratory.

Q.4. Why is the unit of mass not based on atomic standards?

Ans. The reason that unit of mass is not yet based on atomic standards is that the measurement of atomic masses and their comparison with large-scale masses is not yet as precise as the measurement that can be made on large-scale objects such as the standard kilogram.

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3

Measurement of Length, Mass and Time

INTRODUCTION

The observations and experiences that interest the physicists span an immense range. Distances extend from incredibly small dimensions of subatomic particles to thousands of million metres that separate galaxies of the universe. For example, the radius of atomic nucleus is about 10^{-14} m while the radius of the universe is about 10^{25} m. The time intervals encountered in the physical world vary over an extremely wide range. For instance, the time taken by light to cross distance of nuclear size is about 10^{-22} s while the life of the sun is about 10^{18} s. Similarly, enormous range of masses in the physical world engage the attention of the physicists. As an example, the mass of an electron is approximately 10^{-30} kg, whereas the mass of universe is about 10^{55} kg. To measure distances, masses and time intervals of such wide varying range, several methods have been devised and we shall discuss a few of them.

3.1. ORDER OF MAGNITUDE

In doing rough calculations or comparisons, we sometimes round off a number to one or even no significant figures.

A number rounded to the nearest power of 10 is called **order of magnitude**.

For example, the height of an ant might be 8×10^{-4} m ; we would say that the order of magnitude of the height of an ant is 10^{-3} m. If the given number is less than 5, it is taken as 1. If it lies between 5 and 10, it is taken as 10. Thus in the above example, 8 has been regarded as 10 so that the order of magnitude of the height = $10 \times 10^{-4} = 10^{-3}$ m. As another example, the speed of light in vacuum is 3×10^{8} ms^{-1}. We would say that the order of magnitude of the speed of light in vacuum is 10^{8} ms^{-1}. Note that the order of magnitude of a physical quantity does not give its exact value but usually it is accurate to within a factor of 10.

Advantages. The knowledge of the order of magnitude of a physical quantity has the following advantages :

(*i*) It gives the approximate value of the physical quantity and can be assumed to be correct within a factor of 10.

(*ii*) To compare two like physical quantities, we need not know their actual values ; their orders of magnitude provide sufficient information.

(*iii*) Sometimes we want to make a rough estimate in order to check an accurate calculation made on a calculator to make sure that no blunders were made when entering the

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$$\text{In } \Delta ABC, \ \tan\theta = \frac{AB}{BC} \ \ldots (i) \ ; \ \text{In } \Delta DEC, \ \tan\theta = \frac{DE}{EC} \ \ldots (ii)$$

From eqs. (i) and (ii), $\dfrac{AB}{BC} = \dfrac{DE}{EC}$ $\therefore$ Height of pole, $AB = \dfrac{DE}{EC} \times BC$

Since lengths DE, EC and BC can be measured, we can find the height of the pole.

Another method. If it is not a sunny day, the height of the accessible object (*e.g.*, electric pole, tower, etc.) can be determined by a sextant. In this method, the angle θ subtended by the top of tower AB at point C on the ground is measured. In ΔABC (See Fig. 3.4),

$$\tan\theta = \frac{AB}{BC} \qquad \therefore \text{ Height of tower, } AB = BC\tan\theta$$

Since the value of θ and BC are known, we can find the height of the tower. Note that angle θ is called elevation of tower at point C.

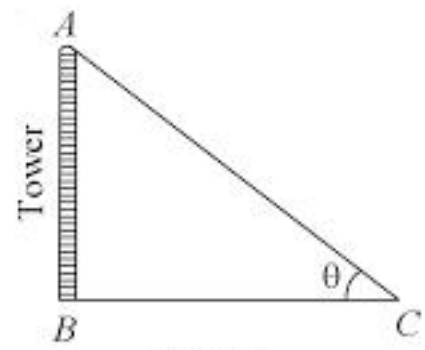


Fig. 3.4

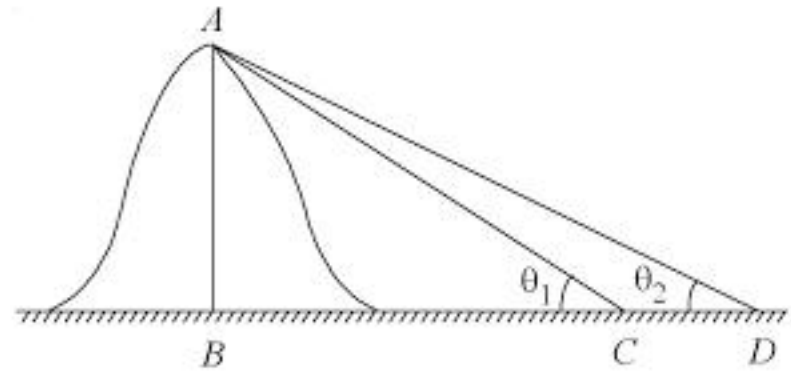


Fig. 3.5

(*viii*) Determination of height of inaccessible objects. The height of an inaccessible object (*e.g.* a mountain) can be determined by triangulation method. Suppose AB is the height of the mountain. The angles of elevation of the peak of the mountain are measured at two convenient points C and D on the ground as shown in Fig. 3.5. Let θ_1 and θ_2 be the angles of elevation of peak of mountain at points C and D respectively.

$$\text{In } \Delta ABC, \ \cot\theta_1 = \frac{BC}{AB} \ ; \ \text{In } \Delta ABD, \ \cot\theta_2 = \frac{BD}{AB}$$

$$\therefore \qquad \cot\theta_2 - \cot\theta_1 = \frac{BD}{AB} - \frac{BC}{AB} = \frac{BD - BC}{AB} = \frac{CD}{AB}$$

or Height of mountain, $AB = \dfrac{CD}{\cot\theta_2 - \cot\theta_1}$

Since the values of θ_1 and θ_2 are known and the distance CD can be measured, we can find the height AB of the mountain.

(*ix*) Determination of width of a river. The width of a river can be determined by triangulation method as shown in Fig. 3.6. Suppose AB is the width of the river; point B being some reference object (*e.g.* a tree). A pole is fixed (perpendicular to the ground) at some point C on the bank of the river so that reference object and the pole at C are in the same line. Now proceeding along the line BC, fix another pole *at* E in such a way that reference object at B and the poles at C and E are in the same line. From E, draw a perpendicular ED on the bank of the river.

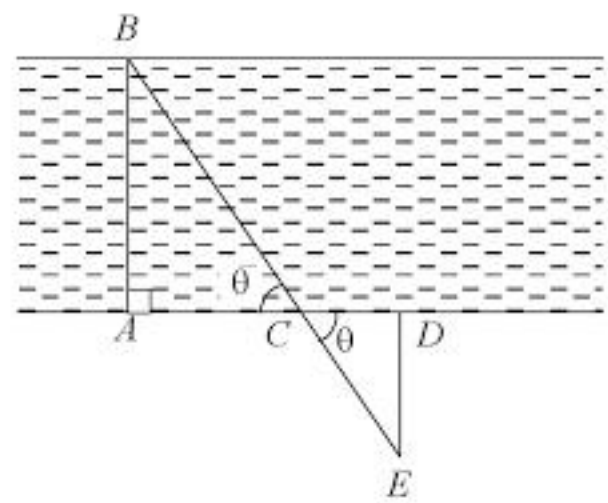


Fig. 3.6

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(*iii*) Size of molecule of a liquid. To measure a very small size like that of a liquid molecule, we have to adopt special methods. A simple method for determining the molecular size of **oleic acid** is given below. Oleic acid is a soapy liquid with large molecular size (10^{-9} m). To find the size of molecule of oleic acid, we form mono-molecular layer (*i.e.*, single-molecule layer) of oleic acid on water surface.

Procedure. First, we prepare a solution of known concentration of oleic acid in alcohol. To prepare a solution of 1/500 concentration (say), 1 c.c. of oleic acid is mixed with 500 c.c. of alcohol. We then pour n drops of this solution on the surface of water contained in a flat and broad vessel. The oleic acid solution spreads very fast on the surface of water and forms a very thin layer of thickness t (say). The alcohol evaporates and oleic acid layer is left on the surface of water. Using a trace paper and a graph paper, we carefully measure the surface area A of the oleic acid layer.

Calculations. If v is the volume of each of the n drops on water surface, then,

Volume of oleic acid in n drops of solution $= n\ v \times \dfrac{1}{500}$

Volume of oleic acid layer $= A \times t$

$$\therefore \qquad nv \times \frac{1}{500} = A \times t \quad \text{or} \quad t = \frac{nv}{500\ A}$$

If the thin layer of oleic acid is assumed to be one molecule thick, the value of t gives the size (*i.e.*, diameter) of a molecule of oleic acid.

Example 3.6. When 96500 C of electricity is passed through silver voltameter, 108 g (1 gram-atomic mass) of silver is deposited on the cathode. What is Avogadro's number N_A ?

Solution. Each silver ion (Ag^+) accepts one electron to become silver atom (Ag) and gets deposited on the cathode. The question becomes: How many electron charges are there in 96500 C? We know that charge on an electron is 1.6×10^{-19} C.

$$\therefore \qquad \text{Avogardo's Number, } N_A = \frac{96500}{1.6 \times 10^{-19}} = \mathbf{6.02 \times 10^{23}}$$

Thus in 108 g of silver, there are 6.02×10^{23} atoms.

Example 3.7. How many water molecules are there in 1g of water?

Solution. The molecular weight of water (H_2O) is that of two hydrogen atoms plus one oxygen atom *i.e.* $2 \times 1.0 + 16.0 = 18.0$. Hence 18 g of water contains N_A ($= 6.02 \times 10^{23}$) molecules.

$$\therefore \qquad \text{Number of molecules in 1g of water} = \frac{N_A}{18} = \frac{6.02 \times 10^{23}}{18} = \mathbf{3.34 \times 10^{22}\ molecules}$$

Example 3.8. How many copper atoms are there in a solid cube of copper which is 1.00 cm on each side ? What is the approximate size of atom? The atomic mass of copper is 63.6 g/mol and density is 8.96 g/cm^3.

Solution.

Mass of copper in the cube = Density × Volume = $8.96 \times 1.00 = 8.96$ g

$$\text{Number of moles} = \frac{\text{Mass}}{\text{Atomic mass}} = \frac{8.96\,\text{g}}{63.6\,\text{g/mol}} = 0.141\,\text{mol}$$

$$\text{Number of atoms} = \text{Number of moles} \times N_A$$

$$= 0.141 \times 6.02 \times 10^{23} = \mathbf{8.49 \times 10^{22}\ atoms}$$

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Procedure. In order to find the mass of a body with a physical balance, the following procedure is adopted :

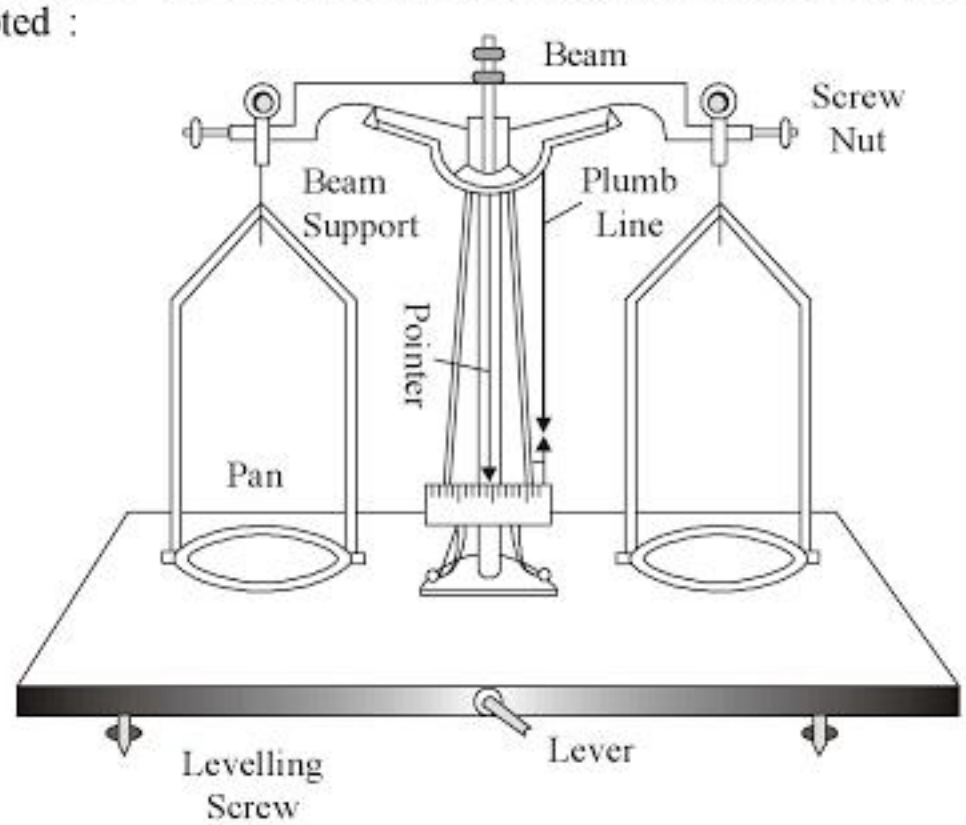


Fig. 3.11

(*i*) The first step is to make the base board horizontal. This can be done by bringing the plumb line just above the pointed mark through the adjustment of levelling screws fixed at the base board.

(*ii*) The next step is to ensure that the beam is horizontal when the pans are empty. For this purpose, the beam is raised gently by the lever and it is ensured that the pointer swings equally on both sides of the zero mark. This can be achieved through the adjustment of screws provided at the ends of the beam.

(*iii*) The beam is now lowered and the body whose mass *(m)* is to be found out is placed on the left pan of the balance. The known weights are placed on the right pan till the pointer remains at zero of the scale or it swings equally on both sides of the zero mark. Under this condition, the balance is in equilibrium. Consequently, the total mass (M) of weights on the right pan gives the mass of the body. It is because weight of the object is equal to weight on the right pan *i.e.*

$$m\,g = M\,g \quad \text{or} \quad m = M$$

Two things are worth noting. First, a physical balance measures the mass of a body at a place and not its weight. Secondly, a physical balance is used for comparing masses of the bodies.

Weight. *The* **weight** *of a body is defined as the gravitational force exerted on it due to earth i.e.,*

$$\text{Weight, } W = \text{gravitational force on the body due to earth}$$
$$= \text{mass of body} \times \text{acceleration due to gravity}$$
$$= m \times g$$
$$\therefore \quad W = m\,g$$

Thus, if a body has a mass of 10 kg, then its weight is $W = m\,g = 10 \times 9.8 = 98$ N. It means that earth is attracting the body with a force of 98 N. The reader may note that mass of an object is different from its weight.

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Q.2. Why parallax method cannot be used for measuring distances of stars more than 100 light years away?

Ans. When a star is more than 100 light years away, the parallax angle is so small that it cannot be measured accurately.

Q.3. What is the difference between mass and weight?

Ans. Refer to Art. 3.7.

Q.4. What is the basic principle of alpha particle scattering method for estimating the size of the nucleus?

Ans. When an alpha particle (positively charged) is projected towards the nucleus, it experiences a repulsive force from the positively charged nucleus. Consequently, α-particle starts slowing down. As the α particle starts slowing down, its kinetic energy starts converting into electric potential energy of the system. At the distance of closest approach (r_0), the entire K.E. of α particle is converted into electric potential energy of the system. The value of r_0 gives us the order of the size of the nucleus.

Q.5. Suggest a distance corresponding to each of the following orders of length: (*i*) 10^7 m (*ii*) 10^4 m (*iii*) 10^3 m (*iv*) 10^2 m (*v*) 10^{-3} m (*vi*) 10^{-6} m (*vii*) 10^{-14} m.

Ans. (*i*) 10^7 m : radius of earth

(*ii*) 10^4 m : height of Mount Everest

(*iii*) 10^3 m : distance travelled by sound in water in 1 s

(*iv*) 10^2 m : length of a college playground

(*v*) 10^{-3} m : thickness of cardboard

(*vi*) 10^{-6} m : mean free path of an air molecule

(*vii*) 10^{-14} m : size of atomic nucleus

Q.6. How will you measure the width of a river?

Ans. Refer to Art. 3.3.

Q.7. When circular scale of a screw gauge carrying 100 divisions is given four complete rotations, the head of the screw moves through 2 mm. Calculate the pitch and least count of the screw gauge.

Ans. No. of divisions on circular scale = 100 ; No. of complete rotations = 4 ; Distance moved = 2 mm.

$$\text{Pitch} = \frac{\text{Distance moved}}{\text{No. of complete rotations}} = \frac{2\,\text{mm}}{4} = 0.5\ \text{mm}$$

$$\text{Least count} = \frac{\text{Pitch}}{\text{No. of divisions on circular scale}} = \frac{0.5\,\text{mm}}{100} = 0.005\ \text{mm}$$

Q.8. What is meant by order of magnitude? Illustrate with examples.

Ans. Refer to Art. 3.1.

Q.9. Mention some repetitive phenomena in nature which could serve as the time standards. Which one is most suitable?

Ans. The following repetitive phenomena occurring in nature can serve as standard for measuring time :

(*i*) Beating of human heart (*ii*) Rotation of earth about its axis

(*iii*) Oscillations of a simple pendulum (*iv*) Vibrations of atoms

Earlier, the rotation of earth was found to be the most appropriate. Now, the vibrations of an atom (cesium–133 atom) is being used to measure time.

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Eq. (*i*) is known as *dimensional equation*. Note that dimensions in every term of the formula must be the same. It is not surprising because we can *add or equate like quantities. Thus in the above formula, the three quantities (*viz. v, u* and *at*) have the same dimensions *i.e.*, $[M^0LT^{-1}]$

4.4. DIMENSIONAL FORMULAS OF PHYSICAL QUANTITIES

Dimensional formula of a physical quantity can be easily written if we know its relation with other physical quantities. Dimensional formulas of a few physical quantities encountered in mechanics are given below in tabular form :

S. No.	Physical Quantity	Relation with other physical quantities	Dimensional Formula	SI Unit
1.	**Area**	length × breadth	$[L]\,[L] = [L^2]$	m^2
2.	**Volume**	length × breadth × height	$[L]\,[L]\,[L] = [L^3]$	m^3
3.	**Density**	$\frac{\text{mass}}{\text{volume}}$	$\frac{[M]}{[L^3]} = [ML^{-3}T^0]$	$kg\ m^{-3}$
4.	**Speed or Velocity**	$\frac{\text{distance}}{\text{time}}$	$\frac{[L]}{[T]} = [M^0LT^{-1}]$	ms^{-1}
5.	**Acceleration**	$\frac{\text{Velocity}}{\text{time}}$	$\frac{[M^0LT^{-1}]}{[T]} = [M^0LT^{-2}]$	ms^{-2}
6.	**Force**	mass × acceleration	$[M][M^0LT^{-2}] = [MLT^{-2}]$	N
7.	**Momentum**	mass × velocity	$[M][M^0LT^{-1}] = [MLT^{-1}]$	$kg\ ms^{-1}$
8.	**Work**	force × distance	$[MLT^{-2}][L] = [ML^2T^{-2}]$	Nm
9.	**Power**	$\frac{\text{work}}{\text{time}}$	$\frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$	W
10.	**Pressure**	$\frac{\text{force}}{\text{area}}$	$\frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$	Nm^{-2}
11.	**Kinetic energy**	$\frac{1}{2}$ × mass × (velocity)^2	$[M][M^0LT^{-1}]^2 = [ML^2T^{-2}]$	Nm
12.	**Potential energy**	mass × g × distance	$[M][M^0LT^{-2}][L] = [ML^2T^{-2}]$	Nm
13.	**Impulse**	force × time	$[MLT^{-2}][T] = [MLT^{-1}]$	Ns
14.	**Torque**	force × distance	$[MLT^{-2}][L] = [ML^2T^{-2}]$	Nm
15.	**Stress**	$\frac{\text{force}}{\text{area}}$	$\frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$	Nm^{-2}
16.	**Strain**	$\frac{\text{extension in length}}{\text{original length}}$	$\frac{[L]}{[L]} = [M^0L^0T^0]$	Number
17.	**Elasticity**	$\frac{\text{stress}}{\text{strain}}$	$\frac{[ML^{-1}T^{-2}]}{[M^0L^0T^0]} = [ML^{-1}T^{-2}]$	Nm^{-2}

* Suppose three quantities A, B and C are related as :

$$A = B + C$$

We can add B and C if they are like quantities *i.e.*, have the same dimensions. Similarly the sum of B and C can be equated to A if they are like quantities.

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(*ii*) length *l* of the string and (*iii*) acceleration due to gravity *g*. We can express the relation as :

$$T = k\, m^a l^b g^c \qquad ...(i)$$

where *k* is a dimensionless constant and *a*, *b* and *c* are the unknown numbers . If we equate the powers of *M*, *L* and *T* on both sides of eq. (*i*), we can find the values of *a*, *b* and *c* and hence the formula for the time period of a simple pendulum.

$$\therefore \quad [T] = [M]^a [L]^b [LT^{-2}]^c \text{ or } [M^0 L^0 T] = [M^a][L^{b+c}][T^{-2c}]$$

Equating the powers of *M*, *L* and *T* on both sides of equation, we have,

$$a = 0\,; \quad b + c = 0 \text{ and } -2\ c = 1$$

$$\therefore \qquad a = 0\,;\ c = -\frac{1}{2} \text{ and } b = \frac{1}{2}$$

Putting the values of *a*, *b* and *c* in eq. (*i*), we have,

$$T = k\, m^0\ l^{1/2} g^{-1/2} \text{ or } T = k\sqrt{\frac{l}{g}}$$

Dimensional analysis does not provide the value of *k* since it is a number. However, we can find the value of *k* by experiment or through mathematical analysis. It is found that $k = 2\pi$.

$$\therefore \qquad \text{Time period, } T = 2\pi\sqrt{\frac{l}{g}}$$

Although we have not been able to find the value of *k* through dimensional analysis, yet we have been able to relate the time period of simple pendulum to the major variables of the situation *viz. l* and *g*. Thus dimensional analysis can be used to find the relation between quantities when the mathematical analysis is too difficult.

4.11. TO CONVERT ONE SYSTEM OF UNITS TO ANOTHER

The magnitude of a physical quantity remains the same irrespective of the system of measurements. This fact enables us to convert one system of units into another.

Illustration. Suppose we have a physical quantity *Q*. Let n_1 and n_2 be numeric values of the physical quantity when measured in two systems having units of size u_1 and u_2 respectively. Then,

$$Q = n_1 u_1 \qquad \text{... on first system of units}$$

$$Q = n_2 u_2 \qquad \text{... on second system of units}$$

$$\text{Clearly,} \quad Q = n_1 u_1 = n_2 u_2$$

Suppose the physical quantity *Q* has the dimensional formula $M^a L^b T^c$. Let the fundamental units be M_1, L_1 and T_1 on the first system and M_2, L_2 and T_2 on the second system. Then,

$$Q = n_1\left[M_1^a L_1^b T_1^c\right] \qquad \text{... on first system of units}$$

$$Q = n_2\left[M_2^a\ L_2^b T_2^c\right] \qquad \text{... on second system of units}$$

$$\therefore \qquad n_1\left[M_1^a L_1^b T_1^c\right] = n_2\left[M_2^a L_2^b T_2^c\right]$$

$$\therefore \qquad n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Thus if we know the value of a physical quantity in one system of units, we can find its value in the other system of units.

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Equating the powers of M, L and T on both sides, we have,

$$a + c = 0\ ;\ a + b = 1\ ;\ -2a = -1 \quad \therefore \quad a = \frac{1}{2}\ ;\ b = \frac{1}{2}\ ;\ c = -\frac{1}{2}$$

Putting the values of a, b and c in eq. (i), we have,

$$v = k F^{1/2} l^{1/2} m^{-1/2}$$

or
$$v = k\sqrt{\frac{Fl}{m}} = k\sqrt{\frac{F}{m/l}} = k\sqrt{\frac{\text{Tension}}{\text{Mass per unit length}}}$$

$\therefore$
$$v \propto \sqrt{\frac{Tension}{Mass\ per\ unit\ length}}$$

Example 4.6. The density ρ of earth is given by :

$$\rho = \frac{3g}{4\pi RG}$$

where R is the radius of earth and G is gravitational constant. Check the dimensional consistency of this relation.

Solution. Dimensions of L.H.S. $= \dfrac{[M]}{[L^3]} = [ML^{-3}]$

Dimensions of R.H.S. $= \dfrac{\left[LT^{-2}\right]}{[L]*\left[M^{-1}L^3T^{-2}\right]} = [ML^{-3}]$

Since the dimensions on the two sides of the equation are the same, the relation is dimensionally consistent.

Example 4.7. Convert 1 km/hr into ms^{-1}.

Solution. $n_2 = n_1\left[\dfrac{L_1}{L_2}\right]\left[\dfrac{T_1}{T_2}\right]^{-1} = 1\left[\dfrac{1\text{km}}{1\text{m}}\right]\left[\dfrac{1\text{hr}}{1\text{s}}\right]^{-1} = 1\left[10^3\right][3600]^{-1} = \dfrac{5}{18}\ \text{m s}^{-1}$

$\therefore$
$$1\text{km/hr} = \frac{\mathbf{5}}{\mathbf{18}}\mathbf{ms^{-1}}$$

Alternatively, $1\text{km/hr} = \dfrac{1\text{km}}{1\text{hr}} = \dfrac{1000\ \text{m}}{60\times 60\ \text{s}} = \dfrac{5}{18}\ \text{ms}^{-1}$

Example 4.8. Find the value of 60 joule/minute on a system which has 100 g, 100 cm and 1 minute as the fundamental units.

Solution. 60 J/min. = 1 J/s = 1 watt

Now watt is the SI unit of power.

S.I. units	Units of new system
$n_1 = 1$ W	$n_2 = ?$
$M_1 = 1$ kg	$M_2 = 100$ g
$L_1 = 1$ m	$L_2 = 100$ cm
$T_1 = 1$ s	$T_2 = 1$ minute

$$[\text{Power}] = \left[ML^2T^{-3}\right] \quad \therefore \quad a = 1\ ;\ b = 2\ ;\ c = -3$$

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* 3 and 4 π are dimensionless and $[G] = [M^{-1}L^3T^{-2}]$.

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4. A quantity X is given by ;

$$X = \varepsilon_0 L \frac{\Delta V}{\Delta t}$$

where ε_0 is the permittivity of free space, L is length, ΔV is the potential difference and Δt is the time interval. The dimensions of X are the same as for **[IIT 2001]**

(a) resistance (b) charge

(c) voltage (d) current

5. If speed of light (c), acceleration due to gravity (g) and pressure (p) are taken as fundamental units, the dimensions of gravitational constant (G) are **[AMU 1999]**

(a) $c^0 g p^{-3}$ (b) $c^2 g^3 p^{-2}$

(c) $c^0 g^2 p^{-1}$ (d) $c^2 g^2 p^{-2}$

6. The dimensions of $\varepsilon_0 E^2/2$ (ε_0 = permittivity of free space, E = electric field) are **[IIT 2000]**

(a) MLT^{-1} (b) ML^2T^{-2}

(c) $ML^{-1}T^{-2}$ (d) ML^2T^{-1}

7. The dimensional formula for μ_0 is given by ; **[Haryana CET 2000]**

(a) M^0LT (b) $MLT^{-2}A^{-2}$

(c) $M^0L^2T^{-1}A^2$ (d) none of the above

8. If L and R denote inductance and resistance respectively, which of the following has the dimensions of frequency? **[MP CET 1999]**

(a) R/L (b) L/R

(c) $\sqrt{R/L}$ (d) $\sqrt{L/R}$

9. The ratio of one micron to one nanometre is **[MNR 1993]**

(a) 10^3 (b) 10^{-3}

(c) 10^{-6} (d) 10^{-9}

10. Which of the following is equal to joule-Ohm/second-volt ? **[Haryana CET 1998]**

(a) ampere (b) watt

(c) tesla (d) volt

11. The unit of permittivity of free space is **[Haryana CET 1998]**

(a) coulomb^2/newton-metre^2

(b) coulomb/(newton-metre)^2

(c) newton-metre^2/coulomb^2

(d) coulomb/newton-metre

12. The physical quantity which has the dimensional formula MT^{-3} is **[Pb CET 1998]**

(a) surface tension (b) solar constant

(c) density (d) compressibility

13. The dimensions of the quantities in one of the following pairs are the same. Identify the pair. **[IIT 1986]**

(a) torque and work

(b) angular momentum and work

(c) energy and Young's modulus

(d) light year and frequency

14. The SI unit of inductance, the henry can be **[IIT 1998]**

(a) weber/ampere

(b) volt-second /ampere

(c) joule/(ampere)^2

(d) all of above

15. A gas bubble from an explosion under water oscillates with a period T proportional to $p^a d^b E^c$ where p is the static pressure, d is the density of water and E is the total energy of explosion. The calculated values of a, b and c on the dimensional analysis will be **[Haryana CET 1994]**

(a) $a = -5/6, b = 2, c = 3$

(b) $a = b = c = 1$

(c) $a = 5,\ b = \frac{1}{2},\ c = \frac{1}{3}$

(d) $a = -\frac{5}{6},\ b = \frac{1}{2},\ c = \frac{1}{3}$

16. In the equation $\left(P + \frac{a}{V^2}\right)(V - b) =$ constant (P = pressure , V = Volume), the units of a are **[MNR 1995]**

(a) dyne × cm^2 (b) dyne × cm^4

(c) dyne/cm^2 (d) dyne × cm^3

17. L, C and R represent the physical quantities inductance, capacitance and resistance respectively. Identify the quantity that represents frequency. **[IIT 1984]**

(a) $\frac{1}{RC}$ (b) $\frac{L}{R}$

(c) $\frac{C}{L}$ (d) $\frac{L}{C}$

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(*ii*) $$\text{Resistance, } R = \frac{\text{Voltage}}{\text{Current}} = \frac{V}{I} \; ; \quad \text{Voltage, } V = \frac{\text{Work done}}{\text{Charge}}$$

$$\therefore \quad R = \frac{\text{Work done}}{\text{Charge} \times \text{Current}}$$

$$\therefore \quad [R] = \frac{[M L^2 T^{-2}]}{[Q][I]} = \frac{[ML^2T^{-2}]}{[IT][I]} = [M\ L^2 I^{-2} T^{-3}]$$

5. In the formula $x = 3\,y\,z^2$, x and z have the dimensions of capacitance and magnetic induction respectively. Find the dimensions of y in MKSQ system. [I.I.T.]

Hint. MKSQ stands for Metre, Kilogram, Second and Charge.

$$x = 3\,y\,z^2$$

or $$y = \frac{x}{3z^2}$$

Now $$\text{Capacitance, } x = \frac{(\text{Charge})^2}{\text{Work done}}$$

$$\therefore \quad [x] = \frac{[Q]^2}{[M L^2 T^{-2}]} = [M^{-1} L^{-2} T^2 Q^2]$$

$$\text{Magnetic induction, } z = \frac{\text{Force}}{\text{Current} \times \text{Length}}$$

$$\therefore \quad [z] = \frac{[M LT^{-2}]}{[Q/T][L]} = [M T^{-1} Q^{-1}]$$

$$\therefore \quad [y] = \frac{[M^{-1} L^{-2} T^2 Q^2]}{[M T^{-1} Q^{-1}]^2} = [M^{-3}\ L^{-2}\ T^4 Q^4]$$

6. Check the correctness of the following equation :

$$h = \frac{2T\cos\theta}{r\,d\,g}$$

where θ is the angle of contact, d is the density of the liquid, r is the radius of the tube, g is acceleration due to gravity, h is the height of the liquid and T is surface tension.

Hint. $$h = \frac{2T\cos\theta}{r\,d\,g} \qquad \ldots (i)$$

The dimensions of h, r, d and g are $[L]$, $[L]$, $[ML^{-3}]$ and $[LT^{-2}]$ respectively.

$$\text{Surface tension, } T = \frac{\text{Force}}{\text{Length}} \quad \therefore \quad [T] = \frac{[M LT^{-2}]}{[L]} = [M T^{-2}]$$

Note that 2 and $\cos\theta$ are dimensionsless.

Putting the dimensions of the various physical quantities in eq. (*i*), we have,

$$[L] = \frac{[MT^{-2}]}{[L][ML^{-3}][LT^{-2}]} = [L]$$

We find that dimensions of L.H.S. are the same as that of R.H.S. Hence, the equation is **correct.**

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Putting the values of a, b and c in eq. (i), we have, $a = k\, r^{-1} v^2 m^0$

or $$a = k\frac{v^2}{r}$$

14. The velocity v of water waves may depend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. Find the relation between these quantities by the method of dimensions. [N.C.E.R.T.]

Hint. As per the conditions of the problem,

$$v \propto \lambda^a \rho^b g^c \quad \text{or} \quad v = k\lambda^a \rho^b g^c \qquad ...(i)$$

where k is a dimensionless constant.

The dimensions of velocity v, wavelength λ, density ρ and acceleration due to gravity g are $[LT^{-1}]$, $[L]$, $[ML^{-3}]$ and $[LT^{-2}]$ respectively. Putting the dimensions of various physical quantities in eq. (i), we have,

$$\left[LT^{-1}\right] = \left[L\right]^a \left[ML^{-3}\right]^b \left[LT^{-2}\right]^c$$

or $$\left[M^0 LT^{-1}\right] = \left[M^b L^{a-3b+c} T^{-2c}\right]$$

Equating the powers of M, L and T on both sides, we have,

$$b = 0\ ;\ a - 3b + c = 1\ ;\ -2c = -1$$

On solving, we get, $a = 1/2$; $b = 0$ and $c = 1/2$.

Putting the values of a, b and c in eq. (i), we have,

$$v = k\,\lambda^{1/2} \rho^0 g^{1/2} = k\,\lambda^{1/2} g^{1/2}$$

or $$v^2 \propto \lambda g$$

15. The position x of a particle depends upon time t according to the equation $x = a\,t + b\,t^2$. Determine the dimensions and the units of a and b. What are the physical quantities denoted by them ? [UPB]

Hint. $$x = a\,t + b\,t^2$$

In this equation, the dimensions of the term x on L.H.S. are $[L]$. Therefore, the dimensions of at as well as $b\,t^2$ on R.H.S. must also be $[L]$.

$\therefore$ Dimensions of $a\,t = [L]$ or Dimensions of $a = \dfrac{[L]}{[T]} = \left[LT^{-1}\right]$

Therefore, the SI unit of a is **ms^{-1}** and it represents the physical quantity **speed**.

Dimensions of $b\,t^2 = [L]$ or Dimensions of $b = \dfrac{[L]}{[T^2]} = \left[LT^{-2}\right]$

Therefore, the SI unit of b is **ms^{-2}** and it represents the physical quantity **acceleration**.

16. The force F is given in terms of time t and displacement x by the equation :

$$F = A\cos Bx + C\sin Dt$$

What are the dimensions of D/B ?

Hint. Bx and Dt are angles and hence dimensionless.

i.e., $$[Bx] = [Dt] \quad \text{or} \quad \left[\frac{D}{B}\right] = \frac{x}{t} = \frac{[L]}{[T]} = \left[LT^{-1}\right]$$

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a least count of 1 mm (or 0.1cm) ; the second observation is made with an instrument having least count of 0.1 mm (or 0.01cm) and the third observation is made with an instrument having least count of 0.01mm (or 0.001cm). Note that last observation is the most accurate.

5.2. LIMIT OF PRECISION OF MEASURING INSTRUMENT

Every measuring instrument has an inherent limit of precision or accuracy. This limit is determined by the least count of the instrument. For example, a distance measured with an ordinary metre rod is correct upto 1 mm or 0.1 cm. No matter how carefully we measure, we can never obtain a result more precise than the limit of our measuring device. A general guideline is that a given measuring instrument has a limit of precision equal to one-half of the smallest division of measurement built into the device. When you make a measurement, you read the measured quantity to the nearest mark on the device. The "true value" of this measurement thus lies within a range of one-half of the smallest division of the device above or below the indicated mark.

The **limit of precision** *of a measuring instrument is* $\pm\frac{1}{2}$ *of the smallest division of measurement the device is able to display.*

For example, a metre rod marked in millimetres (mm) has a limiting precision of ± 0.5 mm or ± 0.05 cm. A vernier calliper that can read to nearest 0.1 mm has a limiting precision of ± 0.005 cm. Thus if the length of a rod as measured by a metre rod is 23.2 cm, the measurement can be written as 23.2 ± 0.05 cm. This means that "true value" of length most likely lies between 23.25 cm and 23.15 cm. *Sometimes limiting precision is taken equal to the least count of the instrument.*

5.3. SIGNIFICANT FIGURES

Suppose you measure the length of a table with a metre rod and find that it lies between 52 cm and 53 cm. Looking carefully at the metre rod, you see that the end of the table lies between 52.3 cm and 52.4 cm marks. You directly read 52.3 cm from the metre rod but cannot tell about further distance because there is no marking between 3 mm and 4 mm marks. You *estimate that this distance is 0.7 mm or 0.07 cm so that you record the length of table as 52.37 cm. Note that you are certain about the digits 5, 2 and 3 but you are doubtful about the last digit (*i.e.*7). We say that number of significant figures in the measurement is 4.

The number of digits in a measurement about which we are certain plus one additional digit which is uncertain are known as **significant figures**.

Thus in the above example, we expressed the length of the table as 52.37 cm. This measurement is expressed to four significant figures – three certain, one estimated. It may be noted that in recording a measurement or calculation, *one and only one doubtful digit is retained.*

The greater the number of significant figures obtained in a measurement, the more accurate is the measurement. Since measuring instruments have a limit of precision, there is a limit on the number of significant figures that can be recorded in a measurement. Let us illustrate this point. Suppose you measure the length of a rod with a metre scale and find it to be 3.24 cm. The measurement is expressed to three significant figures – two certain, one estimated. If you

* It is difficult for the observer to estimate between the smallest divisions of the measuring device. The estimation would vary from observer to observer.

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$$\text{Mean absolute error, } \overline{\Delta T} = \frac{\Sigma|\Delta T|}{5} = \frac{0.01+0.06+0.2+0.09+0.18}{5} = 0.11 \text{ s}$$

$$\therefore \quad \text{Percentage error} = \frac{\overline{\Delta T}}{\overline{T}} \times 100 = \frac{0.11}{2.62} \times 100 = \mathbf{4.2\%}$$

In this experiment, the result should be written as: $T = 2.62 \pm 0.11$ s

Example 5.10. What is the percentage error in the measurement 8.9 ± 0.2 cm?

Solution. Percentage error $= \frac{0.2}{8.9} \times 100 = \mathbf{2.2\%}$

5.9. PROPAGATION OF ERRORS

Any physics experiment involves a series of measurements and each measurement is associated with a certain amount of error. In order to calculate the final result, we may have to perform mathematical operations like addition, subtraction, multiplication, division *etc.* on these measurements. The final error not only depends upon the errors in individual measurements but also on the mathematical operations performed on these measurements. We shall now calculate the *maximum permissible error in different mathematical operations.

(*i*) Error in sum. Suppose the result Q is given as the sum of two observed quantities a and b *i.e.*

$$Q = a + b$$

If Δa and Δb are the absolute errors in the measurements of quantities a and b, the measurements should be recorded as $a \pm \Delta a$ and $b \pm \Delta b$. If ΔQ is the absolute error in the sum, then,

$$Q \pm \Delta Q = (a \pm \Delta a) + (b \pm \Delta b)$$

or $$Q \pm \Delta Q = (a + b) \pm (\Delta a + \Delta b)$$

or $$\pm \Delta Q = \pm (\Delta a + \Delta b)$$

$\therefore$ Maximum error in $Q = \Delta Q = +(\Delta a + \Delta b) = -(\Delta a + \Delta b)$

$\therefore$ **Maximum absolute error in Q = Maximum absolute error in a + Maximum absolute error in b**

Therefore, when two physical quantities are added, the maximum absolute error in the result is equal to the sum of the absolute errors in the physical quantities.

Illustration. Suppose the two lengths a and b are recorded as:

$a = 25.4 \text{ cm} \pm 0.1 \text{ cm}$ and $b = 16.5 \text{ cm} \pm 0.1 \text{ cm}$

We are required to find the sum $Q = a + b$.

(a) Q has its average value when both a and b have their average values.

$\therefore$ Average value of $Q = 25.4 + 16.5 = 41.9$ cm

(b) Q has its maximum value when both a and b have the largest values.

$\therefore$ Maximum value of $Q = (25.4 + 0.1) + (16.5 + 0.1) = 42.1$ cm

$\therefore$ Magnitude of maximum error in $Q = 42.1 - 41.9 = 0.2$ cm

Note that error in Q is 0.2 cm (= 0.1 cm + 0.1 cm) which is simply the sum of the errors in a and b.

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* We generally define error as the maximum amount by which a physical quantity is expected to differ from its true value. Therefore, we indicate maximum error in the result.

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It is desired to find the fractional error and percentage error in the measurement of ρ.

$$\rho = \frac{m}{V} \qquad \therefore \ \frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V} = \frac{0.1}{3.5} + \frac{1}{10} = 0.03 + 0.1 = 0.13$$

Therefore, fractional error in ρ is 0.13 and percentage error = $0.13 \times 100 = 13\%$.

Alternatively; % error in ρ = % error in m + % error in V

$$= 0.03 \times 100 + 0.1 \times 100 = 3\% + 10\% = 13\%$$

(v) Error in power of a quantity. Let $Q = a^2$

If Δa is the absolute error in the measurement of the quantity a, then the measurement should be recorded as $a \pm \Delta a$. If ΔQ is the absolute error in the result, then,

$$Q \pm \Delta Q = (a \pm \Delta a)^2 = a^2 + (\Delta a)^2 + 2a\ \Delta a$$

We can ignore the term $(\Delta a)^2$, since it is the product of two small quantities and is therefore small in comparison with other terms.

$$\therefore \qquad Q \pm \Delta Q = a^2 \pm 2a\ \Delta a$$

or $$\pm \Delta Q = \pm 2a\ \Delta a$$

Dividing both sides by $Q = a^2$, we have,

$$\pm\frac{\Delta Q}{Q} = \pm\frac{2a\Delta a}{a^2} = \pm 2\frac{\Delta a}{a}$$

$\therefore$ Maximum fractional error in $Q = \dfrac{\Delta Q}{Q} = +2 \times \dfrac{\Delta a}{a} = -2 \times \dfrac{\Delta a}{a}$

$\therefore$ Maximum fractional error in $Q = 2 \times$ fractional error in a

In general, if $Q = a^n$, then, $\pm\dfrac{\Delta Q}{Q} = \pm n\dfrac{\Delta a}{a}$

i.e. the fractional error (or percentage error) is n times the fractional error (or percentage error) in a. Note that pure numbers have no errors.

(vi) General case. Let $Q = \dfrac{a^l b^m}{c^n}$

It can be proved that : $\dfrac{\Delta Q}{Q} = l \times \dfrac{\Delta a}{a} + m \times \dfrac{\Delta b}{b} + n\dfrac{\Delta c}{c}$

i.e. maximum fractional error in $Q = l \times$ fractional error in $a + m \times$ fractional error in $b + n \times$ fractional error in c

Example 5.11. The force acting on an object of mass m, travelling at velocity v in a circle of radius r is given by ;

$$F = \frac{mv^2}{r}$$

The measurements are recorded as $m = 3.5$ kg $\pm$ 0.1kg ; $v = 20$ ms^{-1} $\pm$ 1ms^{-1} ; $r = 12.5$ m $\pm$ 0.5 m.

Find the maximum possible (*i*) fractional error (*ii*) percentage error in the measurement of force. How will you record the reading ?

Solution.

(*i*) $$F = \frac{mv^2}{r} \qquad \therefore \ \frac{\Delta F}{F} = \frac{\Delta m}{m} + 2 \times \frac{\Delta v}{v} + \frac{\Delta r}{r}$$

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(*ii*) The bob of a swinging pendulum can be considered as a particle since its size is very small as compared to the length of the string.

(*iii*) A spacecraft travelling towards the moon can be considered as a particle. It is because the length of the spacecraft can be ignored as compared to the distance involved.

Therefore, a particle is not necessary a very small object as commonly imagined. Rather we take particle to be simply an object that is small enough for its size to be unimportant for the scale of our observation. *Therefore, we are free to consider an object or a body as a particle to simplify the problems.*

6.5. LOCATION OF POINT IN SPACE

The space we live in is three dimensional *i.e.*, we can specify the position of any point in space with three numbers. To do so, we choose a convenient point O as the origin. Through O, we draw three mutually perpendicular axes OX, OY and OZ as shown in Fig. 6.1. This is known as rectangular (cartesian) coordinate system. The three numbers – the distances along OX axis, OY axis and OZ axis – will specify the position of the point P in space. We say that coordinates of point P are x, y and z. If all the three coordinates x, y and z of an object remain unchanged with time, we say the object is at rest *w.r.t.* the coordinate system. If any one or more of the object's coordinates change with time, we say that the object is moving *w.r.t.* the coordinate system.

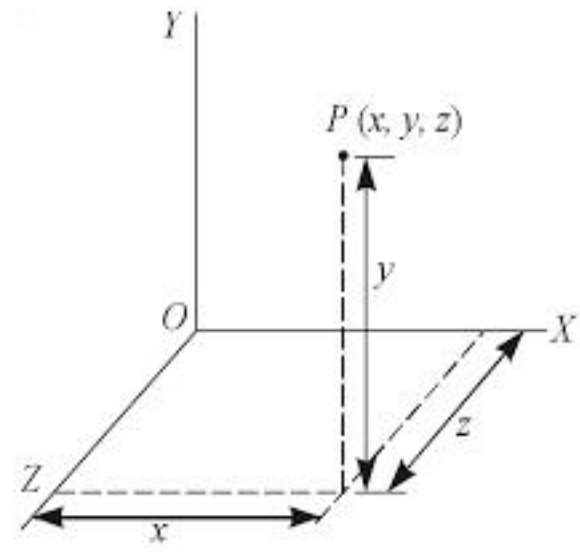


Fig. 6.1

6.6. FRAME OF REFERENCE

When we say that a car is moving at 80 km/hr, what does it mean? It usually means *w.r.t.* the surface of the earth. But earth is moving at 30 km/hr relative to the sun so that car's speed relative to sun is different from 80 km/hr. Statements about motion are meaningful only when we have an answer to the question "motion relative to what"?

The place from which motion is observed and measured is called **frame of reference.**

In order to measure motion (*i.e.*, to locate the position of an object), we need a coordinate system. This coordinate system is attached to some specified body, usually earth's surface. *Unless stated otherwise, frame of reference means coordinate system attached to the earth.* We add clock to the frame of reference to measure time.

6.7. MOTION IN THREE, TWO AND ONE DIMENSIONS

In order to describe the motion of an object, we consider the earth as our frame of reference *i.e.*, the coordinate system is attached to the surface of the earth. We also add clock to our frame of reference to measure time.

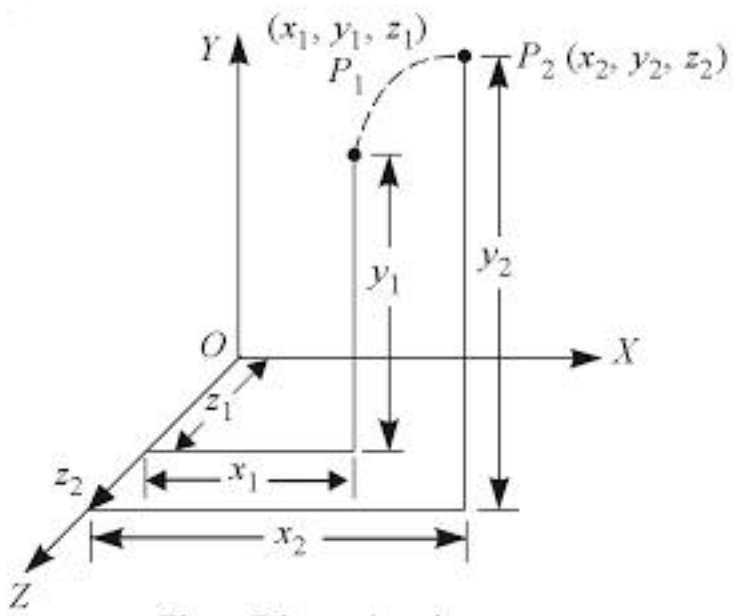
(*i*) **Three dimensional motion.** The motion of an object in space is generally three dimensional motion *i.e.*, all the three coordinates (x, y and z) change with time.

The motion of an object is said to be **three dimensional** *if all the three coordinates* (x, y, z) *of the object change with time.*

Fig. 6.2 illustrates the three dimensional motion of an object. As the object moves from P_1 to P_2, all the three coordinates change *i.e.*, from (x_1, y_1, z_1) to (x_2, y_2, z_2). The examples of three dimensional motion are (*a*) a bird flying in space (*b*) an aeroplane in motion in space *etc.*

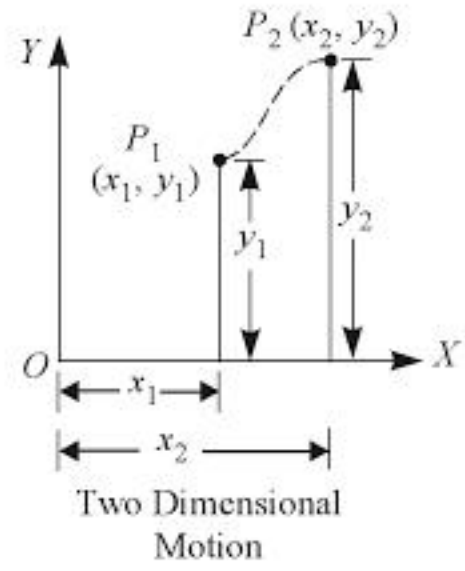
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(*ii*) **Two dimensional motion.** The motion of an object in a plane is two dimensional motion *i.e.*, only two of the three coordinates (say x and y) change with time.



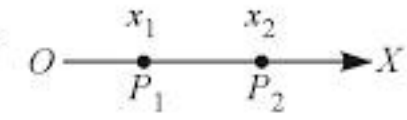
Three Dimensional Motion

Fig. 6.2



Two Dimensional Motion

Fig. 6.3



One Dimensional Motion

Fig. 6.4

The motion of an object is said to be **two dimensional** *if two of the three coordinates of the object change with time.*

Fig. 6.3 illustrates the two dimensional motion of an object. Note that motion of the object is restricted to XY plane. Therefore, as the object moves in this plane, only x and y coordinates change. Thus as the object moves from P_1 to P_2, the coordinates (x_1, y_1) change to (x_2, y_2). The examples of two dimensional motion are (*a*) projectile motion *i.e.*, the motion of a body thrown in air (*b*) motion of a body in a circle with constant speed *etc.*

(*iii*) **One dimensional motion.** The motion of an object along a straight line is one dimensional motion *i.e.*, only one of the three coordinates (say x) changes with time.

The motion of an object is said to be **one dimensional** *if only one of the three coordinates of the object changes with time.*

Fig. 6.4 illustrates the one dimensional motion of an object. Note that motion of the object is along OX axis. As the object moves from P_1 to P_2 along OX axis, the x-coordinate changes from x_1 to x_2. This is the simplest type of motion. Examples of one dimensional motion are (*a*) motion of a train along a straight railway track (*b*) the vertical fall of a stone *etc.*

6.8. SCALAR AND VECTOR QUANTITIES

The physical quantities (*e.g.* mass, length, velocity, force *etc.*) in physics can be divided into two classes *viz.* (*i*) scalar quantities (*ii*) vector quantities.

(*i*) **Scalar quantity.** *A physical quantity which has magnitude only but no direction is called a* **scalar quantity** *or a* **scalar**.

Examples of scalar quantities are mass, length, energy, temperature, speed *etc.* The magnitude or size of a scalar indicates its numerical value and unit of measure *e.g.* mass of 10 kg ; time of 2 hours and so on. The scalars can be added, subtracted, multiplied and divided by using ordinary laws of algebra.

(*ii*) **Vector quantity.** *A physical quantity which has magnitude as well as direction is called a* **vector quantity** *or a* **vector**.

Examples of vector quantities are force, velocity, acceleration, momentum and so on. The magnitude of a vector is a scalar that indicates only how large or small the vector is. Often

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the direction of a vector is as important as its magnitude. For the *addition, subtraction or multiplication of vectors, their directions must be considered. Therefore, rules of ordinary algebra cannot be used to carry out these operations. Instead, we use vector algebra for this purpose.

A vector is represented graphically by a straight line with an arrowhead as shown in Fig. 6.5. The length of the line represents the magnitude of the vector (on suitable scale) and arrowhead indicates its direction. When we write a vector such as in Fig. 6.5 in hand, it is represented as $\vec{F}$. The magnitude of the vector (called modulus of vector) is represented as $|\vec{F}|$ or F (lightface). However, we print its symbol in boldface type as $\boldsymbol{F}$.



Fig. 6.5

6.9. MOTION IN A STRAIGHT LINE

We can describe the motion of an object in two ways : *with mathematical equations* and *with graphs*. In order to study the motion graphically, we must be able to measure the position of the object and time. Since motion along a straight line (*i.e.*, one dimensional motion) is the simplest, we shall use it to describe these concepts.

(*i*) **Position measurement.** In order to locate the position of an object, we need a frame of reference. A convenient way is to fix the coordinate system to earth's surface as shown in Fig. 6.6. Here O is the origin and serves as a reference point for all measurements. *An object moving along X-axis or Y-axis undergoes one dimensional motion.* The positive and negative directions along the axes shown are the usual convention. According to this convention, the measurements to the right of O are taken as positive while the measurements to the left of O are taken negative. Similarly, measurements above O are positive while those below O are negative. It may be noted that the choice of origin is arbitrary. However, once the choice of origin is made in a problem, it should remain unchanged. The SI unit of distance measurement is metre.

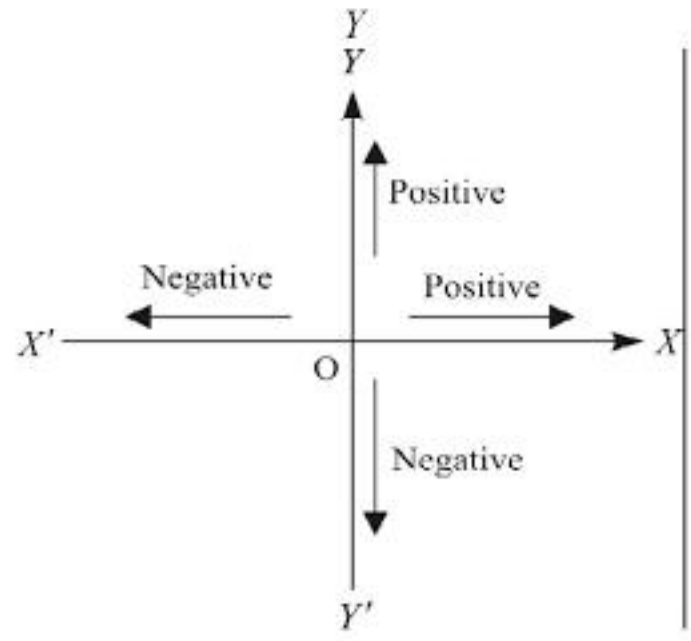


Fig. 6.6

(*ii*) **Time measurement.** We can add a clock to our reference frame to measure time. In order to describe the motion of an object, it is necessary to choose some instant of time as the origin. This instant is assigned zero value of time *i.e.*, $t = 0$. It may be noted that $t = 0$ is not necessarily the beginning of the motion. *In fact $t = 0$ is the instant at which we start the study of motion of the object.* The usual unit of time is second but sometimes it is more convenient to use minute or hour as the unit of time.

Illustration. Consider the one dimensional motion of an object along OX axis as shown in Fig. 6.7. Here O is the origin for position measurement. Suppose we start the observation of motion of the object when it is at the origin O. Therefore, the object is at O, at $t = 0$. Let at any

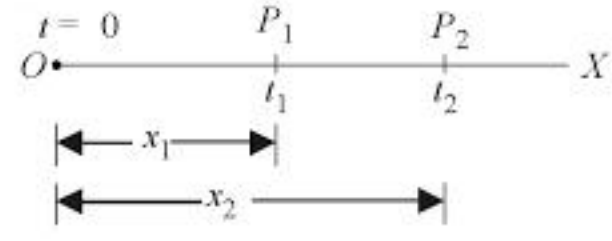


Fig. 6.7

* Division of vector by a vector is not a valid operation in vector algebra. It is because the division of a vector by a direction is not possible.

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time t_1, the object be at point P_1 and covers a distance (= displacement) x_1. We say that the position coordinate of the object is x_1. Similarly, at time t_2, the position coordinate of the object is x_2. Note that when we say that position of an object is x_1, it means that object is on the x-axis and its distance (or displacement) from the origin O is x_1.

Note. In one dimensional motion, magnitude of displacement is equal to distance travelled.

6.10. DESCRIPTION OF MOTION

To describe the motion of an object, we need the concepts of *displacement, velocity* and *acceleration.* These are vector quantities *i.e.*, they possess magnitude as well as direction. For the motion of an object in two or three dimensions, the detailed knowledge of vectors is essential. However, for one dimensional motion, we need only one axis (*e.g.*, x-axis or y-axis) to indicate the position of the object. For such a restricted motion, there are only two *possible directions *viz.* positive and negative. Therefore, vector symbols are not needed for one dimensional motion. The sign (+ or –) indicates the direction of motion along the axis. In this entire unit, we shall study the concepts of displacement, velocity, acceleration and related topics considering one dimensional motion.

6.11. DISTANCE AND DISPLACEMENT

(*i*) **Distance.** *The total length of the path travelled by an object during a given time is called the distance travelled or covered by the object in that time.* It is a scalar quantity and is measured in metres in SI units. The value of distance cannot be negative.

(*ii*) **Displacement.** *The shortest distance from the initial position to the final position of an object is called displacement of the object.* It is a vector quantity *i.e.*, it has magnitude as well as direction. The displacement is measured in metres in SI units and its value can be zero, positive or negative.

Fig. 6.8 shows the concept of displacement. The path of the object may be *ACB, ADB etc.* but the displacement is *AB i.e.*, the shortest distance between the initial position A and the final position B of the object. The magnitude of the displacement is the length of the straight line AB and its direction is from the initial position (A) to the final position (B). The displacement of an object between two points does not tell exactly how the object has actually moved between these points. Since displacement is a vector quantity, its magnitude as well as direction must be specified.

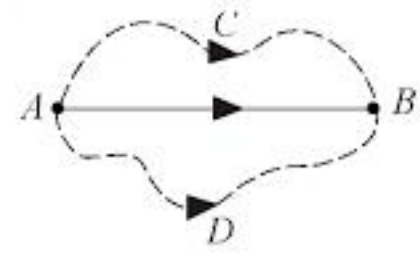


Fig. 6.8

Consider the one dimensional motion of an object along OX axis as shown in Fig. 6.9. Let the time be measured from the instant the object is at the origin O. Suppose at time t_1, the object is at point P_1 with coordinate x_1 and at time t_2, it is at P_2 where its coordinate is x_2. The change in the position of the object (*i.e.*, $x_2 - x_1$) is the *displacement* of the object during time interval (t_2–t_1).

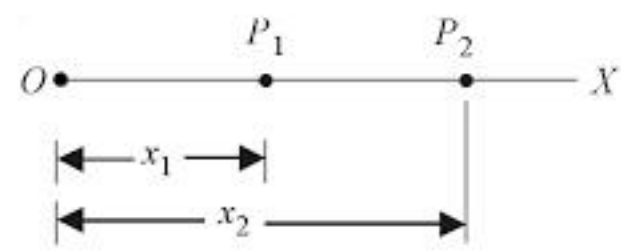


Fig. 6.9

$$\text{Displacement, } \Delta x = x_2 - x_1$$

It is a usual practice to use Greek letter Δ to indicate the change in a quantity. Thus, the change in x is written as **Δx .

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* There are two directions from the origin—left and right or up and down.

** The notation Δx (read 'delta x') stands for a single quantity ; it is not a product of Δ and x.

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6.12. SPEED

The speed of a moving object is defined as the total distance covered divided by the time taken to cover the distance i.e.,

$$\text{Speed} = \frac{\text{Total distance covered}}{\text{Time taken}}$$

The SI unit of speed is ms^{-1} but sometimes the speed of an object is expressed in km/hr.

(*i*) The dimensional formula of speed is $[M^0 LT^{-1}]$.

(*ii*) Speed is a scalar quantity *i.e.*, it has magnitude only but no direction.

(*iii*) The speed of an object can be positive or zero but not negative. It is because distance cannot be negative.

1. **Uniform speed.** *An object is said to be moving with a uniform speed (i.e., constant speed) if it covers equal distances in equal intervals of time, however small these time intervals may be.*

   Thus if a car moves with a uniform speed of 60 km/hr for 2 hours, the distance covered by the car in 2 hours = 60 × 2 = 120km.

2. **Variable speed.** *An object is said to be moving with a variable speed if it covers equal distances in unequal intervals of time, however small these time intervals may be.*

3. **Average speed.** When an object moves with a variable speed, we generally describe its motion in terms of average speed.

   The average speed of an object is the total distance covered divided by the time taken to cover the distance i.e.,

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Time taken}}$$

   For example, if a car covers a distance of 200 km in 4 hours, then average speed of the car = 200/4 = 50 km/hr. Note that average speed tells us nothing about the different speeds and variations that may have taken place during the journey.

4. **Instantaneous speed.** When an object is moving with a variable speed, it has different speeds at different instants of time.

   The speed of an object at a given instant of time is called its instantaneous speed.

The speedometer of a car indicates the instantaneous speed. When we say that the speed of a car at some particular instant is 60 km/hr, we are specifying its instantaneous speed. Note that instantaneous speed is quite different from the average speed.

Example 6.1. An object travels different distances with different speeds in the same direction. Find the expression for the average speed.

Solution. Suppose an object travels distances S_1, S_2, S_3 etc. with speeds v_1, v_2, v_3 etc. in the same direction respectively.

$$\text{Total distance travelled} = S_1 + S_2 + S_3$$

$$\text{Total time taken} = \frac{S_1}{v_1} + \frac{S_2}{v_2} + \frac{S_3}{v_3}$$

$$\therefore \quad \text{Average speed} = \frac{\boldsymbol{S_1 + S_2 + S_3}}{\dfrac{\boldsymbol{S_1}}{\boldsymbol{v_1}} + \dfrac{\boldsymbol{S_2}}{\boldsymbol{v_2}} + \dfrac{\boldsymbol{S_3}}{\boldsymbol{v_3}}}$$

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Special case. If the object travels two equal distances ($S_1 = S_2 = S$) with different speeds v_1 and v_2, then,

$$\text{Average speed} = \frac{S+S}{\frac{S}{v_1}+\frac{S}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$$

Example 6.2. An object travels with different speeds during different time intervals in the same direction. Find the expression for the average speed.

Solution. Suppose an object travels with speeds v_1, v_2, v_3 etc. during time intervals t_1, t_2, t_3 etc. in the same direction.

$$\text{Total distance travelled} = v_1 t_1 + v_2 t_2 + v_3 t_3$$

$$\text{Total time taken} = t_1 + t_2 + t_3$$

$$\therefore \quad \text{Average speed} = \frac{\boldsymbol{v_1 t_1 + v_2 t_2 + v_3 t_3}}{\boldsymbol{t_1 + t_2 + t_3}}$$

Special case. If $t_1 = t_2 = t_3 = t$, then,

$$\text{Average speed} = \frac{v_1+v_2+v_3}{3}$$

6.13. VELOCITY

Velocity of an object is defined as the displacement divided by the time taken for the displacement i.e.,

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time taken}}$$

The SI unit of velocity is ms^{-1}. Velocity is a vector quantity *i.e.*, it has magnitude as well as direction. For example, if a body has a displacement of 50 m in 5 s towards north, then its velocity = 50/5 = 10 ms^{-1} towards north.

(*i*) The dimensional formula of velocity is $[M^0 LT^{-1}]$.

(*ii*) The direction of velocity is the same as that of the displacement.

(*iii*) Since the displacement can be positive, zero or negative, the velocity of an object may be positive, zero or negative.

1. Uniform velocity. *An object is said to be moving with a uniform velocity (i.e., constant velocity) if it undergoes equal displacements in equal intervals of time, however small these time intervals may be.* Uniform velocity means that magnitude as well as direction of the velocity remains constant.

2. Variable velocity. *A body is said to be moving with a variable velocity (i.e., non-uniform velocity) if there is a change in its magnitude or direction or both.*

A body moving with a constant speed in a circle has a variable velocity. Although speed (magnitude of velocity) is constant, the direction of velocity of the body is changing from instant to instant.

3. Average velocity. When an object moves with a variable velocity, we describe its motion in terms of average velocity.

The average velocity of an object is defined as the displacement divided by the time taken for the displacement i.e.,

$$\text{Average velocity, } \bar{v} = \frac{\text{Displacement}}{\text{Time taken}}$$

The bar above the v indicates average velocity.

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Consider one dimensional motion of an object along OX axis as shown in Fig. 6.10. Let at $t = 0$, the object be at the origin O. Suppose at time t_1, the object is at point P_1, with coordinate x_1 and at time t_2, it is at point P_2 where its coordinate is x_2. Therefore, during time interval $(t_2 - t_1)$:

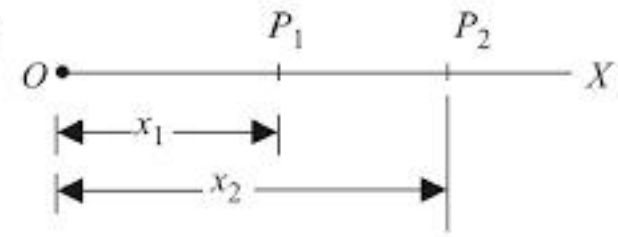


Fig. 6.10

$$\text{Displacement, } \Delta x = x_2 - x_1$$

$$\text{Time interval, } \Delta t = t_2 - t_1$$

$$\therefore \quad \text{Average velocity, } \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Note that displacement and velocity may be positive or negative depending upon the direction of motion of the object. A positive value indicates that the motion is to the right of origin O; a negative value indicates that motion is to the left (*i.e.*, towards the decreasing value of x).

4. Instantaneous velocity. *The velocity of an object at any given instant of time is called its instantaneous velocity.*

Consider an object moving with a uniform velocity of 5 ms^{-1}. It means that the object is moving with a speed (magnitude of velocity) of 5 ms^{-1} in a straight line. The velocity of the object at any instant (*i.e.*, instantaneous velocity) is 5 ms^{-1} and, therefore, its average velocity is also 5 ms^{-1}.

Hence for uniform motion, the average velocity of an object is equal to its instantaneous velocity.

Note. When we are dealing with one dimensional motion (*i.e.*, motion in a straight line), we need not use vector notation for displacement and velocity. Positive and negative signs suffice.

6.14. SPEED VERSUS VELOCITY

Loosely speaking, we often use the words speed and velocity interchangeably. However, the two physical quantities are different. This becomes apparent from the following discussion :

(*i*) Speed is a scalar quantity and has magnitude only. However, velocity is a vector quantity and has magnitude as well as direction.

(*ii*) Speed indicates how fast something is moving whereas velocity tells us how fast and in which direction it is moving.

(*iii*) The distinction between speed and velocity can be made more clear if we refer to Fig. 6.11. Here a car is moving round a circular track at a constant speed of 40 km/hr. At every point on the track, the speed is the same *i.e.*, 40 km/hr. But at every point (such as A, B and C) the velocity is different. Thus the magnitude of velocity at points A, B and C is the same but the direction of velocity is different. Therefore, we note that velocities at all points (*e.g.*, A, B and C) are different, even though the magnitudes of the velocities are same.

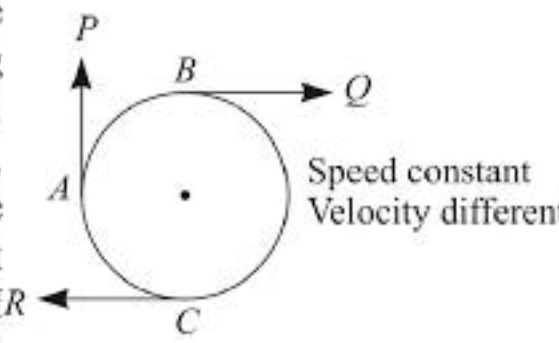


Fig. 6.11

Example 6.3. A particle is moving along X-axis. At time $t_1 = 2$ s, its position is $x_1 = 3$ m and at $t_2 = 7$ s, its position is $x_2 = 18$ m. What is the average velocity of the particle?

Solution. $\text{Displacement, } \Delta x = x_2 - x_1 = 18 - 3 = 15\text{m}$

$$\text{Time interval, } \Delta t = t_2 - t_1 = 7 - 2 = 5 \text{ s}$$

$$\therefore \quad \text{Average velocity, } \bar{v} = \frac{\Delta x}{\Delta t} = \frac{15}{5} = \mathbf{3\ ms^{-1}}$$

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Note that the displacement and average velocity are positive, indicating that the particle moved to the right *i.e.*, towards increasing values of x.

Example 6.4. A train is travelling due east at an average speed of 120 km/hr. How far does this train travel in 6 s?

Solution. Since the time interval is given in seconds, we shall convert the magnitude of velocity in metres per second.

$$120 \text{ km/hr} = \frac{120\,\text{km}}{1\,\text{hr}} = \frac{120\times10^3\,\text{m}}{1\times3600\,\text{s}} = 33.3\,\text{ms}^{-1}$$

$$\text{Average velocity, } \bar{v} = \frac{\Delta x}{\Delta t}$$

$$\therefore \quad \text{Displacement, } \Delta x = \bar{v}\,\Delta t$$

$$\text{Here, } \bar{v} = 33.3\,\text{ms}^{-1};\ \Delta t = 6\,\text{s}$$

$$\therefore \quad \Delta x = 33.3 \times 6 = \mathbf{200\ m}$$

Example 6.5. A runner runs 100 m in 10 s, then turns around and jogs 50 m back toward the starting point in 30 s. Find his (*i*) average speed (*ii*) average velocity.

Solution. (*i*) Total distance travelled = 100 + 50 = 150 m

Total time taken = 10 + 30 = 40 s

$$\therefore \quad \text{Average speed} = \frac{150}{40} = \mathbf{3.75\ ms^{-1}}$$

(*ii*) Total displacement, Δx = 100 – 50 = 50m

Time interval, Δt = 10 + 30 = 40s

$$\therefore \quad \text{Average velocity, } \bar{v} = \frac{\Delta x}{\Delta t} = \frac{50}{40} = \mathbf{1.25\ ms^{-1}}$$

Example 6.6. A particle moves along a circle of radius R as shown in Fig. 6.12. It starts from A and moves in anticlockwise direction. Calculate the distance travelled and displacement (*i*) from A to B (*ii*) form A to C and (*iii*) from A to D.

Solution. (*i*) Distance travelled from A to $B = \frac{2\pi R}{4} = \frac{\pi R}{2}$

$$\text{Displacement, } |AB| = \sqrt{OA^2 + OB^2}$$

$$= \sqrt{R^2 + R^2} = \sqrt{2}\,R$$

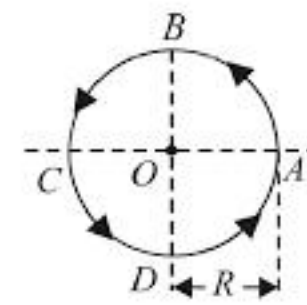


Fig. 6.12

(*ii*) Distance travelled from A to $C = \frac{2\pi R}{2} = \pi R$

Displacement, $|AC| = 2R$

(*iii*) Distance travelled from A to $D = 2\pi R \times \frac{3}{4} = 1.5\pi R$

$$\text{Displacement, } |AD| = \sqrt{R^2 + R^2} = \sqrt{2}\,R$$

Example 6.7. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km/h. What is the magnitude of average velocity and average speed of the man over the interval of time (*i*) 0 – 30 minutes (*ii*) 0 – 50 minutes (*iii*) 0 – 40 minutes?

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Solution. (*i*) **Time interval 0 – 30 minutes.** While going to the market, the man walks at a speed of 5 km/h.

$\therefore$ Displacement in 30 min = 5/2 = 2.5 km

$$\text{Average velocity} = \frac{\text{Net displacement}}{\text{Time}} = \frac{2.5\,\text{km}}{0.5\,\text{h}} = \mathbf{5\ km/h}$$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{2.5\,\text{km}}{0.5\,\text{h}} = \mathbf{5\ km/h}$$

(*ii*) **Time interval 0 – 50 minutes.** While going to the market the man walks at a speed of 5 km/h. Since the market is 2.5 km from his home, he takes 30 minutes to reach the market. While coming back from the market, he walks at a speed of 7.5 km/h so that time taken to reach home = 2.5/7.5 = 1/3 hour = (1/3) × 60 = 20 minutes. This means that in 50 minutes the man walks back home. Therefore, net displacement = 0.

$\therefore$ $$\text{Average velocity} = \frac{\text{Net displacement}}{\text{Time}} = \frac{0}{50\,\text{min}} = \mathbf{0}$$

Now in 50 min, the total distance travelled = 2.5 + 2.5 = 5 km and total time taken = 50/60 hr = 5/6 hr.

$\therefore$ $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{5}{5/6} = \mathbf{6\ km/h}$$

(*iii*) **Time interval 0 – 40 minutes.** In 30 minutes, the man covers a distance of 2.5 km in going from home to the market. While returning home, he covers a distance of 1.25 km in 10 minutes ($\because$ return speed is 7.5 km/h).

$\therefore$ Net displacement during time interval 0 – 40 min

= 2.5 – 1.25 = 1.25 km

$$\text{Average velocity} = \frac{\text{Net displacement}}{\text{Total time}} = \frac{1.25}{40/60} = \mathbf{1.875\ km/h}$$

Total distance travelled in 40 min = 2.5 + 1.25= 3.75 km

$\therefore$ $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time}} = \frac{3.75}{40/60} = \mathbf{5.625\ km/h}$$

PROBLEMS FOR PRACTICE

1. A particle is moving along X-axis. At time t_1 = 3s, its position is x_1 = 6m and at t_2 = 8 s, its position is x_2 = 21 m. What is its average velocity ? **[3 ms^{-1}]**
2. A particle is moving along X-axis. At time t_1 = 4s, its position is x_1 = 40 m and at t_2 = 6 s, its position is x_2 = 20 m. What is its average velocity ? **[–10 ms^{-1}]**
3. A train travels a distance with a speed of 30 km/hr and returns with a speed of 50 km/hr. Find the average speed of the train. **[37.5 km/h]**
4. On a 60 km track, a train travels the first 30 km with a uniform (*i.e.*, constant) speed of 30 km/h. How fast must the train travel the next 30 km so as to average 40 km/h for the entire trip? **[60 km/h]**
5. A stopwatch is used to time the motion of a car. At t_1 = 14 s, the car is at x = 60 m. At t_2 = 17 s, the car is at x = 15 m. What is the average velocity of the car ? **[–15 ms^{-1}]**

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6.15. UNIFORM MOTION IN A STRAIGHT LINE

Uniform motion means that the object moves with a uniform velocity *i.e.*, the *magnitude* and *direction* of the velocity remain the same. Clearly, such a motion can take place *only* if the object moves in a straight line.

An object is said to be in uniform motion if it undergoes equal displacements (in the same direction) in equal intervals of time, however small these time intervals may be.

If an object is moving with a uniform velocity v, then displacement of the object in time t will be $= vt$. This is the key point in uniform motion.

Formulas for Uniform Motion. Consider an object moving with uniform velocity v along the straight line OX as shown in Fig. 6.13. Here O is the origin. Let the time be measured from the instant when the object is at position A *i.e.*, at $t = 0$, $OA = x_0$. Let the object be at positions B and C after times t_1 and t_2 respectively. Then, $OA = x_0$; $OB = x_1$; $OC = x_2$; $AB = vt_1$; $AC = vt_2$.

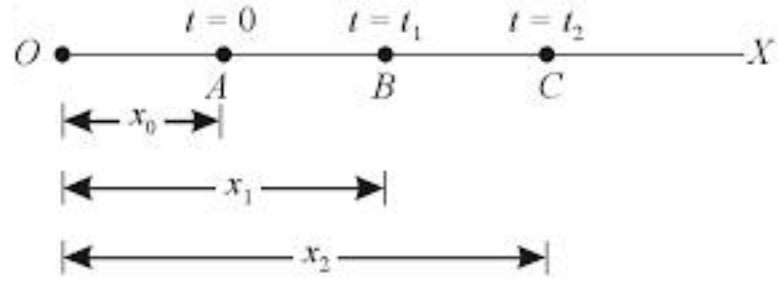


Fig. 6.13

Now $OB = OA + AB$

or $x_1 = x_0 + vt_1$ ...(*i*)

Also $OC = OA + AC$

or $x_2 = x_0 + vt_2$ ...(*ii*)

From eqs. (*i*) and (*ii*), $x_2 - x_1 = v(t_2 - t_1)$ ...(*iii*)

or $x_2 = x_1 + v(t_2 - t_1)$ ...(*iv*)

Eqs. (*i*) to (*iv*) give the formulas for uniform motion. Here eq. (*i*) gives the position of the object at $t = t_1$ while eqs. (*ii*) and (*iv*) give the position of the object at $t = t_2$. However, eq. (*iii*) gives the displacement of the object during the time interval $(t_2 - t_1)$. If $x_2 - x_1 = S$, then displacement during the time $t = (t_2 - t_1)$ is

$$S = vt$$

Note. If the time is measured from the instant the object is at the origin O, then at $t = 0$, $x_0 = 0$. Therefore, the above equations for uniform motion (put $x_0 = 0$) become :

$x_1 = vt_1$; $x_2 = vt_2$; $x_2 - x_1 = v(t_2 - t_1)$; $x_2 = x_1 + v(t_2 - t_1)$

6.16. GRAPHICAL REPRESENTATION OF UNIFORM MOTION

The graphical representation of uniform motion is very helpful because it provides more physical insight than the mathematical relations. We shall study the following graphs :

1. Displacement-time $(x - t)$ graph
2. Velocity-time $(v - t)$ graph

While interpreting the graphs for uniform motion, the following points may be kept in mind :

(*i*) The displacement (x) and velocity (v) will be taken along the Y-axis and time (t) along the X-axis.

(*ii*) Positive velocity (or displacement) means that the motion of the object is to the right of the origin O. Negative velocity (or displacement) means that the motion is to the left of

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the origin O. *Therefore, if $x-t$ or $v-t$ graph is above X-axis (i.e., time axis), the velocity (v) is positive and if it is below X-axis, velocity is negative.*

(*iii*) If at $t = 0$, the initial displacement of the object from the origin is zero (*i.e.*, $x_0 = 0$), the $x-t$ graph will start from the origin of the graph. If at $t = 0$, the initial displacement is x_0, the $x-t$ graph will start at x_0.

(*iv*) Zero velocity of the object means the object is at rest.

6.17. DISPLACEMENT-TIME ($x-t$) GRAPH FOR UNIFORM MOTION

We have seen that in uniform motion, the position x of the object at any time t is related to the uniform velocity v as :

$$x = vt \quad \text{...starting location } O$$

$$x = x_0 + vt \quad \text{...starting location } x_0$$

Both these equations reveal that x is a linear function of t. *Therefore, $x-t$ graph for uniform motion is a straight line.*

(*i*) **Slope of $x-t$ graph.** Fig. 6.14 shows the $x-t$ graph for uniform motion. The graph OA represents the condition when the starting location of the object is O while graph $O'B$ represents the condition when the same object has a starting location x_0. Since the two graphs are parallel ($x \propto t$), they have the same slope.

$$\text{Slope of graph} = \tan\theta = \frac{x}{t} = v$$

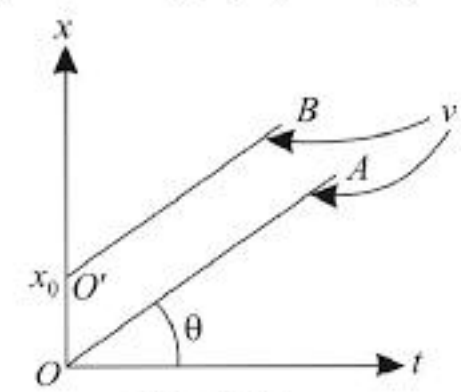


Fig. 6.14

Hence we arrive at a very important conclusion that slope of $x-t$ graph (true for any type of motion) of an object gives the velocity (v) of the object. In the present case, the slope of graph ($= x/t$) gives the uniform velocity v of the object.

(*ii*) **When the object is rest.** Fig. 6.15 shows the $x-t$ graph for an object at rest for two conditions *viz.* for starting locations at O and x_0. Note that displacement does not change with time *i.e.*, velocity is zero. This fact (zero velocity) is also revealed by the graphs being parallel to the X-axis (time axis) *i.e.*, $\tan\theta = \tan 0° = 0$.

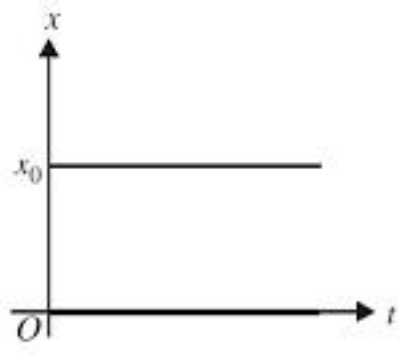


Fig. 6.15

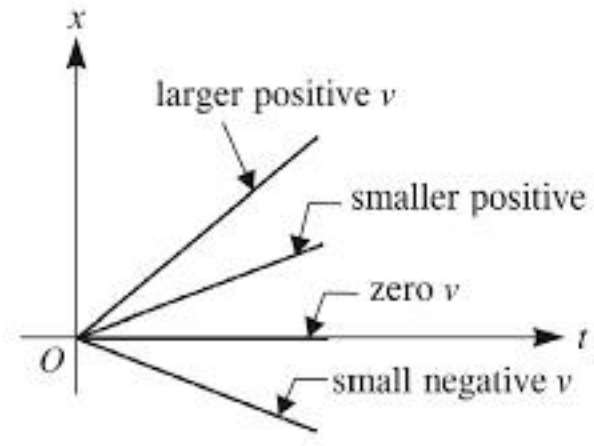


Fig. 6.16

(*iii*) **When object moves with uniform velocity.** Fig. 6.14 (Refer back) shows the situation when two objects have the same uniform velocity v but have different starting locations. Fig. 6.16 shows the $x-t$ graph for different velocities for the same starting location O. Note that if $x-t$ graph is above X-axis (time axis), the velocity is positive. However, if the $x-t$ graph is below X-axis, the velocity is negative. Further, greater the slope ($= \tan\theta$) of the $x-t$ graph, the greater is the velocity and vice-versa.

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6.18. VELOCITY-TIME ($v-t$) GRAPH FOR UNIFORM MOTION

$$x = vt \quad \therefore \quad \frac{dx}{dt} = \frac{d}{dt}(vt) = v = \text{constant}$$

$$x = x_0 + vt \quad \therefore \quad \frac{dx}{dt} = \frac{d}{dt}(x_0 + vt) = v = \text{constant}$$

Therefore, the velocity of the object is constant at all times so that velocity-time graph is a straight line parallel to time axis as shown in Fig. 6.17. Note that $v-t$ graph (solid line) above O indicates positive velocity *i.e,* the object is moving in the positive x-direction. The $v-t$ graph (dotted line) below O indicates negative velocity *i.e.*, the object is moving in the negative x-direction.

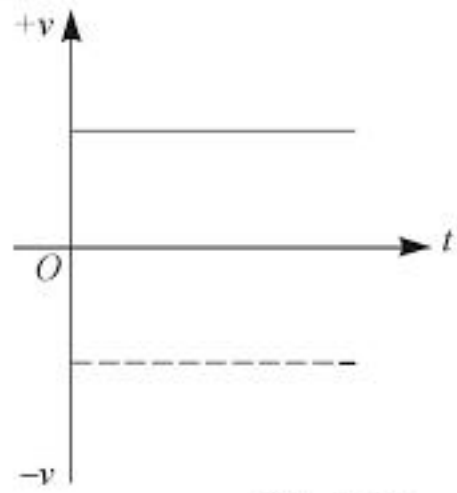


Fig. 6.17

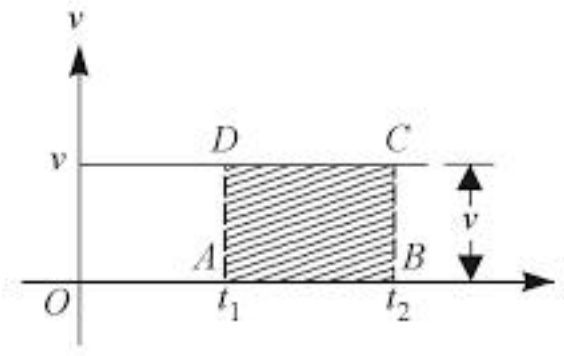


Fig. 6.18

Net displacement from $v-t$ graph. The velocity-time graph of a moving object can be used to calculate the *net* displacement of the object during a given time interval. Thus referring to Fig. 6.18, A and B are the two points on the $v-t$ graph corresponding to times t_1 and t_2. Since velocity of the object is the same (*i.e.*, v) at t_1 and t_2, the area under $v-t$ graph during the time interval t_1 to t_2 is equal to the area of the rectangle $ABCD$.

Area under $v-t$ graph = Area $ABCD = AB \times BC = (t_2 - t_1)v$

But $v(t_2 - t_1)$ is the displacement of the object during the time interval $(t_2 - t_1)$.

Hence the displacement of an object during the time interval $(t_2 - t_1)$ is equal to the area under velocity-time graph during that time interval.

This statement is true for any type of motion. Note that the area of the $v-t$ graph above X-axis (time axis) is positive and that below the X-axis is negative. The *net* displacement will be the algebraic sum of the areas under the $v-t$ graph.

Note. If we are to find distance travelled by the object from the $v-t$ graph, then the negative area of $v-t$ graph (*i.e.*, area of $v-t$ graph below X-axis) is also to be considered positive.

6.19. SUMMARY OF UNIFORM MOTION IN STRAIGHT LINE

The following points are worth noting about uniform motion in a straight line :

(*i*) The magnitude (v) and direction of the velocity remain the same throughout the journey.

(*ii*) The sign of v gives the direction of motion.

(*iii*) The magnitude of velocity gives the time rate of covering distance or displacement.

(*iv*) The displacement-time graph is a straight line. The slope of this graph gives the magnitude of the constant velocity.

(*v*) The position x of the object at any time t is given by $x = x_0 + v\,t$. Here x_0 is the position of the object at $t = 0$ (*i.e.*, its distance from origin at $t = 0$) and v is the constant velocity.

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(*vi*) The velocity-time graph is a straight line parallel to the time axis.

(*vii*) The displacement during any time interval is equal to the area under the velocity-time graph during that time interval.

6.20. GRAPHICAL ANALYSIS OF VARIABLE VELOCITY

In uniform motion, the magnitude and direction of the velocity remain the same. However, in *non-uniform motion,* the velocity of the object is variable. In such a case, the motion of the object is described by its average velocity and instantaneous velocity. Let us discuss these graphically.

(*i*) Geometrical interpretation of average velocity. Consider one dimensional motion of an object along OX axis as shown in Fig. 6.19. Suppose the object is moving with a variable velocity. At time t_1, the object is at point P with coordinate x_1 and at time t_2, it is at Q with coordinate x_2. The graph between x and t is shown in Fig. 6.20. Note that curve in this figure does not represent the path of the object; it is the curve showing the positions of the object at various times. The path of the object is along a straight line as shown in Fig. 6.19. The cordinates of point P are (x_1, t_1) and that of Q are (x_2, t_2). These are represented on the graph. The displacement $\Delta x = x_2 - x_1$ and the time interval $\Delta t = t_2 - t_1$. A straight line drawn from point $P\ (x_1, t_1)$ to point $Q\ (x_2, t_2)$ forms the hypotenuse of the right angled triangle whose sides are Δx and Δt. The ratio $\Delta x/\Delta t$ is the slope of the line PQ. But $\Delta x/\Delta t$ is also the average velocity of the object during the time interval $\Delta t = t_2 - t_1$.

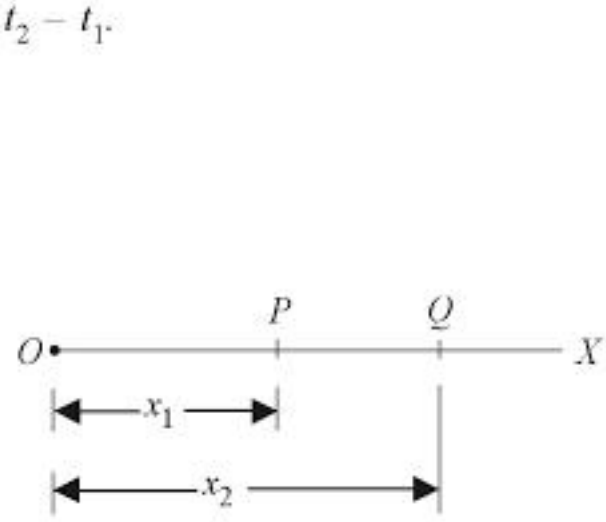


Fig. 6.19

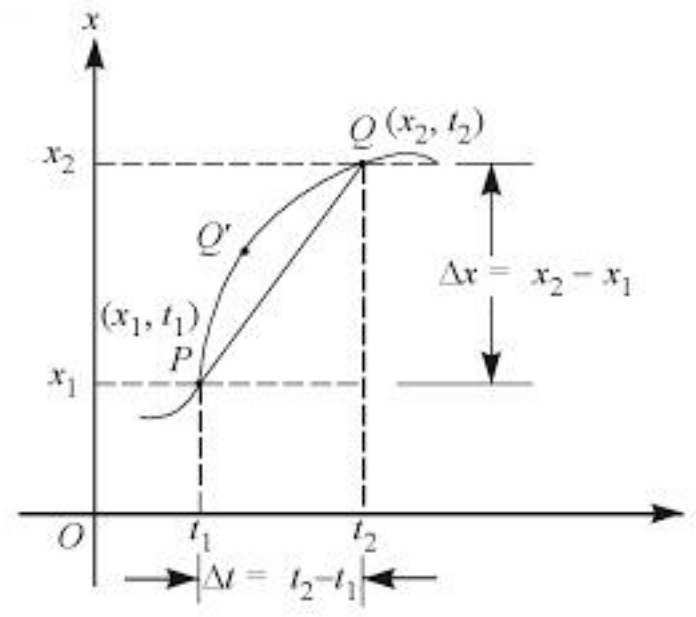


Fig. 6.20

$$\therefore \quad \text{Average velocity} = \frac{\Delta x}{\Delta t} = \text{Slope of line connecting points } P\ (x_1, t_1) \text{ and } Q\ (x_2, t_2)$$

Thus we have the geometrical interpretation of the average velocity.

The average velocity of an object during the time interval $\Delta t = t_2 - t_1$ is equal to the slope of the straight line connecting two points $P(x_1, t_1)$ and $Q(x_2, t_2)$ on the $x - t$ graph.

Note that the average velocity depends upon the time interval choosen. For example in Fig. 6.20, if we choose a smaller time interval, the average velocity would be greater as indicated by the greater steepness of the line connecting P and Q'.

(*ii*) Instantaneous velocity. If you drive a car along a straight road for 100 km in 2 h, your average velocity is 50 km/h. However, it is unlikely that you were moving at 50 km/h at every instant. The velocity at any instant of time is called the instantaneous velocity. This is not a very precise definition. More precisely, the **instantaneous velocity** *at any moment is the average velocity over an indefinitely short time interval.* To make it clear, we shall seek the help of $x - t$ graph.

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Fig. 6.21 shows the same $x - t$ graph as Fig. 6.20. Suppose we want to find the instantaneous velocity at point P *i.e.,* at time t_1. Consider the time interval $\Delta t = t_2 - t_1$. The average velocity during this time interval is slope of the line PQ. Now consider a time t' between t_1 and t_2 at which time the object is at x'. The slope of the line PQ' is greater than the slope of line PQ in this case. As the time interval Δt becomes smaller and smaller, the average velocity approaches the slope of tangent at point P (dotted line).

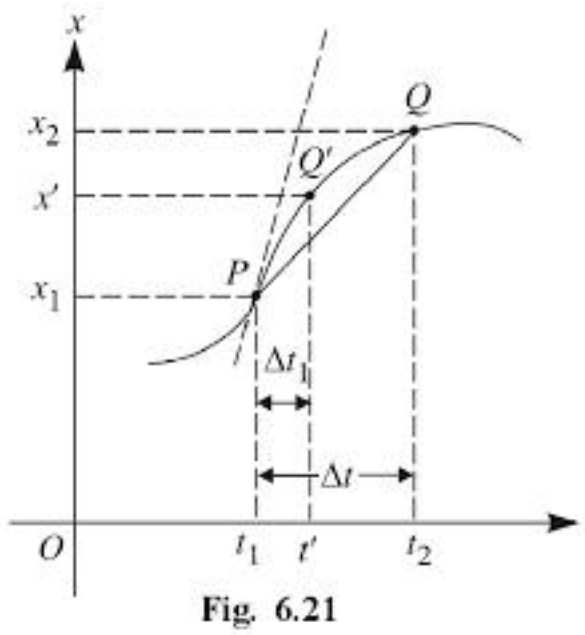


Fig. 6.21

We define the slope of this tangent to be the instantaneous velocity at the time t_1.

Thus instantaneous velocity is the limit of the ratio $\Delta x/\Delta t$ as Δt approaches zero *i.e.,*

$$\text{Instantaneous velocity, } v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

The limit as $\Delta t \to 0$ is written in calculus notation as dx/dt and is called derivative of x with respect to t.

$$\therefore \qquad v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

or
$$v = \frac{dx}{dt}$$

Thus, for a position versus time graph, the slope of the tangent line at a particular time is the object's velocity at that time.

This is the geometrical interpretation of instantaneous velocity. Since the slope may be positive (x increasing) or negative (x decreasing), the instantaneous velocity may be positive or negative in one dimensional motion.

Example 6.8. A car moves at a constant speed of 60 km/h for 1 km and at 40 km/h for next 1 km. What is the average speed of the car ?

Solution. $\text{Average speed} = \dfrac{\text{Total distance covered}}{\text{Total time taken}}$

$x(t_1) = 1$ km ; $v(t_1) = 60$ km/h ; $x(t_2) = 1$ km ; $v(t_2) = 40$ km/h

$$\therefore \qquad t_1 = \frac{x(t_1)}{v(t_1)} = \frac{1}{60} \text{ h} \quad \text{and} \quad t_2 = \frac{x(t_2)}{v(t_2)} = \frac{1}{40} \text{ h}$$

$$\text{Total distance travelled} = x(t_1) + x(t_2) = 1 + 1 = 2 \text{ km}$$

$$\text{Total time taken} = t_1 + t_2 = \frac{1}{60} + \frac{1}{40} = \frac{1}{24} \text{ h}$$

$$\therefore \qquad \text{Average speed} = \frac{2}{1/24} = \mathbf{48 \text{ km/h}}$$

Example 6.9. A car travels 1/3 of the distance on a straight road with a velocity of 10 km/h, next one-third with a velocity 20 km/h and the last one-third with a velocity of 60 km/h. What is the average velocity of the car for the entire journey ?

Solution. Let x be the total distance of the journey.

$x(t_1) = x/3$; $x(t_2) = x/3$; $x(t_3) = x/3$

$v(t_1) = 10$ km/h ; $v(t_2) = 20$ km/h ; $v(t_3) = 60$ km/h

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$$\therefore \quad t_1 = \frac{x(t_1)}{v(t_1)} = \frac{x/3}{10} = \frac{x}{30} \text{ hours}$$

$$t_2 = \frac{x(t_2)}{v(t_2)} = \frac{x/3}{20} = \frac{x}{60} \text{ hours}$$

$$t_3 = \frac{x(t_3)}{v(t_3)} = \frac{x/3}{60} = \frac{x}{180} \text{ hours}$$

$$\text{Total time, } t = t_1 + t_2 + t_3 = \frac{x}{30} + \frac{x}{60} + \frac{x}{180} = \frac{10x}{180}$$

$$\therefore \quad \text{Average velocity} = \frac{x}{t} = \frac{x}{(10x/180)} = \mathbf{18\ km/h}$$

Example 6.10. A particle is moving along the *X*-axis as shown in Fig. 6.22. Its position as a function of time is given by :

$$x = A\,t^2 + B$$

Here $A = 2.1\ \text{ms}^{-2}$; $B = 2.8$ m

Determine (*i*) the displacement of the particle during the time interval from $t_1 = 3$ s to $t_2 = 5$ s (*ii*) the average velocity during this time interval and (*iii*) the magnitude of instantaneous velocity at $t = 5$ s.

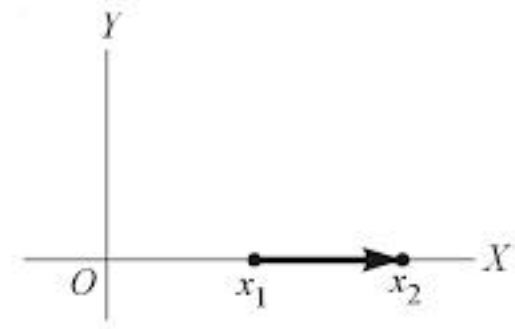


Fig. 6.22

Solution. (*i*) $x = A\,t^2 + B$

At $t_1 = 3$ s, the position of the particle is

$$x_1 = (2.1)\ (3)^2 + 2.8 = 21.7 \text{ m}$$

At $t_2 = 5$ s, the position of the particle is

$$x_2 = (2.1)\ (5)^2 + 2.8 = 55.3 \text{ m}$$

$$\text{Displacement, } \Delta x = x_2 - x_1 = 55.3 - 21.7 = \mathbf{33.6\ m}$$

(*ii*) Magnitude of average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{33.6}{5-3} = \mathbf{16.8\ ms^{-1}}$$

(*iii*) $x = A\,t^2 + B$

$\therefore$ Instantaneous velocity v is given by :

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(At^2 + B\right) = 2At$$

$$\text{At } t = 5 \text{ s, } v = 2 \times 2.1 \times 5 = \mathbf{21\ m\ s^{-1}}$$

Note that instantaneous velocity is quite different from the average velocity. Clearly, the particle is moving with a variable velocity.

Example 6.11. The position of a stone dropped from rest from a cliff is given by $x = 5\,t^2$. What is the magnitude of the velocity of the stone at $t = 2$s ?

Solution. The stone is dropped from rest from the cliff. Therefore, it will fall vertically downward. Clearly, it is one dimensional motion along *Y*-axis.

$$x = 5\,t^2$$

Instantaneous velocity is given by :

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(5t^2\right) = 10t$$

$$\text{At } t = 2 \text{ s ; } v = 10 \times 2 = \mathbf{20\ ms^{-1}}$$

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Example 6.12. The position of a ball rolling in a straight line is given by ;

$$x = 2 + 6.6\ t - 1.1\ t^2$$

What is its velocity at (*i*) $t = 2$ s and (*ii*) $t = 3$ s ?

Solution. $x = 2 + 6.6\ t - 1.1\ t^2$

The instantaneous velocity is given by ;

$$v = \frac{dx}{dt} = \frac{d}{dt}\ (2 + 6.6\ t - 1.1\ t^2) = 6.6 - 2.2\ t$$

(*i*) At $t = 2$ s, $v_1 = 6.6 - 2.2 \times 2 = \mathbf{2.2\ ms^{-1}}$

(*ii*) At $t = 3$ s, $v_2 = 6.6 - 2.2 \times 3 = \mathbf{0\ ms^{-1}}$

Obviously, the ball will come to rest in 3 seconds.

PROBLEMS FOR PRACTICE

1. A car travels 30 km at a uniform speed of 40 km/h and next 30 km at a uniform speed of 20 km/h. Find the average speed of the car. **[26.67 km/h]**
2. A person travelled a distance of 200 km between two cities by a car covering the first half distance of the journey at a constant speed of 40 km/h and the remaining half distance at a constant speed of v km/h. If the average speed during the entire journey is 48 km/h, find the value of v. **[60 km/h]**
3. The displacement at any time t of a particle moving with a uniform velocity along a straight line is $x = 3 + 5t$. What is the velocity of the particle? Displacement is measured in metres and time in seconds. **[5 m s^{-1}]**
4. Draw $x - t$ time graphs for the following cases : (*i*) $x = 3 - 5t$ (*ii*) $x = 2 + 3t$.
5. The position of a particle as a function of time is given by ;

   $x = 6t + 0.5\ t^3$

   What is its velocity at $t = 1$s? **[8.1 ms^{-1}]**
6. The position of a stone dropped from rest from a cliff is given by $x = 5\ t^2$. What is the velocity of the stone at $t = 3$ s? **[30 ms^{-1}]**
7. The position of a particle moving along X-axis is given by ;

   $x = At + 4\ Bt^3$

   What is its velocity at $t = 5$ s? **[120 B]**

6.21. RELATIVE VELOCITY IN ONE DIMENSION

Velocity is not an absolute quantity but is measured relative to other objects. Thus to measure the velocity of an object, we specify the frame of reference. Ordinarily, we consider the surface of earth as our frame of reference and specify the velocity of body with respect to it (*i.e.*, earth). Thus, when we say that a car is travelling at *60 km/h, we mean to say it is going 60 km/h relative to the road or earth's surface.

Suppose the two cars A and B are travelling on a straight level road in the same direction as shown in Fig. 6.23. Let the uniform velocities of A and B be v_A and v_B respectively. We want to find the velocity of car A with respect to car B. Note that v_A is the velocity of car A *w.r.t.* earth. But we want to find v_{AB} – now our frame of reference is car B. If we superpose a velocity $-v_B$ on the two cars, then car B comes to rest (*i.e.*, car B and earth become identical frames of reference), then the velocity of car A *w.r.t.* car B is $v_{AB} = v_A - v_B$.

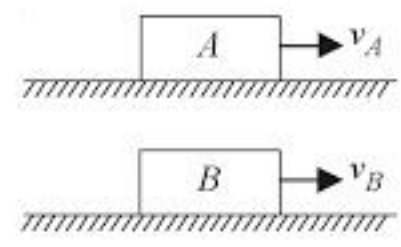


Fig. 6.23

* We generally omit mention of reference frame since we know that it is earth's surface.

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Thus in order to find the relative velocity of body A w.r.t. body B in one dimensional motion, we add negative of velocity of B to both the bodies. This effectively brings the body B to rest The resultant velocity of body A is v_{AB} i.e., relative velocity of A w.r.t. B.

Suppose two cars A and B having velocities v_A and v_B are moving along a straight road in the same direction. In order to find the relative velocity of A *w.r.t.* B, we add velocity $-v_B$ to both the cars. This effectively brings the car B to rest and the resultant velocity of car A becomes $v_A - v_B$.

$\therefore$ Relative velocity of car A *w.r.t.* car B is

$$v_{AB} = v_A - v_B$$

The direction of the relative velocity will depend upon the direction of $(v_A - v_B)$.

Note. $v_{AB} = v_A - v_B$

But v_A and v_B are the velocities of the cars *w.r.t.* earth and can be written as v_{AE} and v_{BE} respectively.

$\therefore$ $v_{AB} = v_{AE} - v_{BE}$

or $v_{AB} = v_{AE} + v_{EB}$ ...(*i*) $(\because v_{BE} = -v_{EB})$

Eq. (*i*) is a very useful relation. By writing the equation in this form, we see that inner subscripts on the right-hand of eq. (*i*) are the same (the two E's) and the outer subscripts on the right of eq. (*i*) (the A and B) are the same as the subscripts of the term v_{AB} on the left-hand side of eq. (*i*).

6.22. MATHEMATICAL TREATMENT OF RELATIVE VELOCITY

We know that velocity of a body is the time rate of change of its position. Therefore, relative velocity of a body A *w.r.t.* body B can be defined as under :

The relative velocity of body A w.r.t. body B (v_{AB}) is the time rate at which body A changes its position w.r.t. body B.

Consider two bodies A and B moving along a straight line in the same direction with uniform velocities v_A and v_B respectively as shown in Fig. 6.24. Let x_{AO} and x_{BO} be their displacements from the origin O at $t = 0$. Clearly, $(x_{AO} - x_{BO})$ is the displacement between the bodies A and B at $t = 0$. If x_A and x_B are their position coordinates in time t, then,

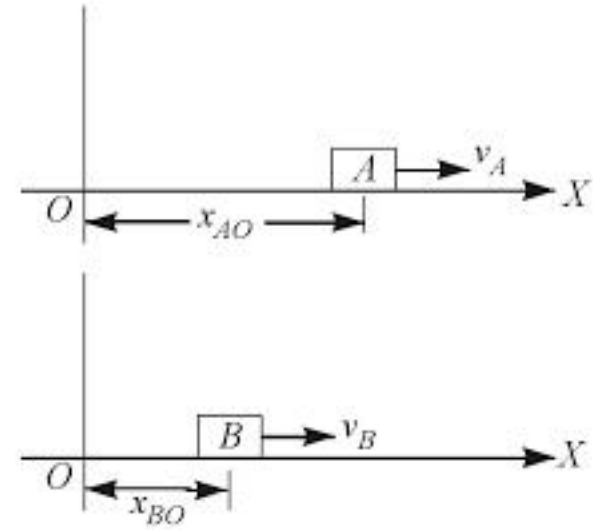


Fig. 6.24

$$x_A = x_{AO} + v_A t$$

and $$x_B = x_{BO} + v_B t$$

$\therefore$ $$x_A - x_B = (x_{AO} - x_{BO}) + (v_A t - v_B t)$$

or $$x_A - x_B = (x_{AO} - x_{BO}) + (v_A - v_B) t$$

Increase in displacements between the two bodies in time t

= Final displacement – Original displacement

$$= [(x_{AO} - x_{BO}) + (v_A - v_B) t] - [(x_{AO} - x_{BO})]$$

$$= (v_A - v_B) t$$

Therefore, relative velocity of A *w.r.t.* B is

$$v_{AB} = \frac{\text{displacement}}{\text{time}} = \frac{(v_A - v_B)t}{t} = v_A - v_B$$

It is left as an exercise for the reader to prove that $v_{BA} = v_B - v_A$.

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6.23. POSITION-TIME GRAPH OF RELATIVE VELOCITY

We have seen above that the total displacement between the bodies A and B in time t is given by ;

$$x_A - x_B = (x_{AO} - x_{BO}) + (v_A - v_B)\, t$$

(*i*) When $v_A = v_B$. When $v_A = v_B$, then,

$$x_A - x_B = x_{AO} - x_{BO}$$

This means that distance between the two bodies will remain the same at all times. Since $^*v_A = v_B$, their position – time graphs are parallel to each other as shown in Fig. 6.25.

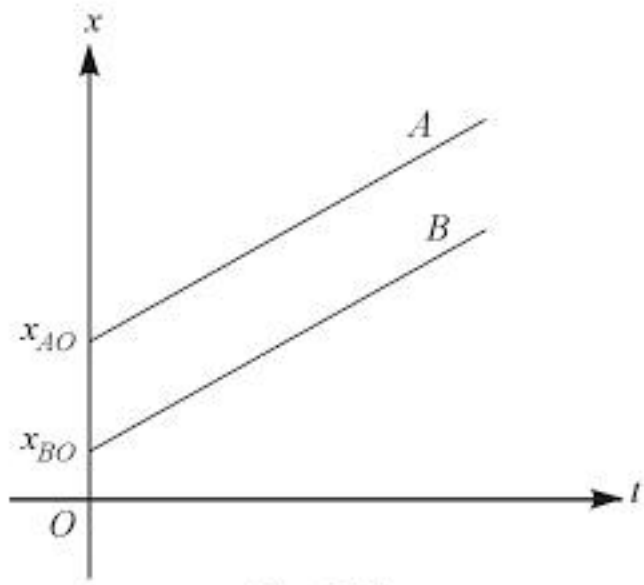


Fig. 6.25

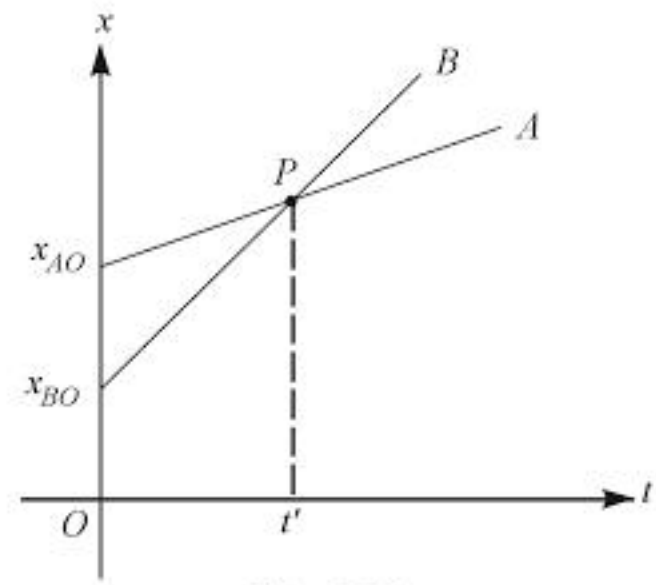


Fig. 6.26.

(*ii*) When $v_A \neq v_B$. In such a case, one of the position-time graphs will be steeper than the other. If $v_B > v_A$, then position-time graph of v_B will be steeper than that of body A as shown in Fig. 6.26. As a result, the two graphs will meet at point P which corresponds to time t'. At t', the two bodies meet *i.e.*, they have the same displacement from the origin. Before t', the body A is ahead of body B and after t', body B will be ahead of body A. On the other hand, if $v_A > v_B$, then $(v_A - v_B)$ will be positive. It means that relative separation between the two bodies will increase by an amount $(v_A - v_B)$ after each second. Therefore, the two graphs will never meet as shown in Fig. 6.27. In fact, the two graphs will open out gradually.

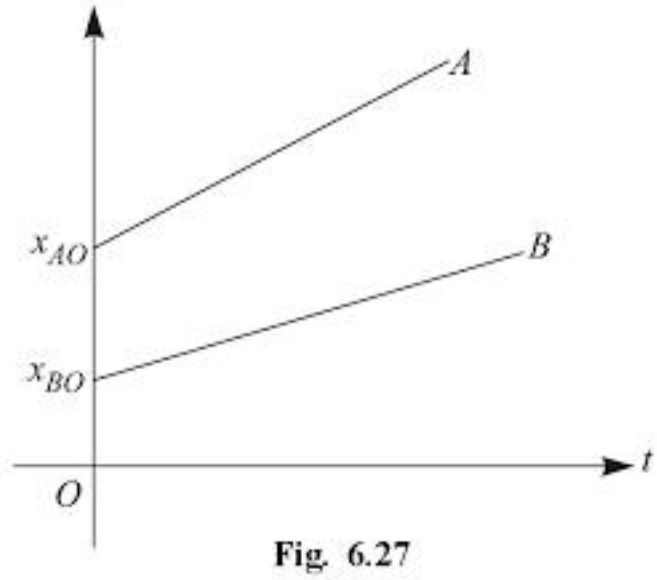


Fig. 6.27

Example 6.13. Two railway tracks are parallel to west-east direction. Along one track, train *A* moves with a speed of 54 km/h from west to east while along the second track, train *B* moves with a speed of 90 km/h from east to west. Find the (*i*) relative velocity of *B* w.r.t. *A* and (*ii*) relative velocity of ground w.r.t. *B*.

Solution. Considering the positive direction of x-axis from west to east, we have,

v_{AE} = Relative velocity of A *w.r.t.* earth = +54 km/h = +15 ms^{-1}

v_{BE} = Relative velocity of B *w.r.t.* earth = –90 km/h = –25 ms^{-1}

(*i*) Relative velocity of B *w.r.t.* A is given by ;

$$v_{BA} = v_{BE} + v_{EA} = v_{BE} - v_{AE} = (-25) - (+15) = \mathbf{-40\ ms^{-1}}$$

* This means their slopes are the same.

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What is the meaning of negative sign? It means that train B appears to move from east to west with a speed of 40 ms^{-1}.

(*ii*) Relative velocity of earth *w.r.t.* B is

$$v_{EB} = v_{EE} + v_{EB} = v_{EE} - v_{BE} = 0 - (-25) = \mathbf{25\ ms^{-1}}$$

Positive sign shows that the ground or earth's surface will appear to move west to east with a speed of 25 ms^{-1}.

Example 6.14. Two trains, each of length 100 m, are running on parallel tracks. One overtakes the other in 20 seconds and one crosses the other in 10 seconds. Calculate the velocity of each train.

Solution. Let u and v be the velocities of the trains in ms^{-1}.

Relative velocity of overtaking $= u - v$

Relative velocity of crossing $= u + v$

Total distance $= 100 + 100 = 200$ m

$$\therefore \quad 20 = \frac{200}{u - v}$$

or $$u - v = 10 \qquad ...(i)$$

Also $$10 = \frac{200}{u + v}$$

or $$u + v = 20 \qquad ...(ii)$$

Solving eqs. (*i*) and (*ii*), we have, $u = \mathbf{15\ ms^{-1}}$; $v = \mathbf{5\ ms^{-1}}$

Example 6.15. Two trains along the same straight rails are moving with constant velocity of 60 km/h and 30 km/h towards each other. If at time $t = 0$, the distance between them is 90 km, find the time when they collide.

Solution. Relative velocity of approach $= 60 - (-30) = 60 + 30 = 90$ km/h

Distance between trains $= 90$ km

$$\therefore \quad \text{Time when they will collide} = \frac{90}{90} = \mathbf{1h}$$

Example 6.16. Two cars A and B are at positions 100 m and 200 m from the origin at $t = 0$. They start simultaneously with velocities 10 ms^{-1} and 5 ms^{-1} respectively. Calculate the time and position at which they will overtake one another. Assume that they are moving in the same direction.

Solution. $x_{BO} = 200$ m ; $x_{AO} = 100$ m ; $v_A = 10\ ms^{-1}$; $v_B = 5\ ms^{-1}$

$$x_B - x_A = (x_{BO} - x_{AO}) + (v_B - v_A)t$$

For the solution of the problem, $x_B - x_A = 0$.

$$\therefore \quad 0 = (200 - 100) + (5 - 10)t \quad \text{or} \quad 0 = 100 - 5t \quad \therefore \quad t = 100/5 = \mathbf{20\ s}$$

Also $$x = 100 + 10 \times 20 = 100 + 200 = \mathbf{300\ m}$$

Example 6.17. The distance between two towns M and N is 400 km. Two cars A and B set off simultaneously from the towns M and N towards each other. The car A from M travels at a speed of $v_A = 60$ km/h and car B from N at a speed of $v_B = 40$ km/h. Find the time that will elapse before they meet and the point where they will meet.

Solution. $v_A = 60\ kmh^{-1}$; $v_B = -40\ kmh^{-1}$

The negative sign is given because car B is moving in a direction opposite to that of the car A.

$\therefore$ Relative velocity of car A *w.r.t.* to car B is

$$v_{AB} = v_A - v_B = 60 - (-40) = 100\ kmh^{-1}$$

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Distance between the towns M and N is $x = 400$ km.

$\therefore$ Time after which the two cars meet is

$$t = \frac{x}{v_{AB}} = \frac{400}{100} = \mathbf{4\ h}$$

The distance S from the town M when the two cars meet is

$$S = v_A \times t = 60 \times 4 = \mathbf{240\ km}$$

6.24. ACCELERATION

The acceleration of a moving object is defined as the change in velocity divided by the time taken for the change in velocity i.e.,

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time interval}}$$

The SI unit of acceleration is ms^{-2}. Acceleration is a vector quantity *i.e.*, it has magnitude as well as direction.

(*i*) The dimensional formula of acceleration is $[M^0\ LT^{-2}]$.

(*ii*) The direction of acceleration is the same as that of the velocity.

(*iii*) The acceleration is positive if the velocity is increasing and is negative if the velocity is decreasing. The negative acceleration is also called **retardation** or **deceleration.**

1. **Uniform acceleration.** *An object is said to be moving with uniform acceleration (i.e., constant acceleration) if its velocity changes by equal amounts in equal intervals of time, however small these time intervals may be.*

   Note that uniform acceleration means that magnitude as well as direction of acceleration remains the same. For example, objects falling freely under gravity near earth's surface have uniform acceleration. In this case, the magnitude of acceleration in 9.8 ms^{-2} and is always directed vertically downward.

2. **Variable acceleration.** *An object is said to be moving with variable acceleration (or non-uniform acceleration) if its velocity changes in magnitude or direction or both.* The problems involving variable acceleration are solved either graphically or by the use of calculus.

3. **Average acceleration.** *The average acceleration of an object is defined as the change in velocity Δv $(= v_2 - v_1)$ divided by the time interval Δt $(= t_2 - t_1)$ for the change in velocity i.e.,*

   $$\text{Average acceleration, } \bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

   Suppose a body starts from rest and moving in a straight line attains a velocity of 20 ms^{-1} in 2 seconds. The change in velocity, $\Delta v = 20 - 0 = 20$ ms^{-1}.

   $$\therefore \quad \text{Average acceleration, } \bar{a} = \frac{\Delta v}{\Delta t} = \frac{20}{2} = 10\ ms^{-2}$$

Example 6.18. A car travels in the positive x direction at 20 ms^{-1} and needs to pass a truck . The car's velocity is increased to 25 ms^{-1} in 2 s. Find the average acceleration of the car.

Solution. Since the motion is in the positive x direction, both initial (v_1) and final (v_2) velocities are positive.

$$\text{Change in car's velocity, } \Delta v = v_2 - v_1 = 25 - 20 = 5\ ms^{-1}$$

$$\text{Time interval, } \Delta t = t_2 - t_1 = 2\ s$$

$$\therefore \quad \text{Average acceleration, } \bar{a} = \frac{\Delta v}{\Delta t} = \frac{5}{2} = \mathbf{2.5\ ms^{-2}}$$

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What does average acceleration mean? In this case, it means that on the average, the velocity of the car increases by 2.5 m s^{-1} every second.

Example 6.19. A car is moving in the positive x direction at 20 ms^{-1} and comes up behind a truck and is unable to pass. The car slows to 15 ms^{-1} in a time 2 s. Calculate car's average acceleration.

Solution. Change in car's velocity, $\Delta v = v_2 - v_1 = 15 - 20 = -5\ ms^{-2}$

Note that the change in velocity is negative because the car's velocity in the positive x direction is decreased by 5 ms^{-1}.

$$\text{Time interval, } \Delta t = t_2 - t_1 = 2s$$

$$\therefore \quad \text{Average acceleration, } \bar{a} = \frac{\Delta v}{\Delta t} = \frac{-5}{2} = \mathbf{-2.5\ ms^{-2}}$$

The average acceleration is negative because the car's velocity along the positive x direction is decreased. It, on the average, lost 2.5 ms^{-1} each second.

6.25. GRAPHICAL ANALYSIS OF VARIABLE ACCELERATION

In uniform acceleration, the magnitude and direction of the acceleration remain the same. However, in non-uniform acceleration, the acceleration of the object is variable. In such a case, the motion of the object is described by its average acceleration and instantaneous acceleration. Let us discuss these graphically.

(*i*) Geometrical interpretation of average acceleration. A graphical approach gives further insight into the relationship between average acceleration, velocity and time. Consider one dimensional motion of an object along the positive x direction as shown in Fig. 6.28. At time t_1, the object is at point P and has a velocity v_1 and at a later time t_2, the object is at Q and has a velocity v_2. The velocity-time graph of the object is shown in Fig. 6.29. Note that *curve in this figure does not represent the path of the object ; it is the curve showing the velocities of the object at various times. The path of the object is along a straight line as shown in Fig. 6.28.

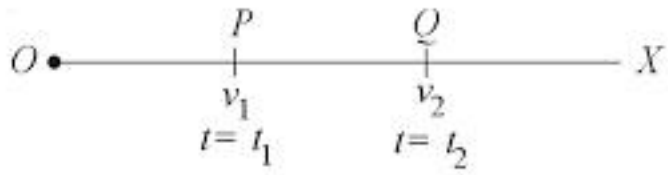


Fig. 6.28

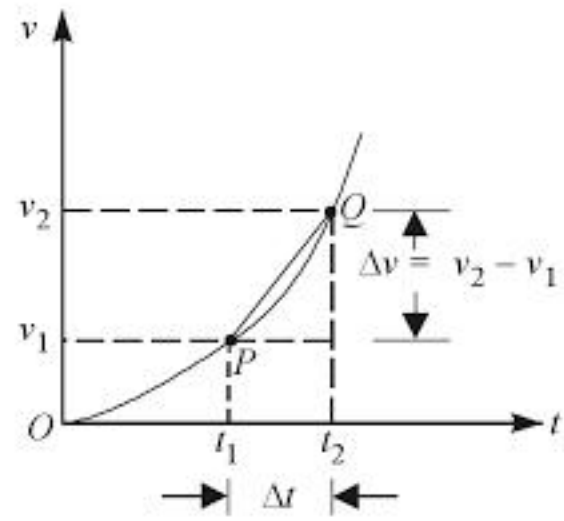


Fig. 6.29

The coordinates of point P are (v_1, t_1) and that of Q are (v_2, t_2). The change in velocity $\Delta v = v_2 - v_1$ and the time interval $\Delta t = t_2 - t_1$. The straight line PQ forms the hypotenuse of the right angled triangle whose sides are Δv and Δt. The ratio $\Delta v / \Delta t$ is the slope of the line PQ. But $\Delta v / \Delta t$ is also the average acceleration of the object during the time interval $\Delta t = t_2 - t_1$.

$$\therefore \text{ Average acceleration, } \bar{a} = \frac{\Delta v}{\Delta t} = \text{Slope of the line connecting points } P(v_1, t_1) \text{ and } Q(v_2, t_2)$$

Thus we have the geometrical interpretation of the average acceleration.

---

* The graph is a curve (not a straight line) because the object is moving with a variable velocity.

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The average acceleration of an object during the time interval $\Delta t = t_2 - t_1$ is equal to the slope of the straight line connecting two points $P(v_1, t_1)$ and $Q(v_2, t_2)$ on the $v - t$ graph.

The reader may note that the geometrical interpretation of the average acceleration is defined in a manner similar to the way we defined the average velocity. In case of average velocity, we used $x - t$ graph and in case of average acceleration, we have used $v - t$ graph.

(*ii*) Instantaneous acceleration. We can define instantaneous acceleration in a manner similar to the way we defined the instantaneous velocity. Fig. 6.30 shows a typical $v - t$ graph for the one dimensional motion of an object. The average acceleration of the object during the time interval $\Delta t = t_2 - t_1$ is the slope of the line QP. The instantaneous acceleration at any time t_1 is the slope of the tangent to *$v - t$ graph at that time. Thus at time t_1,

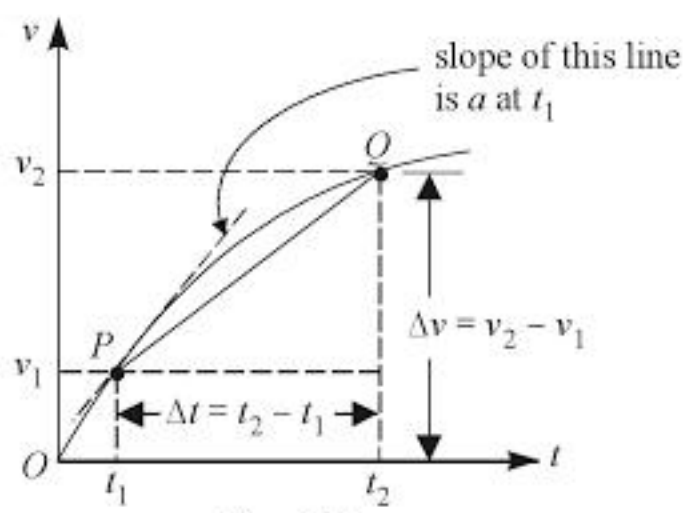


Fig. 6.30

$$\text{Instantaneous acceleration, } a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

$$\therefore \qquad a = \frac{dv}{dt}$$

Like velocity, acceleration is a rate. The velocity of an object is the rate at which displacement changes with time, its acceleration is the rate at which its velocity changes with time. Therefore, acceleration can also be expressed as :

$$a = \frac{dv}{dt} = \frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$\therefore \qquad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Here d^2x/dt^2 is called the *second derivative* of x *w.r.t.* time. It is because we first take the derivate of x *w.r.t.* time (*i.e.*, dx/dt) and then we again take the derivative *w.r.t.* time [d/dt (dx/dt)] to get acceleration.

6.26. GRAPHICAL REPRESENTATION OF ACCELERATION

The acceleration of an object is very important in physics because it is related to the forces acting on the object. We shall see in chapter 10 that the acceleration is directly proportional to the net force acting on the object. In order to have more insight into this important physical quantity, we discuss its graphical interpretation.

(*i*) When acceleration of the object is zero.

$$a = \frac{dv}{dt} \quad \text{or} \quad 0 = \frac{dv}{dt}$$

This means that rate of change of velocity with time is zero. In other words, the velocity of the object is **constant. Therefore, velocity-time graph of an object moving with zero acceleration is a straight line parallel to the time axis as shown in Fig. 6.31. Note that graph corresponds to zero slope. This is expected because acceleration is the slope of $v - t$ graph.

---

* Note that as we go from t_1 to t_2, the velocity is increasing but acceleration is decreasing since slope of the curve is decreasing.

** If the acceleration of an object is zero, the object is either moving with constant velocity in a straight line or remains at rest. We are considering here the former case.

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(*ii*) When acceleration of the object is constant. Consider the one dimensional motion of an object. Suppose at $t = 0$, the object is at the origin and starts with constant acceleration 'a'. Then,

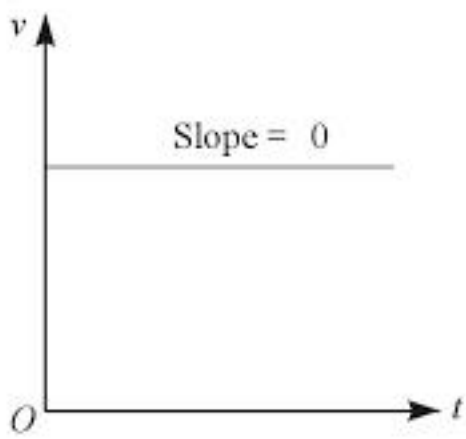


Fig. 6.31

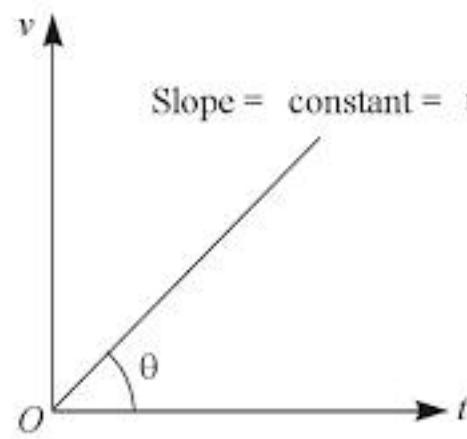


Fig. 6.32

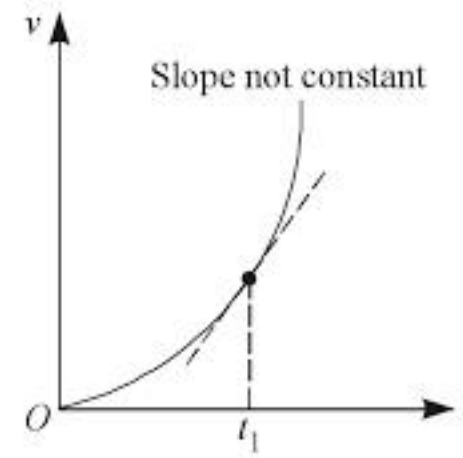


Fig. 6.33

$$a = \frac{dv}{dt} \quad \text{or} \quad dv = a\,dt$$

We integrate velocity from the initial velocity zero to the final value v corresponding to time t.

$$\int_0^v dv = a\int_0^t dt$$

$$\therefore \qquad v = at$$

Since a is constant, $v \propto t$. Therefore, the velocity–time graph of an object moving with constant acceleration is a straight line as shown in Fig. 6.32. Note that slope of line is constant and is equal to the acceleration of the object. Also note that slope of the line is everywhere the same, indicating that velocity is increasing at a constant rate.

Note. The motion in a straight line with constant acceleration is a very important type of motion. It is called *uniformly accelerated linear motion.*

(*iii*) When acceleration of the object is variable. For zero acceleration or constant acceleration, the graphical analysis of acceleration is simple. We can easily find the acceleration of the object (= slope of graph) from its $v - t$ graph. But if the velocity of the object changes continuously with time (See Fig. 6.33), how can we determine the acceleration at a particular instant of time? In other words, the acceleration is variable. We have already seen that instantaneous acceleration given by ;

$$a = \frac{dv}{dt}$$

The instantaneous acceleration at any time t_1 is equal to the slope of the tangent (shown dotted in Fig. 6.33) to the $v - t$ graph at that time. In order to deal with variable acceleration, we use the techniques of calculus.

Note. Like velocity, acceleration can also be positive, negative or zero. Position–time graphs for motion with positive, negative and zero acceleration are shown in Figs. 6.34 (*i*), (*ii*) and (*iii*) respectively. Note that the graph curves upward for positive acceleration, downward for negative acceleration and is a straight line for zero acceleration.

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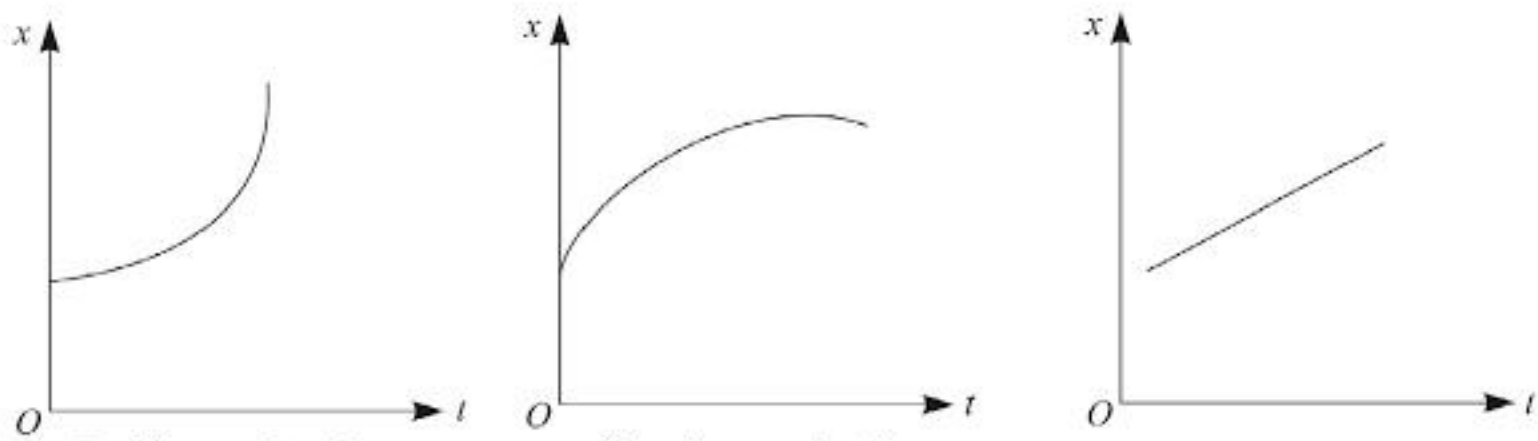


Fig. 6.34

Example 6.20. The position of stone dropped from a cliff is given by $x = 5\ t^2$ where x is in metres and t is in seconds. What is the acceleration of the stone at $t = 2$ s?

Solution. $x = 5\ t^2 \quad \therefore \quad \frac{dx}{dt} = 10\ t$ or $\frac{d^2x}{dt^2} = 10\ \text{ms}^{-2}$

But d^2x / dt^2 is acceleration of the stone. Therefore, $a = 10\ \text{m s}^{-2}$

Example 6.21. The position of a particle is given by $x = K\ t^3$ where K is a constant with the dimensions of m s^{-2}. Find (*i*) velocity (*ii*) acceleration of the particle as a function of time.

Solution. (*i*) $x = K\ t^3 \quad \therefore \quad v = \frac{dx}{dt} = \frac{d}{dt}\left(Kt^3\right) = 3\ Kt^2$ or $v = 3\ Kt^2$

Note that value of v varies as the square of time. Clearly, it is a case of variable velocity.

(*ii*) Now, $a = \frac{dv}{dt} = \frac{d}{dt}(v) = \frac{d}{dt}\left(3Kt^2\right) = 6\ Kt \quad \therefore \quad a = 6\ Kt$

Note that value of a varies directly with time. Clearly, the acceleration is not constant but varies with time.

Example 6.22. The position of a particle moving along a straight line is given by ;

$$x = 2 - 5t + 6t^2$$

Find the acceleration of the particle at $t = 2$ s.

Solution. $x = 2 - 5t + 6\ t^2$ or $\frac{dx}{dt} = \frac{d}{dt}(2 - 5t + 6\ t^2) = -5 + 12\ t$

$\therefore \quad a = \frac{d^2x}{dt^2} = \frac{d}{dt}\ (-5 + 12\ t) = 12$

Note that acceleration of the particle is constant and is independent of time.

CONCEPTUAL QUESTIONS

Q.1. What is translation motion?

Ans. Rigid objects that move without rotation are said to undergo translation motion. When we are interested only in translation motion and the object's size is not significant, we can treat the object as a particle.

Q.2. Which is generally our frame of reference for studying motion of objects?

Ans. Ordinarily our frame of reference for studying motion of objects is a coordinate system attached to the surface of earth. Unless stated otherwise, this is our frame of reference.

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Q.3. An aeroplane is flying in the sky. Is its motion three dimensional?

Ans. Yes. It is because it may be changing its direction as well as the height from the ground during its flight.

Q.4. Which is the simplest type of motion?

Ans. The uniform motion in a straight line (one dimensional motion) is the simplest type of motion.

Q.5. Why is time period repeated twice in the unit of acceleration?

Ans. The velocity of a particle is the rate at which its displacement changes with time; its acceleration is the rate at which its velocity changes with time. Thus acceleration is the rate of a rate. So time is repeated twice.

Q.6. Fig. 6.35 shows $v - t$ graph for various situations. What does each graph indicate?

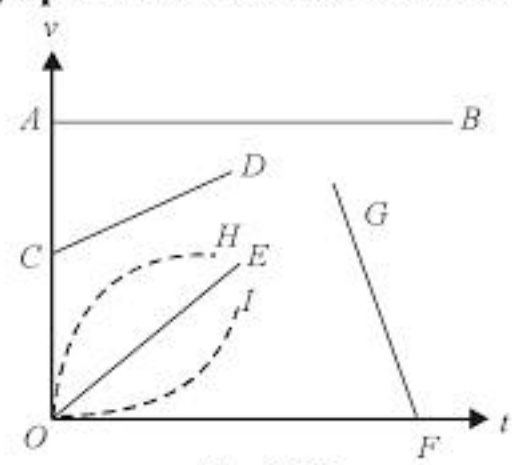


Fig. 6.35

Ans. Acceleration = Slope of $v - t$ graph

(*i*) Straight line AB indicates that the acceleration of the moving body is *zero*. Clearly, the body is moving with constant velocity.

(*ii*) Straight line CD indicates that the body has *constant positive acceleration* with initial velocity OC. In this case, the velocity of the body is increasing.

(*iii*) Straight line OE indicates that the body has positive constant acceleration with zero initial velocity.

(*iv*) Dotted curve OI shows the increasing acceleration. Here the slope of the graph increases with time.

(*v*) Dotted curve OH indicates decreasing acceleration. Here the slope of the graph decreases with time.

(*vi*) The straight line FG indicates that the body is moving with constant negative acceleration. Here the slope of the graph is negative. It means the velocity of the body is decreasing at a constant rate.

Q.7. Is the time variation of position as shown in Fig. 6.36 observed in nature?

Ans. The graph shown in Fig. 6.36 cannot be observed in nature. It is because with the increase of position, the time first increases and then decreases which is impossible.

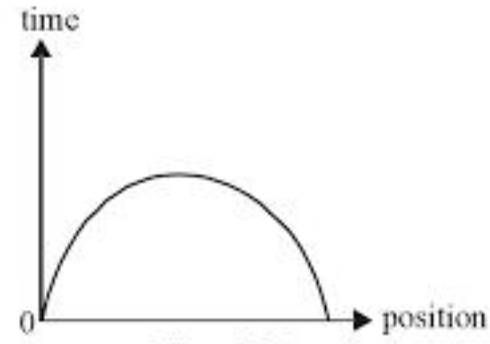


Fig. 6.36

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VERY SHORT ANSWER QUESTIONS

Q.1. Are rest and motion absolute or relative terms?

Ans. Rest and motion are relative terms.

Q.2. Which speed is measured by the speedometer of a car?

Ans. Instantaneous speed *i.e.*, speed at a given instant of time.

Q.3. A body is moving with a uniform velocity. What does it mean?

Ans. It means that the body is moving with a constant speed in a straight line.

Q.4. Can a body have a constant velocity but a variable speed?

Ans. No. It is because speed is equal to the magnitude of velocity.

Q.5. Can a body have a constant speed but variable velocity?

Ans. Yes. When a body is rotated along a circular path with constant linear speed, the velocity is changing (because direction of motion is changing) but speed is constant.

Q.6. What does the slope of position-time graph of a body represent?

Ans. The slope of $x - t$ graph of a body represents its velocity.

Q.7. Under what condition will the distance and displacement of a moving object will have the same magnitude?

Ans. When the object moves in a straight line in the same direction.

Q.8. What is the nature of position-time graph for a stationary object?

Ans. The $x - t$ graph for a stationary object is a straight line parallel to time-axis.

Q.9. Can the speed of a body be negative?

Ans. No. It is because speed = distance/time and distance can never be negative.

Q.10. Can the slope of $x - t$ graph be negative?

Ans. Yes. It is so when the velocity of the body is negative.

Q.11. What is meant by a point object in physics?

Ans. An object is considered as a point object when its size is negligible compared to the scale of observation.

Q.12. What does slope of velocity-time graph represent?

Ans. The slope of velocity-time graph represents acceleration.

Q.13. How will you calculate distance travelled from velocity-time graph?

Ans. The area under $v - t$ graph for a given time interval gives the distance covered during that time interval.

Q.14. What will be the nature of $x - t$ graph for a uniform motion?

Ans. It will be a straight line inclined to the time-axis.

Q.15. Can an object be accelerated without speeding up or slowing down?

Ans. Yes, it can be so in case of uniform circular motion of an object.

Q.16. If the acceleration of a body is zero, what is the nature of motion?

Ans. If the acceleration of a body is zero, the body is either moving with constant velocity in a straight line or remains at rest.

Q.17. What is the basic difference between speed and velocity?

Ans. Speed indicates how fast the body is moving whereas velocity tells us how fast and in which direction it is moving.

Q.18. What does area under velocity-time graph represent?

Ans. The net area under velocity-time graph represents displacement for a given time interval.

Q.19. Can the displacement be greater than distance travelled by an object?

Ans. Displacement is the shortest distance between two positions. Therefore, displacement cannot be greater than the distance travelled.

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Q.20. What will be the nature of velocity-time graph for a uniform motion?

Ans. It will be a straight line parallel to time-axis.

Q.21. When will the relative velocity of two moving objects be zero?

Ans. When the two objects are moving with same speed in the same direction (*i.e.*, equal velocities).

Q.22. How can a body be simultaneously at rest and in motion?

Ans. In relative motion, it is possible. A passenger in a train is at rest *w.r.t.* train but is in motion *w.r.t.* ground.

SHORT ANSWER QUESTIONS

Q.1. If the displacement of a body is zero, is the distance covered by it necessarily zero? Comment.

Ans. No. For example, when a body is thrown vertically upward, it comes back to the same point after some time. In this case, displacement of the body is zero but the distance travelled by the body is $2h$ where h is the maximum height to which the body rises. Similarly, when a body moving in a circle of radius r completes one revolution, the displacement of the body is zero but distance travelled by it is $2\pi r$.

Q.2. Explain that rest and motion are always relative.

Ans. Rest and motion are relative terms. For example, a book on the table is at rest *w.r.t.* table and other objects in the room. If an observer is located on the moon, he will observe that the book and other objects in the room are moving. Thus the book is at rest if viewed from the room but is moving if viewed from the moon.

Q.3. A body can have zero average velocity but not zero average speed. Comment.

Ans. The statement is true.

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}; \quad \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

If a body moving in a circular path of radius r completes one revolution in t seconds, then displacement is zero but distance travelled is $2\pi r$. Therefore, the average velocity of the body is zero while average speed is $2\pi r/t$.

Q.4. A car covers a distance S_1 with velocity v_1 and distance S_2 with velocity v_2. What is the average velocity of car?

Ans. $$\text{Average velocity} = \frac{S_1 + S_2}{(S_1 / v_1) + (S_2 / v_2)}$$ ... Refer to Example 6.1.

Q.5. During a given interval of time, can the average velocity of a body be greater than its average speed?

Ans. No. It is because during a given time interval, the total displacement can be equal to or smaller than the total distance travelled.

Q.6. A cyclist completes one round of circular track of radius $R = 20$ m in 20 seconds. What will be the displacement at the end of 20 s and 30 s?

Ans. At the end of 20 s, the cyclist completes one revolution and the displacement is zero. However, at the end of 30 s, the cyclist completes 1.5 revolutions and the displacement = diameter of circle $= 2R = 2 \times 20 = 40$ m.

Q.7. In the above question, what is the average speed and average velocity?

Ans. After 20 s, average speed $= 2\pi R/t = 2\pi \times 20/20 = 2\pi$ m/s and average velocity = 0 m/s.

After 30 s, average speed $= 1.5 \times 2\pi \times 20/30 = 2\pi$ m/s and average velocity $= 2R/t = 40/30 = 4/3 \text{ms}^{-1}$.

Q.8. The velocity-time graph of two bodies A and B make angles of 30° and 60° with the time-axis. What is the ratio of their accelerations?

Ans. $a_A/a_B = \tan 30°/\tan 60° = 1/3$.

Q.9. The displacement-time graph of two bodies A and B make angles of 45° and 30° with time-axis. What is the ratio of their velocities?

Ans. $v_A/v_B = \tan 45°/\tan 30° = \sqrt{3}$.

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Q.10. A car travels half the distance with velocity v_1 and the second half with a velocity v_2. If the total distance is S, what is the average velocity of the car?

Ans. $$\text{Average velocity} = \frac{2v_1v_2}{v_1+v_2}$$ ... Refer to Example 6.1.

Q.11. Does velocity or acceleration decide the direction of motion of a body?

Ans. It is the velocity and not acceleration that decides the direction of motion of a body. Thus when a body is thrown upward, the direction of motion of the body and its velocity are in the upward direction. However, acceleration due to gravity (g) acts vertically downward. Therefore, direction of motion of the body is in the direction of velocity and not that of acceleration.

Q.12. If the distance (x) covered by a moving body is directly proportional to time (t), what conclusions you draw?

Ans. $x \propto t$ or $x = kt$ when k is constant of proportionality.

Now, $$v = \frac{dx}{dt} = \frac{d}{dt}(kt) = k = \text{constant}$$

Therefore, the body is moving with uniform velocity *i.e.*, with a constant speed in a straight line.

Q.13. If the displacement of a body is directly proportional to the square of time, what conclusions you draw?

Ans. The body is moving with uniform acceleration (Refer to Example 6.20).

Q.14. If the displacement of a body is proportional to t^3, what conclusions you draw?

Ans. Acceleration $\propto t$ (Refer to Example 6.21).

Q.15. A ship S_1 is sailing due east with a velocity of 30 km/h and another ship S_2 is sailing due south with the same velocity. What is the velocity of S_2 relative to S_1?

Ans. Impress a velocity of 30 km/h due west on both ships. This brings ship S_1 to rest. The velocity v of S_2 *w.r.t.* S_1 is the resultant of two velocities, each 30 km/h, one due west and the other due south.

$\therefore$ $$v = \sqrt{30^2+30^2} = 30\sqrt{2} \text{ km/h south-west}$$

Q.16. When two cars A and B move towards each other with constant velocities v_A and v_B, the distance between them decreases by 10 m/s. When they move in same direction with the same speed, the distance between them increases by 5 m/s. What are their velocities ?

Ans. $v_A + v_B = 10$ m/s and $v_A - v_B = 5$ m/s. Therefore, $v_A = 7.5$ m/s ; $v_B = 2.5$ m/s.

LONG ANSWER QUESTIONS

Q.1. Explain the terms : distance, displacement, average speed and average velocity. **[Refer to Arts. 6.11, 6.12 and 6.13]**

Q.2. Explain difference between speed and velocity. **[Refer to Art. 6.14]**

Q.3. What is the geometrical interpretation of average velocity? **[Refer to Art. 6.20]**

Q.4. Prove that displacement of an object in a given time interval is equal to the area under velocity-time graph in that time interval. **[Refer to Art. 6.18]**

Q.5. What is the importance of $x - t$ graph? **[Refer to Art. 6.17]**

Q.6. Explain the $x - t$ and $v - t$ graphs of uniform motion. **[Refer to Arts. 6.17 and 6.18]**

Q.7. Draw position-time graphs of two objects moving along a straight line when their relative velocity is (*i*) zero (*ii*) non-zero. **[Refer to Art. 6.23]**

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COMPETITION SUCCESS MATERIAL

Useful Concepts/Information

1. Distance is a scalar while displacement is a vector. Both have the same dimensions and units.
2. The displacement of a moving particle in a given time interval can be positive, negative or zero but distance travelled by it cannot be negative or zero.
3. An object or body can be considered as a particle if its size is small compared to the scale of observation.
4. In general, the magnitude of displacement of a moving body is not equal to the distance travelled. However, if the body moves in a straight line without changing direction, the magnitude of displacement is equal to the distance travelled.
5. The average velocity of a moving body in a given time interval can be positive, negative or zero but its average speed cannot be negative or zero.
6. For a given time interval, a moving body can have a single average velocity. However, it can have a number of average speeds depending on the path followed.
7. The average velocity of a body over a parituclar time interval is equal to the displacement Δx divided by time interval Δt.

   $$\text{Average velocity, } \bar{v} = \frac{\Delta x}{\Delta t}$$

   On a displacement-time graph, the slope of the chord over a given time interval gives the average velocity over that time interval.
8. The instantaneous velocity v (velocity at a given instant) is defined as the limit of the average velocity at a particular time as $\Delta t \to 0$.

   $$\therefore \quad v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

   On a displacement-time graph, the instantaneous velocity is equal to the slope of the tangent to the graph at that instant of time.
9. If a particle travels distances s_1, s_2 and s_3 with speeds v_1, v_2 and v_3 respectively, then,

   Total distance travelled, $s = s_1 + s_2 + s_3$

   $$\text{Total time taken, } t = \frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3}$$

   $$\therefore \quad \text{Average speed} = \frac{s}{t}$$
10. If a particle travels with speeds v_1, v_2 and v_3 during time intervals t_1, t_2 and t_3 respectively, then,

    Total distance travelled, $s = v_1t_1 + v_2t_2 + v_3t_3$

    Total time taken, $t = t_1 + t_2 + t_3$

    $$\therefore \quad \text{Average speed} = \frac{s}{t}$$
11. For a body at rest, displacement-time graph is a striaght line parallel to time axis.
12. Uniform velocity means magnitude of velocity as well as its direction remains the same.
13. The velocity of a body is positive if it is moving to the right of the origin and the velocity is negative if the body is moving to the left of the origin.

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14. The displacement-time graph of a body moving with uniform velocity is a straight line inclined at an angle θ to the time axis. If $\theta < 90°$, the uniform velocity is positive and if $\theta > 90°$, the uniform velocity is negative.
15. The velocity-time graph of a body moving with uniform velocity is a straight line parallel to time axis.
16. Acceleration is the time rate of change of velocity. The average acceleration over a time interval Δt is equal to change of velocity during the time interval divided by Δt.

Average acceleration, $\bar{a} = \dfrac{\Delta v}{\Delta t}$

17. The instantaneous acceleration a of a body is defined as the limit of the average acceleration at a particular time as $\Delta t \to 0$.

$$\therefore \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

18. If the velocity the time graph of a body is a striaght line parallel to time axis, it means the accelration of the body is zero. The body is, therefore, either at rest or moving with uniform velocity.
19. If the velocity-time graph of a body is a striaght line inclined at an angle θ to the time axis, the body is moving with uniform (constant in magnitude and direction) acceleration. If $\theta < 90°$, the acceleration is positive and if $\theta > 90°$, the acceleration is negative.
20. If the slope of velocity-time graph is changing, it means the body is moving with variable acceleration.

MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. The displacement-time graph for two particles A and B are straight lines inclined at angles of $30°$ and $60°$ with time axis. The ratio of velocities of $V_A : V_B$ is **[CPMT 1990]**

(*a*) 1 : 2 (*b*) 1 : $\sqrt{3}$

(*c*) $\sqrt{3}$: 1 (*d*) 1 : 3

2. For a particle, displacement-time relation is $t = \sqrt{x} + 3$. Its displacement when its velocity is zero is **[CBSE PMT 1999]**

(*a*) 2m (*b*) 4m

(*c*) 0 (*d*) none of these

3. A car covers the first half of the distance between two places at 40 km/hr and the other half at 60 km/hr. The average speed of the car is **[CBSE PMT 1990]**

(*a*) 40 km/h (*b*) 48 km/h

(*c*) 50 km/h (*d*) 60 km/h

4. The area under acceleration-time graph represents **[MP PMT 1993]**

(*a*) the displacement (*b*) velocity

(*c*) change in velocity (*d*) distance travelled

5. A car moves along a straight line whose equation is given by $s = 12t + 3t^2 - 2t^3$ where s is in metres and t is in seconds. The velocity of the car at start will be **[CBSE PMT 1998]**

(*a*) 7 m/s (*b*) 9 m/s

(*c*) 12 m/s (*d*) 16 m/s

6. The displacement-time graph of a moving particle is shown in Fig. 6.37. **[CBSE PMT 1994]**

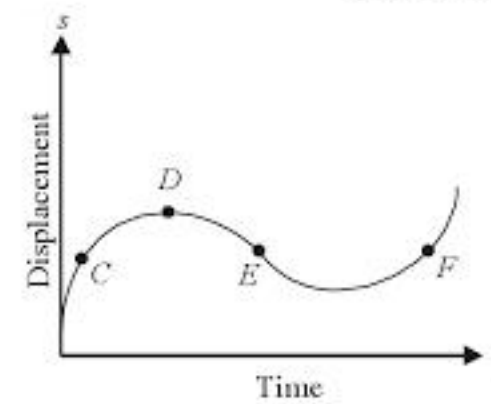


Fig. 6.37

The instantaneous velocity of the particle is negative at the point

(*a*) D (*b*) F

(*c*) C (*d*) E

7. The position x of a particle varies with time t as $x = at^2 - bt^3$. The time t when the acceleration is equal to zero is **[CBSE PMT 1997]**

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(*a*) $\frac{2a}{3b}$ (*b*) $\frac{a}{b}$

(*c*) $\frac{a}{3b}$ (*d*) zero

8. A particle moves along a straight line such that its displacement at any time t is given by: **[CBSE PMT 1994]**

$x = (t^3 - 6t^2 + 3t + 4)$ metres

The velocity when the acceleration is zero is

(*a*) –9 m/s (*b*) –15 m/s

(*c*) 3 m/s (*d*) 42 m/s

9. Which of the following curve in Fig. 6.38 does not represent motion in one dimension? **[CBSE PMT 1992]**

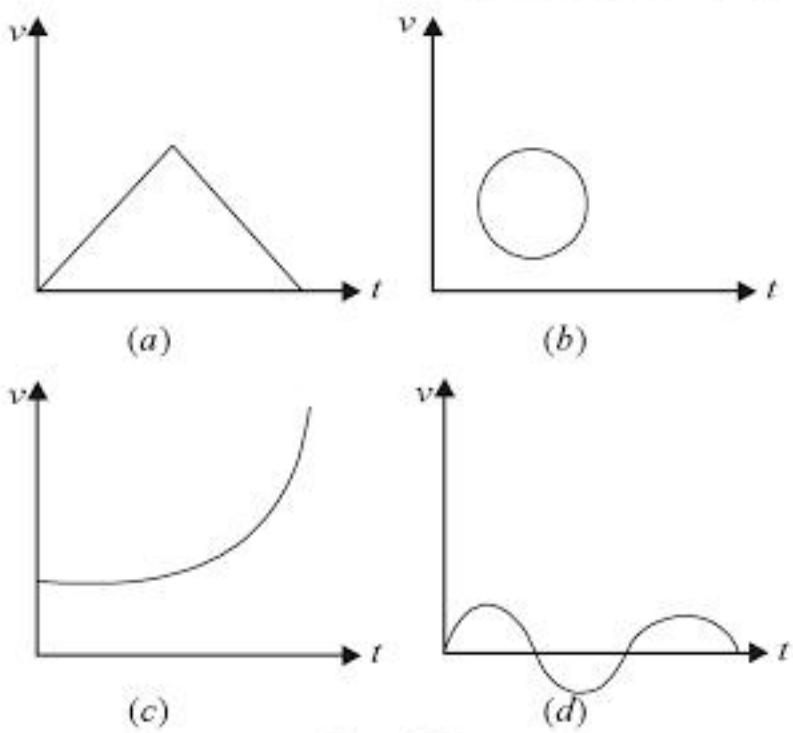


Fig. 6.38

10. In 1.0 s, a particle goes from point A to point B, moving in a semicircle of radius 1.0 m as shown in Fig. 6.39. The magnitude of average velocity is **[CPMT 2000]**

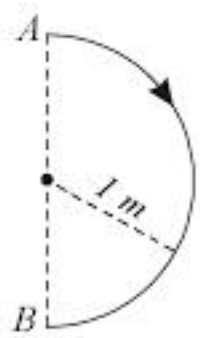


Fig. 6.39

(*a*) 3.14 m/s

(*b*) 2.0 m/s

(*c*) 1.0 m/s

(*d*) zero

11. One train A 120 m long is moving in a direction with a speed of 20 m/s. Another train B 130 m long is moving with a speed of 30 m/s in the opposite direction. The train B crosses the train A in a time **[CPMT 1996]**

(*a*) 5 s (*b*) 36 s

(*c*) 38 s (*d*) None of these

12. The displacement of a particle is given by ; $y = a + bt + ct^2 - dt^4$

The initial velocity and acceleration are respectively **[CPMT 1999]**

(*a*) $b, -4d$ (*b*) $-b, 2c$

(*c*) $b, 2c$ (*d*) $2c, -4d$

13. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will **[CBSE 2005]**

(*a*) go on decreasing with time

(*b*) be independent of α and β

(*c*) drop to zero when $\alpha = \beta$

(*d*) go on increasing with time

14. Two boys are standing at the ends A and B of a ground, where $AB = a$. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other boy in a time t, where t is : **[CBSE 2005]**

(*a*) $a/\sqrt{v^2 + v_1^2}$ (*b*) $\sqrt{a^2/(v^2 - v_1^2)}$

(*c*) $a/(v - v_1)$ (*d*) $a/(v + v_1)$

15. Which of the following velocity-time $(v - t)$ graph shows a realistic situation for a body in motion (See Fig. 6.40)? **[AIIMS 2005]**

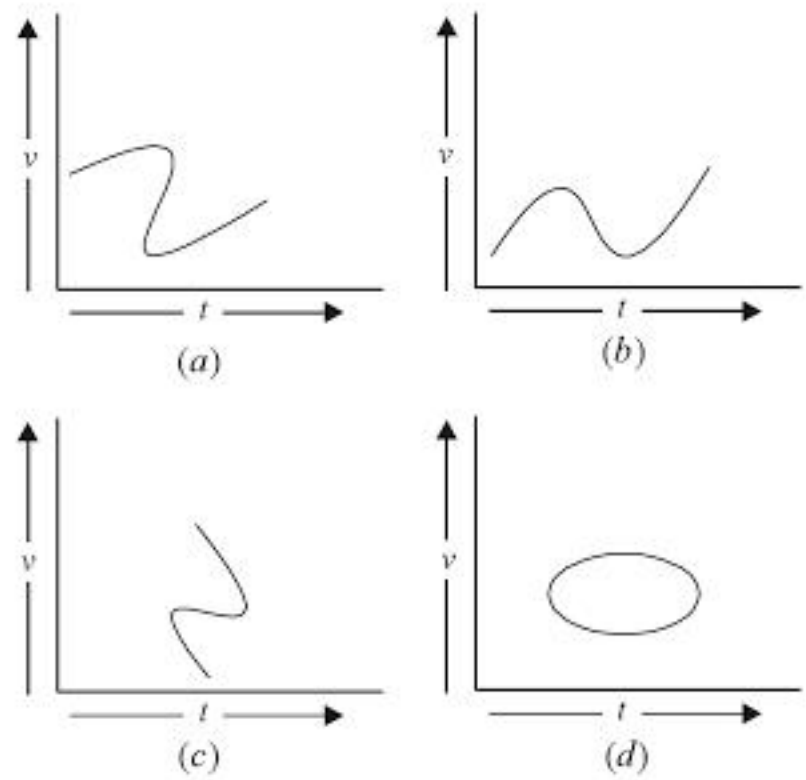


Fig. 6.40

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ANSWERS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. (*d*)	**2.** (*c*)	**3.** (*b*)	**4.** (*c*)	**5.** (*c*)
6. (*d*)	**7.** (*c*)	**8.** (*a*)	**9.** (*b*)	**10.** (*b*)
11. (*a*)	**12.** (*c*)	**13.** (*d*)	**14.** (*b*)	**15.** (*b*)

HINTS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. Velocity of particle is equal to the slope of displacement-time graph. (See Fig. 6.41).

$$\therefore \quad \frac{V_A}{V_B} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{\mathbf{1}}{\mathbf{3}}$$

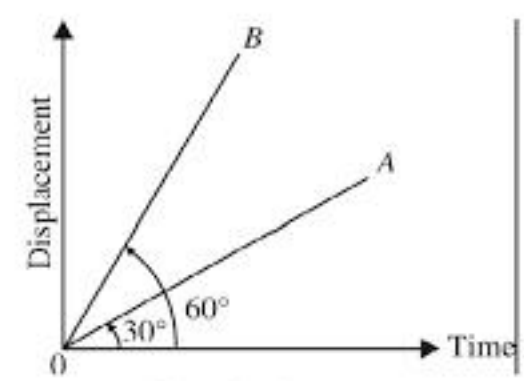


Fig. 6.41

2. $$t = \sqrt{x} + 3$$

Differentiating both sides *w.r.t.* t, we get,

$$1 = \frac{1}{2\sqrt{x}} \frac{dx}{dt}$$

Putting $dx/dt = 0$, we have $x = \mathbf{0}$.

3. Let the total distance be s.

$$\text{Total time taken} = \frac{s/2}{40} + \frac{s/2}{60} = \frac{s}{48} h$$

$$\therefore \quad \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{s}{s/48} = \mathbf{48\ km/h}$$

4. Area under acceleration – time graph

= acceleration × time

= **Change in velocity**

5. The velocity at start means the velocity at $t = 0$.

$$s = 12t + 3t^2 - 2t^3$$

$$\therefore \quad \frac{ds}{dt} = 12 + 6t - 6t^2$$

$$\therefore \quad \left(\frac{ds}{dt}\right)_{t=0} = 12 + 6 \times 0 - 6 \times (0)^2 = \mathbf{12\ m/s}$$

6. Instantaneous velocity of a particle is the slope of the tangent to the displacement-time graph at a point. At point E, the slope is negative. Therefore, the instantaneous velocity of the particle is negative at point ***E***.

7. $$x = at^2 - bt^3$$

or $$\frac{dx}{dt} = 2at - 3bt^2$$

or $$\frac{d^2x}{dt^2} = 2a - 6bt$$

Let at time t_1 the acceleration of the particle become zero. Then as per given conditions:

$$0 = 2a - 6bt_1$$

$$\therefore \quad t_1 = \frac{2a}{6b} = \frac{\boldsymbol{a}}{\boldsymbol{3b}}$$

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8. $$x = t^3 - 6t^2 + 3t + 4$$

or $$\frac{dx}{dt} = 3t^2 - 12t + 3$$

or $$\frac{d^2x}{dt^2} = 6t - 12$$

When the acceleration (d^2x/dt^2) of the particle is zero, the time t is given by :

$$0 = 6t - 12 \quad \therefore \quad t = 2 \text{ sec.}$$

The velocity of the particle at $t = 2$ sec is

$$\left(\frac{dx}{dt}\right)_{t=2\text{sec}} = 3 \times (2)^2 - 12 \times 2 + 3 = \mathbf{-9 m/s}$$

9. In **Fig. 6.38 (b)**, the particle has two velocities at the same time. This is not possible in one dimensional motion.

10. $$\text{Displacement} = 2 \times \text{radius} = 2 \times 1.0 = 2.0 \text{ m}$$

$$\therefore \quad \text{Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{2 \text{m}}{1 \text{s}} = \mathbf{2 m/s}$$

11. Total length of two trains, $x = 130 + 120 = 250$ m

Relative velocity of train B *w.r.t.* train A

$$= 30 - (-20) = 50 \text{ m/s}$$

$\therefore$ Time taken by train B to cross train A

$$= \frac{x}{50} = \frac{250}{50} = \mathbf{5 s}$$

12. $$y = a + bt + ct^2 - dt^4$$

or $$\frac{dy}{dt} = b + 2ct - 4dt^3$$

or $$\frac{d^2y}{dt^2} = 2c - 12dt^2$$

$$\text{Initial velocity} = \left(\frac{dy}{dt}\right)_{t=0} = b + 2c \times 0 - 4d \times (0)^3 = \mathbf{b}$$

$$\text{Initial acceleration} = \left(\frac{d^2y}{dt^2}\right)_{t=0} = 2c - 12d(0)^2 = \mathbf{2c}$$

13. $$x = ae^{-\alpha t} + be^{\beta t}$$

$$\therefore \quad v = \frac{dx}{dt} = \frac{d}{dt}\left(ae^{-\alpha t} + be^{\beta t}\right) = -a\alpha e^{-\alpha t} + b\beta e^{\beta t} = A + B$$

$$\text{where} \quad A = -a\alpha e^{-\alpha t} \quad \text{and} \quad B = b\beta e^{\beta t}$$

As time t increases, the term A approaches zero. However, as time t increases, the term B (positive) approaches infinity. As a result, velocity **goes on increasing with time.**

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14. Fig. 6.42 shows the conditions of the problem In time t, the distance covered by the boy starting at point A is $AC = vt$. In time t, the distance covered by the boy starting at point B is $BC = v_1 t$.

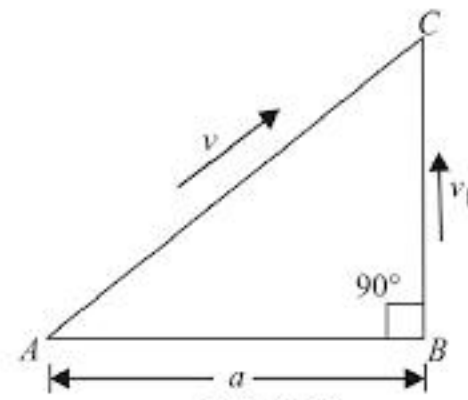


Fig. 6.42

Now $(AC)^2 = (AB)^2 + (BC)^2$

or $(vt)^2 = a^2 + (v_1 t)^2$

or $v^2t^2 = a^2 + v_1^2 t^2$

$\therefore \quad t^2(v^2 - v_1^2) = a^2 \quad$ or $\quad t = \sqrt{\dfrac{a^2}{\left(v^2 - v_1^2\right)}}$

15. We know that a body in motion has only one velocity at a particular time. However, this velocity can decrease or increase with time. In the given $v - t$ graphs, only graph (b) has only one velocity at a particular time. But graphs (a), (c) and (d) have two velocities at same times. Therefore, **$v - t$ graph (b) shows a realistic situation for the body in motion.**

MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. A car travels for half of its time at 80 km/h and for the rest half time at 40 km/h. Total distance covered is 60 km. What is the average speed of the car? **[Rajasthan PET 1996]**

(a) 60 km/h (b) 80 km/h

(c) 120 km/h (d) 180 km/h

2. The displacement of a body is given to be proportional to the cube of time elapsed. The magnitude of the acceleration of the body is **[NCERT 1990]**

(a) increasing with time

(b) decreasing with time

(c) constant but not zero

(d) zero

3. The correct statement from the following is **[MP PET 1993]**

(a) A body having zero velocity will not necessarily have zero acceleration.

(b) A body having zero velocity will necessarily have zero acceleration.

(c) A body having uniform speed can have only uniform acceleration.

(d) A body having non-uniform velocity will have zero acceleration.

4. A particle moving in a straight line covers half the distance with a speed of 3m/s. The other half of the distance is covered in two equal time intervals with speed of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during the entire motion is **[IIT 1992]**

(a) 4 m/s (b) 5 m/s

(c) 5.5 m/s (d) 4.8 m/s

5. Which one of the following four statements is false? **[Manipal 1995]**

(a) A body can have zero velocity and still be accelerated.

(b) A body can have constant velocity and still have a varying speed.

(c) A body can have a constant speed and still have a varying velocity.

(d) The direction of a body can change when magnitude of its acceleration is constant.

6. Figs. 6.43 (i) and (ii) below show the displacement-time graph of two particles moving along x-axis. We can say that **[Haryana CEET 1996]**

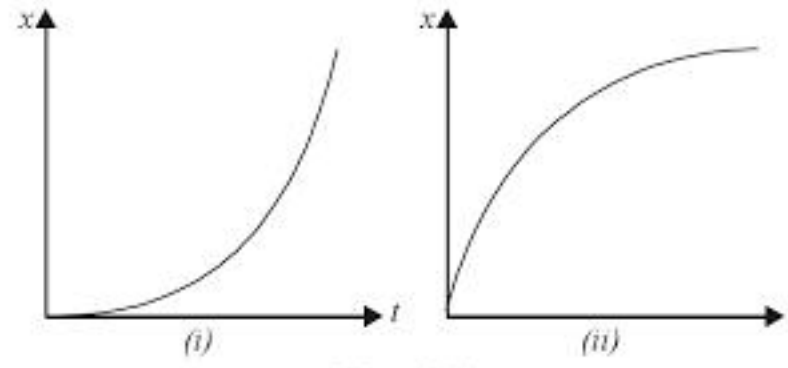


Fig. 6.43

(a) both the particles are having a uniformly accelerated motion.

(b) both the particles are having a uniformly retarded motion.

(c) particle (i) is having a uniformly accelerated motion while the particle (ii) is having a uniformly retarded motion.

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(*d*) particle *(i)* is having a uniformly retarded motion while the particle *(ii)* is having a uniformly accelerated motion.

7. The displacement of a particle starting from rest ($t = 0$) is given by $s = 6t^2 - t^3$. The time in seconds at which the particle will attain zero velocity is **[SCRA 1998]**

(*a*) 2 (*b*) 4

(*c*) 6 (*d*) 8

8. Which of the following in Fig. 6.44 cannot be speed-time graph ? **[Manipal 1999]**

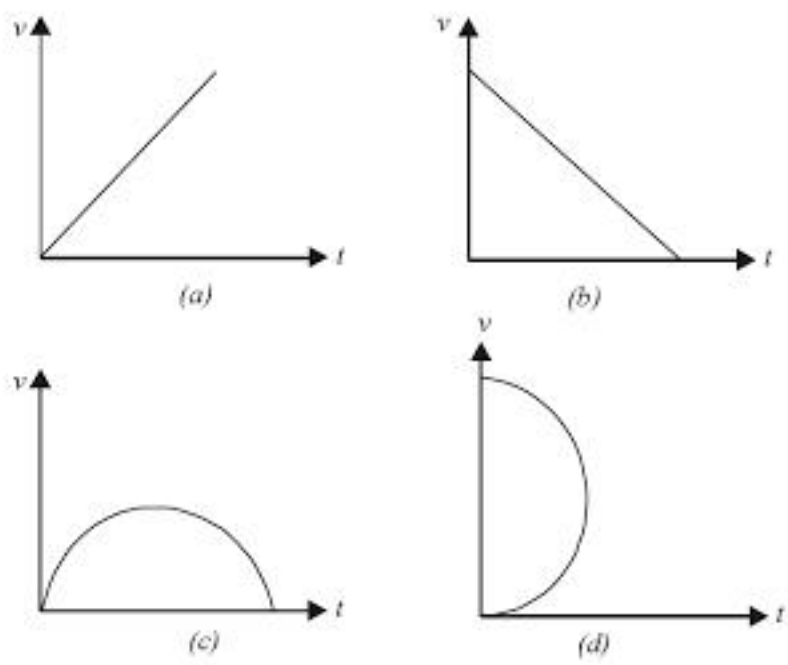


Fig. 6.44

9. An aeroplane is flying with a uniform speed of 100km/h along the circumference of a circle. The change in velocity in half the revolution will be (km/h) **[KCET 2000]**

(*a*) 100 (*b*) 200

(*c*) 300 (*d*) 150

10. The velocity of a body depends upon time according to the equation $v = 20 + 0.1t^2$. The body is undergoing **[MNR 1995]**

(*a*) uniform acceleration

(*b*) uniform retardation

(*c*) non-uniform acceleration

(*d*) zero acceleration

11. An athelete completes one round of a circular track of radius R in 40 seconds. What will be his displacement at the end of 2 min and 20 seconds? **[NCERT 1990]**

(*a*) zero (*b*) $2R$

(*c*) $2\pi R$ (*d*) $7\pi R$

12. The acceleration 'a' in m/s^2 of a particle is given by $a = 3t^2 + 2t + 2$ where t is the time in seconds. If the particle starts out with a velocity $v = 2$ m/s at $t = 0$, then the velocity at the end of 2 seconds is **[SCRA 1994]**

(*a*) 12 m/s (*b*) 18 m/s

(*c*) 27 m/s (*d*) 36 m/s

13. A body starts from rest at time $t = 0$, the acceleration–time graph is shown in Fig. 6.45. The maximum velocity attained by the body will be : **[IIT 2004]**

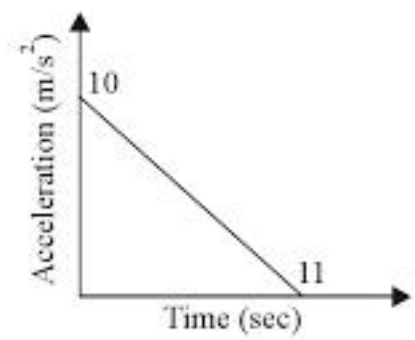


Fig. 6.45

(*a*) 110 m/s (*b*) 55 m/s

(*c*) 650 m/s (*d*) 550 m/s

14. The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is **[AIEEE 2005]**

(*a*) $-2abv^2$ (*b*) $2bv^3$

(*c*) $-2av^3$ (*d*) $2av^2$

15. Which of the following option is correct for the object having a straight line motion represented by the following graph (See Fig. 6.46)? **[DCE 2004]**

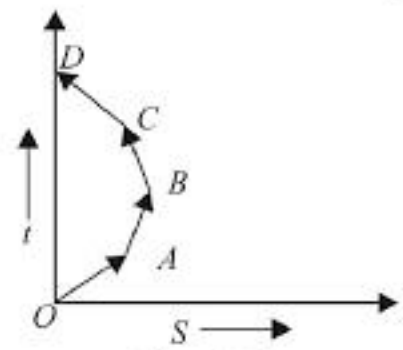


Fig. 6.46

(*a*) average velocity is zero

(*b*) velocity of the object increases uniformly

(*c*) the object moves with constantly increasing velocity from O to A and then it moves with constant velocity

(*d*) the graph shown is impossible

ANSWERS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. (*a*)	**2.** (*a*)	**3.** (*a*)	**4.** (*a*)	**5.** (*b*)
6. (*c*)	**7.** (*b*)	**8.** (*d*)	**9.** (*b*)	**10.** (*c*)
11. (*b*)	**12.** (*b*)	**13.** (*b*)	**14.** (*c*)	**15.** (*a*)

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HINTS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. $$\text{Average speed} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2} = \frac{80 \times t/2 + 40 \times t/2}{t/2 + t/2} = \mathbf{60\ km/h}$$

2. $$x = kt^3 \quad \therefore \quad \frac{dx}{dt} = 3kt^2 \quad \text{or} \quad \frac{d^2x}{dt^2} = 6kt$$

$\therefore$ Acceleration $\propto t$

3. When a body is projected vertically upward, at the highest point of motion, its velocity is zero. However, the body is acted upon by acceleration due to gravity (downward).

4. Let s metres be the total distance travelled by the particle. Suppose t_1 seconds is the time for first half journey and t_2 seconds two equal time intervals to cover the second half of journey.

$$\therefore \quad t_1 = \frac{s/2}{3} = \frac{s}{6} \text{ seconds}; \quad s_1 = 4.5\ t_2 \quad \text{and} \quad s_2 = 7.5\ t_2$$

$$\text{Now} \quad s_1 + s_2 = \frac{s}{2} = 4.5t_2 + 7.5t_2 = 12t_2$$

$$\therefore \quad t_2 = s/24$$

$$\text{Total time} = t_1 + 2t_2 = \frac{s}{6} + 2 \times \frac{s}{24} = s/4$$

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{s}{s/4} = \mathbf{4\ m/s}$$

5. The constant velocity of a body means that its magnitude (speed) as well as direction is unchanged throughout its motion.

6. The slope of $x - t$ graph at any time gives the velocity of the particle at that time. In Fig. 6.43 (i), the slope of $x - t$ graph increases and hence the body has accelerated motion. The reverse is the case for Fig. 6.43 (ii).

7. $$s = 6t^2 - t^3$$

$$\text{Velocity} = \frac{ds}{dt} = \frac{d}{dt}(6t^2 - t^3) = 12t - 3t^2$$

When velocity is zero, we have, $0 = 12t - 3t^2 \quad \therefore\ t = \mathbf{4\ s}$

8. In **Fig. 6.44 (*d*)**, there will be two speeds at one time. But a particle can have one speed at a time.

9. In half revolution, the velocity of the plane will be opposite to the initial velocity.

$\therefore$ Change in velocity $= 100 - (-100) = \mathbf{200\ km/h}$

10. $$\text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}(20 + 0.1t^2) = 0.2t$$

Since $a \propto t$, the body is undergoing **non-uniform acceleration.**

11. Total time of motion $= 2 \times 60 + 20 = 140$ seconds

Time/revolution = 40 seconds. In 120 seconds, the athelete will complete 3 revolutions and the displacement will be zero.

$\therefore$ Displacement in 140 sec = Displacement in 20 sec.

$$= R + R = \mathbf{2R}$$

12. $$a = \frac{dv}{dt} = 3t^2 + 2t + 2$$

or $$dv = (3t^2 + 2t + 2)dt$$

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or $$\int_u^v dv = \int_0^t (3t^2 + 2t + 2)dt$$

or $$v - u = t^3 + t^2 + 2t$$

or $$v = u + t^3 + t^2 + 2t$$

At the end of 2 seconds, the velocity is

$$(v)_{t = 2 \text{ sec}} = 2 + (2)^3 + (2)^2 + 2 \times 2 = \mathbf{18\ m/s}$$

13. The area under acceleration-time graph gives the change in velocity.

$$\therefore \quad v_{max} = \frac{1}{2} \times 10 \times 11 = 55 \text{ m/s}$$

Since the initial velocity is zero, the maximum velocity attained = **55 m/s**

14. $$t = ax^2 + bx$$

Differentiating *w.r.t.* t, we have,

$$1 = 2a\,x\,\frac{dx}{dt} + b\frac{dx}{dt} = \frac{dx}{dt}(2ax + b)$$

$$\therefore \quad \frac{dx}{dt} = v = \frac{1}{2ax + b} = (2ax + b)^{-1} \qquad \ldots (i)$$

Again differentiating eq. (*i*) *w.r.t.* t, we have,

$$\frac{d^2x}{dt^2} = -1 \times (2ax + b)^{-2} \times 2a\,\frac{dx}{dt}$$

$$= \frac{-1}{(2ax + b)^2} \times \frac{2a}{(2ax + b)} \qquad \left(\because \frac{dx}{dt} = \frac{1}{(2ax + b)}\right)$$

$$= \frac{-2a}{(2ax + b)^3} = -2av^3 \qquad \left(\because v = \frac{1}{(2ax + b)}\right)$$

15. It is clear from the graph that the displacement of the object is zero. Therefore, **average velocity of the object is zero.**

NUMERICAL PROBLEMS FOR COMPETITIVE EXAMINATIONS

1. A person travelled a distance of 200 km between two cities by a car covering the first half distance of the journey at a constant speed of 40 km/h and the remaining half distance at a constant speed of v km/h. If the average speed of the person for the entire journey is 48 km/h, find v.

Hint. $$t_1 = \frac{100}{40} = 2.5 \text{ hours}\,;\ t_2 = \frac{100}{v} \text{ hours}$$

$$\text{Average speed} = \frac{\text{Total distance}}{t_1 + t_2}$$

or $$48 = \frac{200}{2.5 + (100/v)}$$

On solving, v = **60 km/h**

2. A train 100 m long is moving with a uniform velocity of 45 km/h. What is the time it will take to cross a bridge 1 km long ?

Hint. Total distance to be covered

= Length of bridge + Length of train = 1 + 0.1 = 1.1 km

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$\therefore$ Time taken by the train to cross the bridge is

$$t = \frac{\text{Total distance}}{\text{Velocity}} = \frac{1.1}{45}\text{hour} = \frac{1.1}{45} \times 3600\,\text{s} = \mathbf{88\ s}$$

3. A motor ship covers a distance of 300 km between two localities on a river in 10 hours downstream and in 12 hours upstream. Find the velocity of the river and the velocity of the motor ship in still water, assuming that these velocities are constant.

Hint. Suppose v_1 and v_2 are the velocities of the ship and water flow respectively in km/h. In case of downstream, the resultant velocity is $(v_1 + v_2)$ while in case of upstream, the resultant velocity is $(v_1 - v_2)$.

$$\text{In downstream, } t_1 = \frac{x}{v_1 + v_2} \quad \text{or} \quad 10 = \frac{300}{v_1 + v_2}$$

$$\therefore \quad v_1 + v_2 = 30 \qquad \ldots(i)$$

$$\text{In upstream, } t_2 = \frac{x}{v_1 - v_2} \quad \text{or} \quad 12 = \frac{300}{v_1 - v_2}$$

$$\therefore \quad v_1 - v_2 = 25 \qquad \ldots(ii)$$

On solving eqs. (i) and (ii), $v_1 = \mathbf{27.5\ km/h}$; $v_2 = \mathbf{2.5\ km/h}$

4. An electron starting from rest has a velocity v given by ;

$$v = k\,t$$

where $k = 2\ \text{ms}^{-2}$ and t is the time in seconds. What will be the distance covered by the electron in first 3 seconds ? **[NCERT]**

Hint.

$$v = \frac{dx}{dt} = kt$$

$$\text{Integrating, we get, } x = \frac{kt^2}{2}$$

$$\text{Here } k = 2\ \text{ms}^{-2}\ ;\ t = 3\text{s}$$

$$\therefore \quad x = \frac{2 \times (3)^2}{2} = \mathbf{9\ m}$$

5. Two cars started simultaneously towards each other from towns A and B which are 480 km apart. It took first car travelling from A to B 8 hours to cover the distance and the second car travelling from B to A took 12 hours. Determine when the cars meet after starting and at what distance from town A. Assume that both the cars travel at constant speed.

Hint. Speed of car travelling from town A is

$$v_A = 480/8 = 60\ \text{km/h}$$

Speed of car travelling from town B is

$$v_B = 480/12 = 40\ \text{km/h}$$

Let the two cars meet after time t hours. In this time, the distance travelled from A is $60t$ and the distance travelled from B is $40t$. When the cars meet, the total distance travelled = 480 km.

$$\therefore \quad 60t + 40\,t = 480$$

$$\text{or} \quad t = \frac{480}{60 + 40} = \mathbf{4.8\ hours}$$

The distance from town A at the meeting point

$$= v_A \times t = 60 \times 4.8 = \mathbf{288\ km}$$

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6. **Two cars are moving in the same direction with the same speed of 30 km/h. They are separated by a distance of 5 km. A truck moving in the opposite direction meets these two cars at an interval of 4 minutes. Find the speed of the truck.**

Hint. Suppose the truck is moving with a speed of v km/h as shown in Fig. 6.47. Apply a velocity of 30 km/h on all the vehicles in the direction of motion of the truck. The two cars come to rest with a fixed distance of 5 km while the velocity of truck becomes $(v + 30)$km/h. The time t hours taken by the truck to cover a distance of 5 km with a speed of $(v + 30)$km /h is given by ;

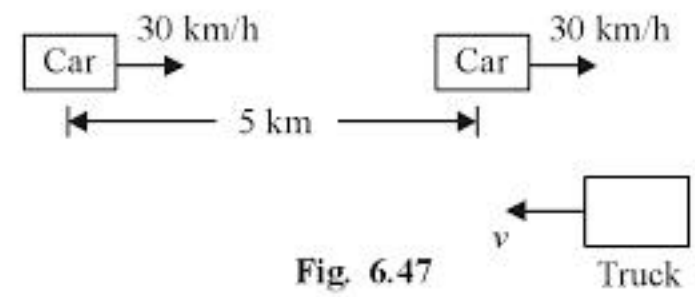


Fig. 6.47

$$t = \frac{\text{Distance}}{\text{Velocity}} = \frac{5}{v+30}$$

or $$\frac{1}{15} = \frac{5}{v+30} \qquad \left(\because t = 4\,\text{minutes} = \frac{1}{15}\,\text{hours}\right)$$

$\therefore$ $$v = \mathbf{45\ km/h}$$

7. **A body is moving in a straight line along X-axis. Its distance x in metres from the origin is given by ; $x = 8\ t - 3\ t^2$**
The time t is in seconds. Find the average speed of the body in time interval $t = 0$ to $t = 1$ s.

Hint. $$x = 8\ t - 3\ t^2$$

$$\text{Velocity, } v = \frac{dx}{dt} = \frac{d}{dt}(8t - 3t^2) = 8 - 6t$$

$$\text{At } t = 0 \text{ s, } v_0 = 8 - 6 \times 0 = 8 \text{ ms}^{-1}$$

$$\text{At } t = 1 \text{ s, } v_1 = 8 - 6 \times 1 = 2 \text{ ms}^{-1}$$

$\therefore$ $$\text{Average speed} = \frac{8+2}{2} = \mathbf{5\ ms^{-1}}$$

8. **Two trains are moving in opposite directions. Train A moves east with a speed of 72 km/h and train B moves west with a speed of 90 km/h. What is the (*i*) relative velocity of B *w.r.t.* A ? (*ii*) the relative velocity of ground w.r.t. B (*iii*) the velocity of a dog running on the roof of the train A against its motion (with a velocity of 54 km/h w.r.t. to train A) as observed by a man standing on the ground.**

Hint. If not mentioned, the speeds are understood to be *w.r.t.* earth or ground. Therefore, speeds of trains A and B are *w.r.t.* earth.

(*i*) Speed of train A, $v_{AE} = 72\,\text{km/h} = \dfrac{72 \times 1000}{60 \times 60} = 20\,\text{ms}^{-1}$

Speed of train B, $v_{BE} = 90\,\text{km/h} = \dfrac{90 \times 1000}{60 \times 60} = 25\,\text{ms}^{-1}$

Let us choose the positive direction from west to east.

$\therefore$ $$v_{AE} = +\ 20\ \text{ms}^{-1}\ ;\ v_{BE} = -\ 25\ \text{ms}^{-1}$$

Relative velocity of B *w.r.t.* A is given by ;

$$v_{BA} = v_{BE} + v_{EA} = v_{BE} - v_{AE} = -25 - (+\ 20)$$
$$= \mathbf{-\ 45\ ms^{-1}\ or\ 45\ ms^{-1}\ east\ to\ west}$$

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(*ii*) The relative velocity of ground (*i.e.*, earth) *w.r.t.* B is

$$v_{EB} = v_{EE} + v_{EB} = v_{EE} - v_{BE} = 0 - (-25) = \mathbf{25\ ms^{-1}}$$

Thus, velocity is from west to east.

(*iii*) Velocity of dog *w.r.t.* to train A is

$$v_{DA} = 54\,\text{km/h} = \frac{54\times1000}{60\times60} = 15\,\text{ms}^{-1}$$

Since our positive direction is from west to east, $v_{DA} = -15\ \text{ms}^{-1}$.

Now $\quad v_{DA} = v_{DE} + v_{EA} = v_{DE} - v_{AE}$

$\therefore \quad v_{DE} = v_{DA} + v_{AE} = -15 + 20 = \mathbf{5\ ms^{-1}}$

9. The motion of a particle along a straight line is described by the equation :

$$x = 6 + 4\,t^2 - t^4$$

where x is in metres (m) and t is the positive time in seconds (s).

(*i*) Find the position, velocity and acceleration at $t = 2$ s.

(*ii*) During what time interval is the velocity positive ?

(*iii*) During what time interval is x positive ?

(*iv*) What is the maximum possible velocity attained by the particle ?

Hint. (*i*) $\quad x = 6 + 4\,t^2 - t^4$

$$\text{Velocity,}\quad v = \frac{dx}{dt} = \frac{d}{dt}\left(6+4t^2-t^4\right) = 8t - 4t^3$$

$$\text{Acceleration,}\quad a = \frac{dv}{dt} = \frac{d}{dt}\left(8t-4t^3\right) = 8 - 12t^2$$

At $t = 2$ s, $x = 6 + 4\,(2)^2 - (2)^4 = \mathbf{6\ m}$

At $t = 2$ s, $v = 8 \times 2 - 4\,(2)^3 = \mathbf{-16\ ms^{-1}}$

At $t = 2$ s, $a = 8 - 12\,(2)^2 = \mathbf{-40\ ms^{-2}}$

(*ii*) The velocity v is positive if $8t - 4t^3 \geq 0$. That is if

$$4\,t\,(2 - t^2) > 0$$

i.e., $\qquad 0 < t < \sqrt{2} \qquad$ or $\qquad \mathbf{0 < t < 1.41\ s}$

(*iii*) The position x is positive if $6 + 4\,t^2 - t^4 \geq 0$. That is, $t^2 \leq 2 \pm \sqrt{10}$ or $t^2 \leq 2 \pm 3.16$ s. Since t is positive, t lies between 0 and $\sqrt{5.16}$ or **0 and 2.27 s.**

(*iv*) The velocity is maximum for t given by ;

$$\frac{dv}{dt} = 0 \quad \text{or} \quad 8 - 12t^2 = 0 \quad \text{or} \quad t = \sqrt{\frac{2}{3}} = 0.816\text{s}$$

$\therefore \quad$ Maximum velocity, $v_{max} = (8t - 4t^3)_{t\,=0.816\ \text{s}}$

$$= [8 \times 0.816 - 4 \times (0.816)^3] = \mathbf{4.355\ ms^{-1}}$$

10. Two cars A and B are at positions 100 m and 200 m from the origin at $t = 0$. They start simultaneously with velocities 10 ms^{-1} and 5 ms^{-1} respectively. Calculate the time and position at which they will overtake one another. Assume that they are moving in the same direction.

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Hint. $x_{BO} = 200$ m ; $x_{AO} = 100$ m ; $v_A = 10$ ms^{-1} ; $v_B = 5$ ms^{-1}

$$x_B - x_A = (x_{BO} - x_{AO}) + (v_B - v_A)t$$

For the solution of the problem, $x_B - x_A = 0$.

$\therefore$ $0 = (200 - 100) + (5 - 10)t$

or $0 = 100 - 5t$

$\therefore$ $t = 100/5 = \mathbf{20\ s}$

$x = 100 + 10 \times 20 = 100 + 200 = \mathbf{300\ m}$

11. The motion of a particle along X-axis is given by the equation $x = 20 + 4\,t^2$ where x is in metres and t in seconds. Find (*i*) the displacement of the particle in time interval $t_1 = 2$ s and $t_2 = 5$ s (*ii*) the average velocity in the same time interval (*iii*) the instantaneous velocity at t = 2 s.

Hint. $x = 20 + 4\,t^2$

(*i*) The position of the particle at $t_1 = 2$ s is $x_1 = 20 + 4\,(2)^2 = 36$ m

The position of the particle at $t_2 = 5$ s is $x_2 = 20 + 4\,(5)^2 = 120$ m

$\therefore$ Displacement of the particle during time interval 2 s to 5 s

$= x_2 - x_1 = 120 - 36 = \mathbf{84\ m}$

(*ii*) Average velocity $= \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{84}{5-2} = \dfrac{84}{3} = \mathbf{28\ ms^{-1}}$

(*iii*) $x = 20 + 4\,t^2$

Instantaneous velocity is given by ;

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(20 + 4t^2\right) = 8t$$

At $t = 2$ s ; $v = 8 \times 2 = \mathbf{16\ ms^{-1}}$

12. If velocity (m/s) varies with time as $v = 4t$, find the distance travelled by the body in the interval of 2 to 4 seconds. **[CBSE 2004]**

Hint. $v = 4t$ or $\dfrac{dx}{dt} = 4t$

$\therefore$ $\displaystyle\int_0^x dx = \int_2^4 4t\,dt$

or $\displaystyle x = 4\left[\frac{t^2}{2}\right]_2^4 = 2[4^2 - 2^2] = \mathbf{24\ m}$

13. The position of a particle as a function of time is described as $y(t) = 5\cos(10t + 15°)$ in metres. What is the average velocity of the particle from $t = 0$ to $t = 3$ seconds?

Hint. $y\,(t) = 5\cos(10t + 15°)$

When $t = 0$ s ; $y\,(0) = 5\cos\,(10 \times 0 + 15°) = 5\cos 15° = 4.83$ m

When $t = 3$ s ; $y\,(3) = 5\cos\,(10 \times 3 + 15°) = 5\cos 45° = 3.53$ m

$\therefore$ Average velocity, $\bar{v} = \dfrac{\text{Total displacement}}{\text{Total time}} = \dfrac{4.83 + 3.53}{3} = \mathbf{2.79\ m/s}$

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7
Uniformly Accelerated Linear Motion

INTRODUCTION

When an object *moves with a constant acceleration in a straight line,* it is said to have **uniformly accelerated linear motion.** It is the simplest kind of accelerated motion. There are many practical situations in which the acceleration is uniform (*i.e.,* constant). For example, near the surface of earth, all objects fall vertically with a constant acceleration of gravity g (= 9.8 ms^{-2}). There are only *four mathematical equations* that describe the linear motion of an object with constant acceleration. In this chapter, we shall deal with mathematical and graphical aspects of uniformly accelerated linear motion.

7.1. EQUATIONS FOR UNIFORMLY ACCELERATED LINEAR MOTION

Consider an object moving with a uniform acceleration 'a' along a straight line OX with origin at O as shown in Fig. 7.1. Let at $t = 0$, the position of the object be x_0 (initial displacement from O) and its velocity v_0 (initial velocity). Let at $t = t$, the position of the object be x and its velocity v (final velocity). Clearly, in time t, the displacement of the object is $x - x_0$.

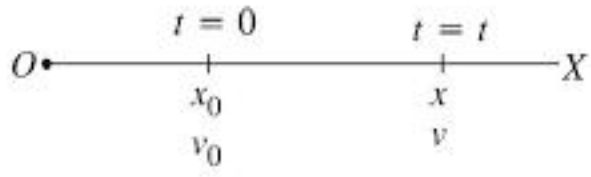


Fig. 7.1

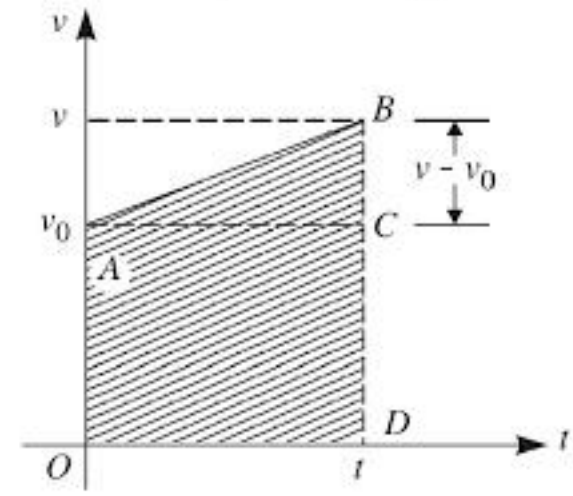


Fig. 7.2

(*i*) Displacement – Average Velocity relation. The velocity-time ($v - t$) graph for the motion in Fig. 7.1 is AB as shown in Fig. 7.2. The velocity increases uniformly from v_0 at $t = 0$ to v in time t (= OD). We know that displacement during any time interval is equal to the area under velocity-time graph during that time interval. In this case, the time interval is from 0 to t so that displacement is equal to the shaded area in Fig. 7.2.

$\therefore$ Displacement = Shaded area = Area of trapezium $OABD$

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$$= \frac{AO + BD}{2} \times OD = \frac{v_0 + v}{2} \times t$$

$\therefore$ Average velocity, $\bar{v} = \dfrac{\text{Displacement}}{\text{Time interval}} = \dfrac{v_0 + v}{2} \times t \div t$

or $$\bar{v} = \frac{v_0 + v}{2}$$

Note that for the case of constant acceleration *and only for this case,* the average velocity is equal to one-half the sum of initial and final velocities.

Now Displacement = Average velocity × Time interval

or $$x - x_0 = \left(\frac{v_0 + v}{2}\right) t \qquad \ldots(i)$$

(*ii*) Velocity–Time relation. Since object's velocity has changed by an amount $v - v_0$ during time t, the constant acceleration 'a' during this time is

$$a = \frac{v - v_0}{t}$$

or $$v = v_0 + at \qquad \ldots(ii)$$

(*iii*) Position–Time relation. This can be found as under :

Average velocity, $\bar{v} = \dfrac{\text{Displacement}}{\text{Time}} = \dfrac{x - x_0}{t}$

or $$x - x_0 = \bar{v}\, t$$

or $$x - x_0 = \left(\frac{v_0 + v}{2}\right) t \qquad \left(\because \bar{v} = \frac{v_0 + v}{2}\right)$$

or $$x - x_0 = \left(\frac{v_0 + v_0 + at}{2}\right) t \qquad (\because v = v_0 + at)$$

or $$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

or $$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad \ldots(iii)$$

If $x - x_0 = S$, the displacement of the object in time t, then eq. (*iii*) can be written as :

$$S = v_0 t + \frac{1}{2}\, at^2$$

(*iv*) Velocity–Displacement relation. This can be found as under :

$$x - x_0 = \left(\frac{v_0 + v}{2}\right) t \qquad \text{[From eq. (}i\text{)]}$$

or $$x - x_0 = \left(\frac{v_0 + v}{2}\right) \left(\frac{v - v_0}{a}\right) \qquad \left[\text{From eq. (}ii\text{), } t = \frac{v - v_0}{a}\right]$$

or $$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

$\therefore$ $$v^2 - v_0^2 = 2a(x - x_0) \qquad \ldots (iv)$$

If $x - x_0 = S$, the displacement of the object in time t, then eq. (*iv*) can be written as :

$$v^2 - v_0^2 = 2\, aS$$

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7.2. SUMMARY OF EQUATIONS FOR UNIFORM ACCELERATION

The *four equations* of motion for uniform acceleration are given in the table below. Each equation is also identified by the variable (x, v, a or t) that *does not* appear in it.

Equation	Variable not appearing
$v = v_0 + at$	x
$x - x_0 = v_0 t + \frac{1}{2} at^2$	v
$x - x_0 = \left(\frac{v_0 + v}{2}\right) t$	a
$v^2 - v_0^2 = 2a(x - x_0)$	t

(*i*) The *first two equations* respectively give the final velocity v and final displacement x in terms of their initial values v_0 and x_0, the acceleration 'a' and time interval t.

(*ii*) The *last two equations* are very important because they are usually sufficient to solve problems for constant acceleration.

7.3. DISPLACEMENT IN nth SECOND FOR UNIFORM ACCELERATION

Consider an object moving with a uniform acceleration 'a' along a straight line OX with origin at O as shown in Fig. 7.3. Let at $t = 0$, the position of the object be x_0 and its velocity v_0 (initial velocity). Let at $t = t$, the position of the object be x and its velocity v. Clearly, in time t, the displacement of the object is $x - x_0 = S$. The displacement of the object in time t is given by ;

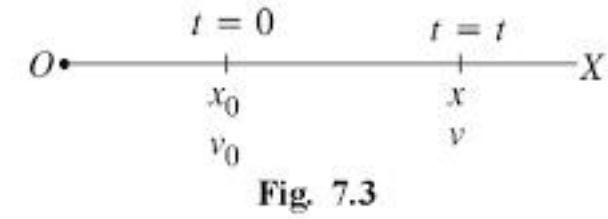


Fig. 7.3

$$S = v_0 t + \frac{1}{2} at^2$$

If S_n and S_{n-1} are the displacements of the object in n and $n - 1$ seconds, then displacement of the object in the nth second is

$$\begin{aligned} S_{nth} &= S_n - S_{n-1} \\ &= \left[v_0 n + \frac{1}{2} an^2\right] - \left[v_0 (n-1) + \frac{1}{2} a(n-1)^2\right] \\ &= \left[v_0 n + \frac{1}{2} an^2\right] - \left[v_0 n - v_0 + \frac{1}{2} an^2 - an + \frac{a}{2}\right] \\ &= v_0 (n - n + 1) + \frac{a}{2}(n^2 - n^2 + 2n - 1) \\ &= v_0 + \frac{a}{2}(2n - 1) \end{aligned}$$

$$\therefore \quad S_{nth} = v_0 + \frac{a}{2}(2n - 1) \qquad \ldots (i)$$

Eq. (*i*) gives the expression for the displacement in the nth second for uniform acceleration.

7.4. EQUATIONS OF UNIFORM ACCELERATION FROM v–t GRAPH

Fig. 7.4 shows the velocity-time graph for an object moving along a straight line with constant acceleration a. The velocity-time graph is a straight line because a (= slope of graph) is constant. Note that graph starts from v_0 because at $t = 0$, the initial velocity is assumed to be v_0.

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(*i*) Velocity at any time *t*. From velocity-time graph, we can derive the relation for final velocity v of the object at any time t. Thus in Fig. 7.4, the final velocity v is the sum of two parts — initial velocity v_0 and the velocity change at during the time t. (Note that time interval $= t - 0 = t$).

$$\therefore \qquad v = v_0 + at$$

It is the same result that we derived earlier from algebraic considerations. Note that how clearly this relation is indicated in the $v - t$ graph.

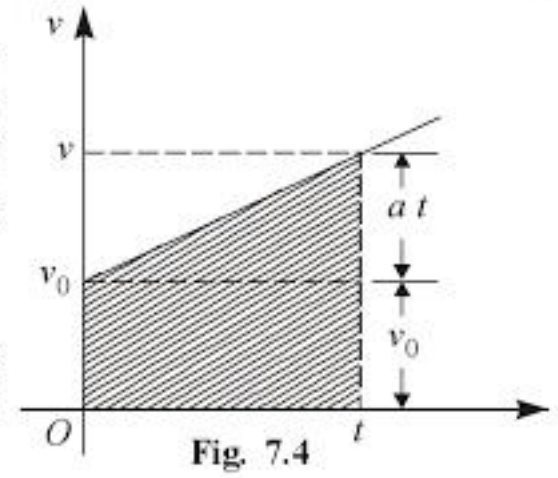


Fig. 7.4

(*ii*) Final position *x* at any time *t*. From the velocity-time graph in Fig. 7.4, we can derive the relation for the position x of the object as a function of time. We know that displacement $*x - x_0$ of the object during the time t is equal to the area of the graph for that time interval (*i.e.*, 0 to t in this case). The area under the graph for time interval 0 to t is the shaded area. It consists of a rectangle of width t and height v_0 plus right angled triangle of base t and height $a\,t$.

$$\therefore \qquad x - x_0 = \text{Shaded area}$$

$$\text{or} \qquad x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$\text{or} \qquad x = x_0 + v_0 t + \frac{1}{2} at^2$$

This is the same result which we derived earlier from algebraic considerations. Note the ease with which we have found the above relation from the $v - t$ graph. Indeed, graphical study of motion provides more insight than the mathematical equations.

(*iii*) $\qquad v^2 - v_0^2 = 2aS$

We shall derive the above relation graphically. Referring to Fig 7.4, we have,

$$x - x_0 = \text{Area of trapezium (shaded portion)}$$

$$= \frac{1}{2}(v + v_0)t \qquad \ldots(i)$$

$$\text{Acceleration, } a = \text{Slope of velocity-time graph}$$

$$= \frac{v - v_0}{t}$$

$$\text{or} \qquad t = \frac{v - v_0}{a}$$

Putting the value of t in eq. (*i*), we have,

$$x - x_0 = \frac{1}{2}(v + v_0) \times \frac{v - v_0}{a}$$

$$\therefore \qquad v^2 - v_0^2 = 2a\,(x - x_0)$$

If $x - x_0 = S$, the displacement of the particle in time t, then, $v^2 - v_0^2 = 2aS$.

(*iv*) Average velocity. Compare the velocity-time graphs in Fig. 7.5 (*i*) and in Fig. 7.5 (*ii*). The shaded area under $v - t$ graph is the same for both cases for the same time interval.

In Fig. 7.5 (*i*), the area is under the trapezoid and in Fig. 7.5 (*ii*), the same area is under the rectangle. Both areas are **equal to the change in displacement $x - x_0$. The horizontal line

---

* At $t = 0$, velocity is v_0 and the position of the object (*i.e.*, its displacement from the origin) is x_0.

** The velocity-time graphs are with identical areas under the graph. In Fig. 7.5 (*i*), the area is with constant acceleration and in Fig. 7.5 (*ii*), the same area is shown with constant velocity.

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(solid line) in Fig. 7.5 (*ii*) has the value $(v_0 + v)/2$; it is in fact the average velocity $\bar{v}$.

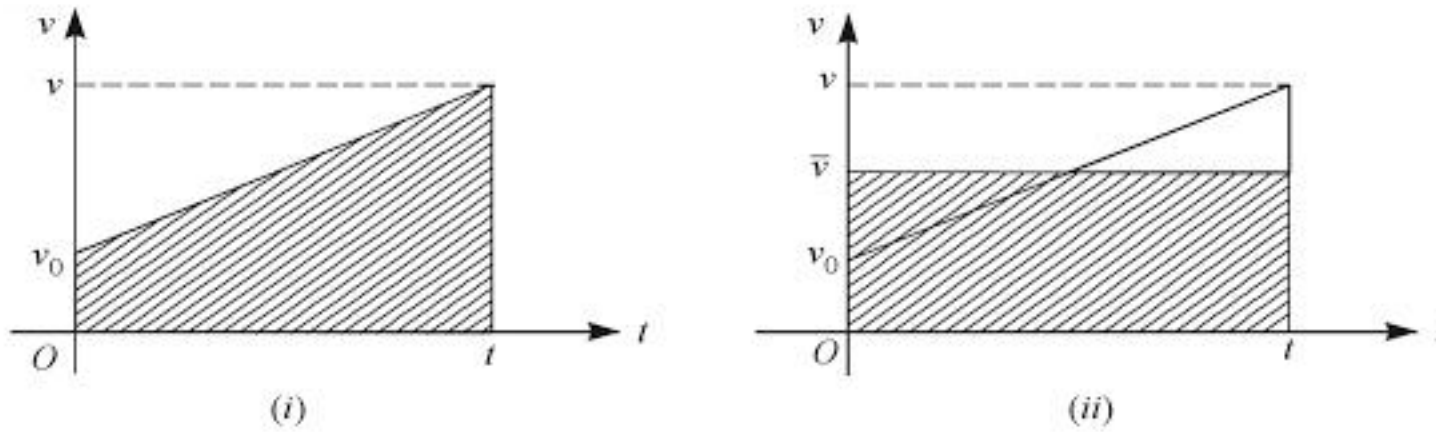


Fig. 7.5

$$\therefore \qquad \text{Average velocity, } \bar{v} = \frac{v_0 + v}{2}$$

Referring to Fig. 7.5 (*ii*), the area under the $v - t$ graph for $0 - t$ time interval is equal to the width t of the rectangle multiplied by height $\bar{v}$ $[= (v_0 + v)/2]$.

$$\therefore \qquad x - x_0 = \left(\frac{v_0 + v}{2}\right) t$$

7.5. POSITION – TIME GRAPH FOR CONSTANT +VE ACCELERATION

The position–time relation for uniformly accelerated motion in a straight line is given by ;

$$x = x_0 + v_0 t + \frac{1}{2} at^2 \qquad \ldots(i)$$

Eq. (*i*) shows that x is a quadratic function of time t. Therefore, the x –t graph is a ***parabola** (See Fig. 7.6). Note that slope of the tangent at $t = 0$ is equal to the initial velocity v_0 and slope of the tangent at time t is equal to the velocity v at time t. It is clear that the slope of the graph is continuously increasing (*i.e.*, velocity is continuously increasing). The measurements would show that the rate of increase of slope is constant.

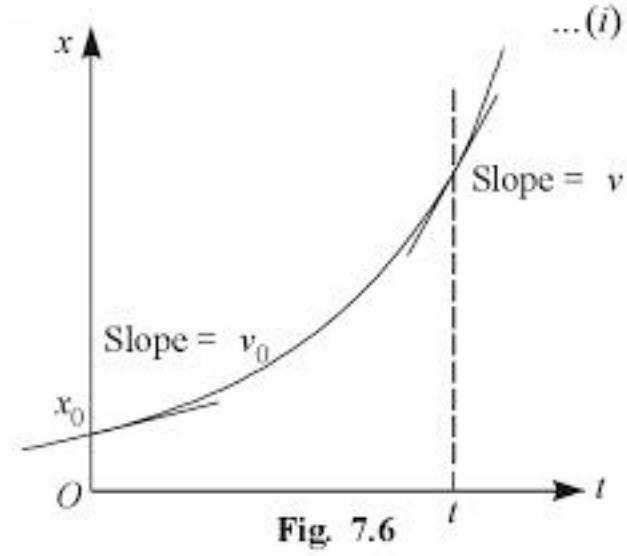


Fig. 7.6

Note. $$x = x_0 + v_0 t + \frac{1}{2} at^2$$

At first sight, it may appear that in this equation, acceleration is not constant. However, by differential calculus, it can be easily shown that acceleration is constant.

$$\frac{dx}{dt} = \frac{d}{dt}\left(x_0 + v_0 t + \frac{1}{2} at^2\right) = v_0 + at$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}(v_0 + at) = a$$

Note that acceleration a is independent of time and, therefore, it is constant. The reader may note the important role of mathematics in physics. From the **$x - t$ graph, it is not clear that acceleration a is constant. But the use of differential calculus readily proves it.

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* If x is ploted against t, the curve is a parabola whose symmetry axis is parallel to the direction of the constant acceleration.

** However, from the v–t graph, it is clear that acceleration is constant in this case.

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7.6 VERTICAL MOTION UNDER GRAVITY

One important example of linear motion with constant acceleration is the vertical motion (downward or upward) of an object under gravity. Objects that move under gravity alone are called freely falling bodies. *A freely falling body always has an acceleration of 9.8 ms^{-2} vertically downward.* Note that magnitude (9.8 ms^{-2}) as well as direction (vertically downward) of the acceleration is constant.

A freely falling body has a constant acceleration of 9.8 ms^{-2} acting vertically downward. It does not matter whether the body is rising or falling; its acceleration is still g (= 9.8 ms^{-2}) downward. When a body is rising, its velocity decreases at the rate of 9.8 ms^{-1} every second until it reaches at the top of its motion. At this instant, the velocity of the body is zero and it starts downward journey. As the body is falling, its velocity increases at the rate of 9.8 ms^{-1} every second until it hits the ground.

Equations of Motion. Since vertical motion under gravity alone is the linear motion with constant acceleration, all the equations of motion derived in Art. 7.2 are applicable to freely falling bodies. The only thing to be done is to change the nomenclature *i.e.*, displacement S is replaced by h (vertical displacement) and a by g. Therefore, equations of motion for a freely falling body are :

$$(i) \qquad h = \left(\frac{v_0 + v}{2}\right) t$$

$$(ii) \qquad v = v_0 + g\,t$$

$$(iii) \qquad h = v_0\,t + \frac{1}{2}\,g\,t^2$$

$$(iv) \qquad v^2 - v_0^2 = 2\,g\,h$$

$$(v) \qquad h_{nth} = v_0 + \frac{g}{2}(2n - 1)$$

Note. It may be noted that eq. (*v*) is not the basic equation for uniform acceleration but is only a special case of eq. (*iii*).

Sign convention. While solving problems regarding freely falling bodies, it is very important to adopt one sign convention for displacements and velocities. *It is a usual practice to choose h to be positive in the upward direction from the selected origin and negative in the downward direction from the origin.* Therefore, you must specify the origin of measurement in problems involving motion under gravity.

Example 7.1. A car starts from rest and accelerates uniformly for 10 s to a velocity of 36 km/h. It then runs at constant velocity and is finally brought to rest in 50 m with a constant retardation. The total distance covered by the car is 500 m. Find the value of acceleration, retardation and total time taken.

Solution. Fig. 7.7 shows the velocity-time graph for the motion. The car uniformly accelerates for time t_1 (= 10 s) and attains a constant velocity v (= 10 ms^{-1}). In the time inverval t_1 to t_2, it moves with this constant velocity (*i.e.*,10 ms^{-1}). In time interval t_2 to t_3, it retards uniformly and comes to stop.

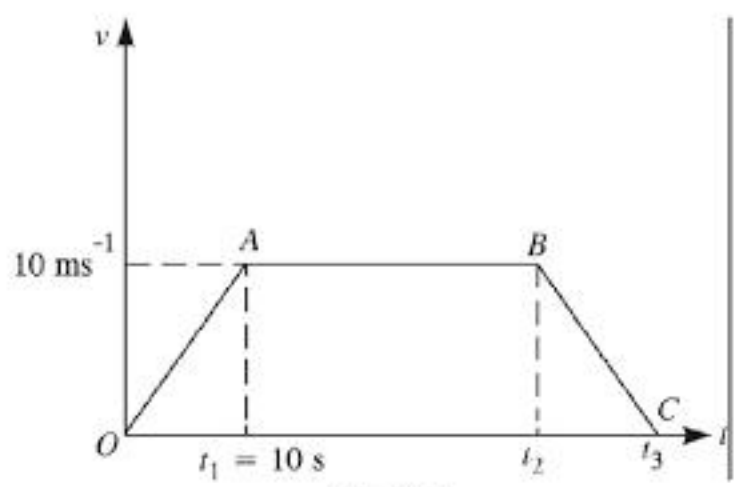


Fig. 7.7

(*i*) Constant Acceleration part (*OA*)

$v_0 = 0$; v = 36 km/h = 10 ms^{-1} ; t_1 = 10 s

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$\therefore$ Constant acceleration, $a = \dfrac{v - v_0}{t_1} = \dfrac{10 - 0}{10} = \mathbf{1 ms^{-2}}$

Average velocity, $\bar{v} = \dfrac{v_0 + v}{2} = \dfrac{0 + 10}{2} = 5 \text{ ms}^{-1}$

Distance covered, $S_1 = \bar{v}\, t_1 = 5 \times 10 = 50 \text{ m}$

(*ii*) Constant Retardation part (*BC*)

$v_0 = 10 \text{ ms}^{-1}$; $v = 0$; $t_3 - t_2 = ?$; $S_3 = 50 \text{ m}$

Average velocity, $\bar{v} = \dfrac{v_0 + v}{2} = \dfrac{10 + 0}{2} = 5 \text{ ms}^{-1}$

Distance covered, $S_3 = \bar{v}\,(t_3 - t_2)$

$\therefore$ $t_3 - t_2 = \dfrac{S_3}{\bar{v}} = \dfrac{50}{5} = 10 \text{ s}$

Retardation, $a = \dfrac{v - v_0}{t_3 - t_2} = \dfrac{0 - 10}{10} = \mathbf{-1 ms^{-2}}$

(*iii*) Constant-velocity part (*AB*)

Distance covered, $S_2 = S - S_1 - S_3 = 500 - 50 - 50 = 400 \text{ m}$

$\therefore$ Time taken, $t_2 - t_1 = \dfrac{S_2}{\text{constant velocity}} = \dfrac{400}{10} = 40 \text{ s}$

$\therefore$ Total time for journey $= t_1 + (t_2 - t_1) + (t_3 - t_2)$

$= 10 + 40 + 10 = \mathbf{60 \text{ s}}$

Note. The statement of the problem suggests that magnitudes of acceleration and retardation must be same. For the same reason, times for acceleration and retardation must be same ; also the distances covered during acceleration and retardation periods must be the same.

Example 7.2. Fig. 7.8 shows the speed-time graph of a car.

(*i*) What is the acceleration of the car in first two hours, in the next two hours and in the last two hours?

(*ii*) What is the total distance travelled by the car?

(*iii*) What is the average speed of the car?

Solution. (*i*) Since the speed-time graph during acceleration (AB) period is a straight line, the car moves with constant acceleration during this time interval.

$\therefore$ Constant acceleration, a = Slope of line AB

$$= \frac{BD}{AD} = \frac{60}{4} = \mathbf{15 \text{ km/hr}^2}$$

The car retards uniformly (graph BC) from 4 hours to 8 hours. Therefore, retardation is constant and is given by :

$$a = \text{Slope of line } BC = -\frac{BD}{DC} = -\frac{60}{4} = \mathbf{-15 \text{ km/hr}^2}$$

(*ii*) Total distance = Area of ΔABC $= \dfrac{1}{2} \times 60 \times 8 = \mathbf{240 \text{ km}}$

(*iii*) Average speed $= \dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{240}{8} = \mathbf{30 \text{ km/h}}$

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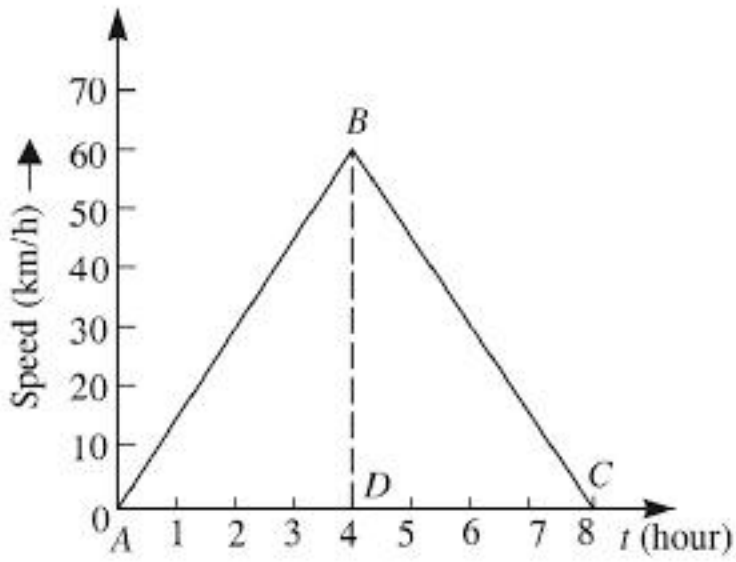


Fig. 7.8

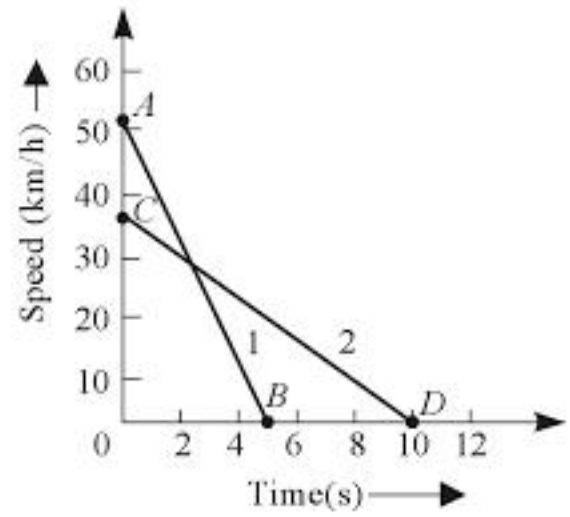


Fig. 7.9

Example 7.3. A driver of a car travelling at 52 km/h applies the brakes and decelerates uniformly. The car stops in 5 seconds. Another driver going at 34 km/h applies his brakes slower and stops after 10 seconds. On the same graph paper, plot speed-time graphs for the two cars. Which of the two cars travelled more distance after the brakes were applied?

Solution. Since both the cars decelerate uniformly, their speed-time graphs will be straight lines as shown in Fig. 7.9. We know that distance travelled by an object during any time interval is equal to the area under speed-time graph during that time interval.

Distance travelled by car 1 = Area OAB

$$= \frac{1}{2} \times *OA \times OB = \frac{1}{2} \times \frac{52 \times 1000}{60 \times 60} \times 5 \quad = \mathbf{36.1\ m}$$

Distance travelled by car 2 = Area OCD

$$= \frac{1}{2} \times OC \times OD = \frac{1}{2} \times \frac{34 \times 1000}{60 \times 60} \times 10 \quad = \mathbf{47.2\ m}$$

Therefore, car 2 travels more distance after the brakes were applied.

Example 7.4. A body moving with uniform acceleration covers 24 m in the 4th second and 36 m in the 6th second. Calculate acceleration and initial velocity.

Solution. The distance covered in the nth second is given by :

$$S_{nth} = v_0 + \frac{a}{2}(2n-1) \quad \therefore \ 24 = v_0 + \frac{a}{2}(2 \times 4 - 1) \qquad ...(i)$$

Also
$$36 = v_0 + \frac{a}{2}(6 \times 2 - 1) \qquad ...(ii)$$

Subtracting eq. (i) from eq. (ii), we get,

$$12 = 2\,a \quad \therefore \ a = 12/2 = \mathbf{6\ ms^{-2}}$$

Putting the value of a in eq. (i), we get,

$$24 = v_0 + \frac{6}{2}(2 \times 4 - 1) \qquad \therefore \ v_0 = \mathbf{3\ ms^{-1}}$$

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* $OA = 52\text{km/h} = \dfrac{52 \times 1000}{60 \times 60}\text{ms}^{-1}$

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Example 7.5. A body is moving with uniform acceleration. Its velocity after 5 seconds is 25 ms^{-1} and after 8 seconds, it is 34 ms^{-1}. Calculate the distance it will travel in 12th second.

Solution. $v = v_0 + a\,t$

For the first case : $v = 25\ ms^{-1}\ ;\ t = 5\ s$

$\therefore$ $25 = v_0 + 5a$ ... (*i*)

For the second case : $v = 34\ ms^{-1}\ ;\ t = 8\ s$

$\therefore$ $34 = v_0 + 8a$ ... (*ii*)

Solving eqs. (*i*) and (*ii*), we get, $v_0 = 10\ ms^{-1}\ ;\ a = 3\ ms^{-2}$.

The distance travelled in the 12th second is given by ;

$$S_{12th} = v_0 + \frac{a}{2}(2n - 1) = 10 + \frac{3}{2}(2 \times 12 - 1) = \mathbf{44.5\ m}$$

Example 7.6. A tennis ball is dropped on to the floor from a height of 10 m. It rebounds to a height of 2.5 m. If the ball is in contact with the floor for 0.01 second, what is the average acceleration during the contact ? Take $g = 10\ ms^{-2}$.

Solution. Let v_1 be the velocity of the ball just before striking the floor and v_2 be the velocity of the ball just after striking the floor. Since the two velocities are acting in the opposite directions, the change in velocity $\Delta v = v_1 - (-v_2) = v_1 + v_2$ and time for the change $\Delta t = 0.01$ second.

$\therefore$ Average acceleration, $\bar{a} = \dfrac{v_1 + v_2}{\Delta t}$

Now $v_1 = \sqrt{2gh_1} = \sqrt{2 \times 10 \times 10} = \sqrt{200}\ ms^{-1}$; $v_2 = \sqrt{2gh_2} = \sqrt{2 \times 10 \times 2.5} = \sqrt{50}\ ms^{-1}$

$\therefore$ $\bar{a} = \dfrac{\sqrt{200} + \sqrt{50}}{0.01} = \mathbf{2121\ ms^{-2}}$

Example 7.7. A car starts from rest and accelerates unifomiily at a rate of 2 ms^{-2} for 20 seconds. It then maintains a constant velocity for 10 seconds. The brakes are then applied and the car is uniformly retarded and comes to rest in 5 seconds. Draw the velocity-time graph for the motion and find (*i*) the maximum velocity (*ii*) the retardation in the last 5 seconds (*iii*) total distance travelled and (*iv*) average velocity.

Solution. Fig. 7.10 shows the velocity-time graph for the motion of the car. The graph is *OABC*.

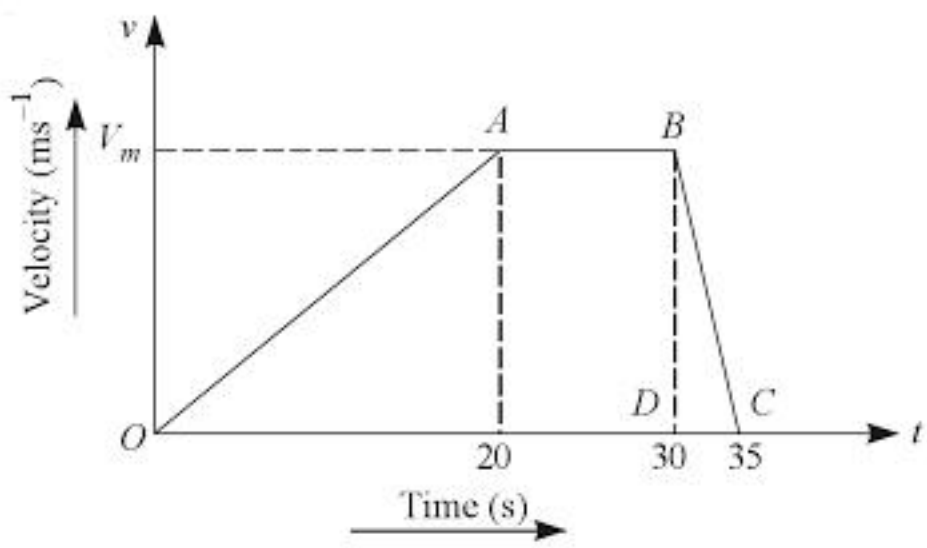


Fig. 7.10

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(*i*) $a = 2\ \text{ms}^{-2}$; $v_0 = 0$; $t = 20\ \text{s}$; $v_m = ?$

$v_m = v_0 + at = 0 + 2 \times 20 = \mathbf{40\ ms^{-1}}$

(*ii*) Retardation = Slope of line $BC = -\dfrac{BD}{DC} = -\dfrac{40}{5} = \mathbf{-8\ ms^{-2}}$

Alternatively; Retardation $= \dfrac{v - v_m}{t} = \dfrac{0 - 40}{5} = -8\ \text{ms}^{-2}$

(*iii*) Total distance travelled, S = Area of trapezium $OABC$

$$= \frac{1}{2}(AB + OC) \times BD$$

$$= \frac{1}{2}(10 + 35) \times 40 = \mathbf{900\ m}$$

(*iv*) Average velocity $= \dfrac{S}{t} = \dfrac{900}{35} = \mathbf{25.71\ ms^{-1}}$

Example 7.8. A ball is thrown vertically upward with a velocity of 29.4 ms^{-1}. Describe the motion of the ball.

Solution. Let us take origin at the ground. Then upward displacement and velocity will be positive and downward displacement and velocity will be negative. As g is always directed downward, $g = -9.8\ \text{ms}^{-2}$.

Upward Journey. Since the ball is under constant acceleration ($= -9.8\ \text{m s}^{-2}$), its velocity will decrease by 9.8 ms^{-1} every second.

Velocity of ball after 1s = 29.4 – 9.8 = 19.6 ms^{-1}

Velocity of ball after 2s = 19.6 – 9.8 = 9.8 ms^{-1}

Velocity of ball after 3s = 9.8 – 9.8 = 0

Thus the ball will reach the highest point in 3 s. At the highest point, the velocity of the ball is zero. Now the ball starts *downward journey.

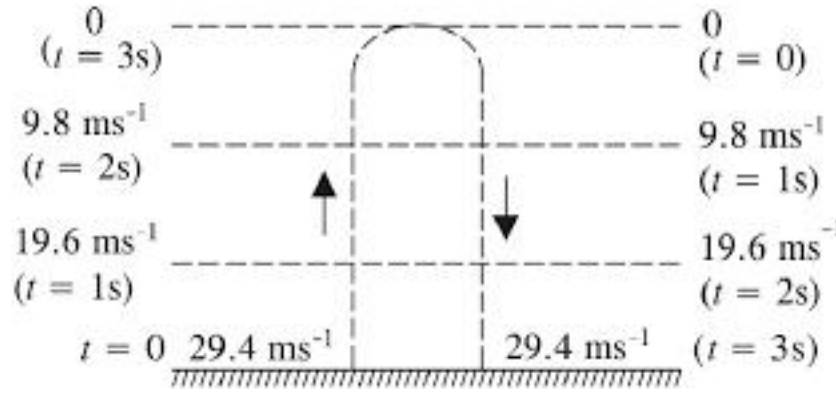


Fig. 7.11

Downward Journey. Note that during the downward journey of the ball, g is again $-9.8\ \text{ms}^{-2}$. But displacement and velocity are negative.

Velocity after 1 s = 0 – 9.8 = $-9.8\ \text{ms}^{-1}$

Velocity after 2 s = $-(v_0 + g\,t) = -(9.8 + 9.8) = -19.6\ \text{ms}^{-1}$

Velocity after 3 s = $-(19.6 + 9.8) = -29.4\ \text{ms}^{-1}$

Thus the ball reaches the ground in 3 s and strikes it with a velocity of 29.4 m s^{-1}.

Conclusions. Three important conclusions can be drawn about a freely falling body.

(*i*) The time taken for upward journey is equal to the time taken for the downward journey.

(*ii*) The magnitude of velocity (*i.e.* speed) with which the ball was thrown upward, it hits the ground with the same speed.

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* In fact, the downward journey of the ball will be along the same path along which it performed upward journey. But in Fig.7.11, the two paths are shown separate for explaining the behaviour of a freely falling body.

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(*iii*) The magnitude of velocity of the ball at any point in its upward and downward journey is the same.

The above behaviour of a freely falling body is due to the fact that value of g is constant.

Example 7.9. A person throws a ball vertically upward with an initial velocity of 15 ms^{-1}. Calculate (*i*) how high it goes and (*ii*) how long the ball is in air before it comes to his hand.

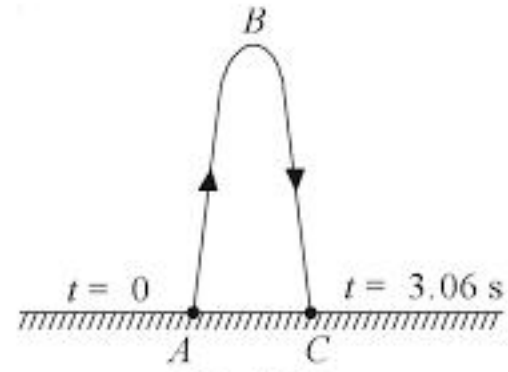


Fig. 7.12

Solution. We select earth as the origin so that $g = -9.8\ ms^{-2}$.

(*i*) At the highest point, velocity is zero.

Now $$v^2 - v_0^2 = 2\,g\,h$$

Here $$v = 0\ ;\ v_0 = +15\ ms^{-1}\ ;\ g = -9.8\ ms^{-2}$$

$\therefore$ $$(0)^2 - (+15)^2 = 2 \times (-9.8) \times h$$

$\therefore$ Maximum height, $$h = \frac{-(+15)^2}{2\times(-9.8)} = \mathbf{11.5\ m}$$

(*ii*) $$h = v_0\,t + \frac{1}{2}gt^2$$

Here *$h = 0\ ;\ v_0 = +15\ ms^{-1}\ ;\ g = -9.8\ m\ s^{-2}$

$\therefore$ $$0 = 15t - \frac{1}{2}\times 9.8\ t^2 \quad \text{or} \quad 0 = 15\,t - 4.9\,t^2$$

$\therefore$ $$0 = t\,(15 - 4.9\,t)$$

$\therefore$ $$t = 0 \quad \text{and} \quad 15 - 4.9\,t = 0 \quad \text{or} \quad t = \frac{15}{4.9} = \mathbf{3.06\ s}$$

Note that there are two solutions for t ; 0 s and 3.06 s. The first solution corresponds to initial point A (See Fig. 7.12) and second solution corresponds to the return–point C. Therefore, the ball is in the air for 3.06 s.

Example 7.10. An object is thrown vertically upward from the ground at 30 ms^{-1}.

(*i*) What is the displacement after 4 s? (*ii*) What is the velocity after 4 s?

(*iii*) What is the maximum height it attains ? (*iv*) What is the time of flight ?

Solution. We select earth as the origin. Then $g = -9.8\ ms^{-2}$. For upward journey, $v_0 = +30\ ms^{-1}\ ;\ g = -9.8\ ms^{-2}$.

(*i*) $$h = v_0\,t + \frac{1}{2}g\,t^2$$

Here $v_0 = +30\ ms^{-1}\ ;\ g = -9.8\ ms^{-2}\ ;\ t = 4\ s\ ;\ h = ?$

$\therefore$ $$h = 30 \times 4 + \frac{1}{2}(-9.8) \times (4)^2 = \mathbf{42\ m}$$

(*ii*) $$v = v_0 + g\,t$$

Here $v_0 = +30\ ms^{-1}\ ;\ g = -9.8\ ms^{-2}\ ;\ t = 4\ s$

$\therefore$ $$v = 30 + (-9.8) \times 4 = \mathbf{-9\ ms^{-1}}$$

Thus four seconds after the object is thrown, it is 42 m above the ground and is travelling downward (negative sign shows downward motion) with an instantaneous speed of 9 ms^{-1}.

* The net displacement for entire journey is zero.

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(*iii*) At the highest point, the velocity of the object is zero.

Now $v^2 - v_0^2 = 2\,g\,h$

Here $v = 0\ ;\ v_0 = +30\ \text{ms}^{-1}\ ;\ g = -9.8\ \text{ms}^{-2}$

$\therefore \quad (0)^2 - (30)^2 = 2\,(-9.8)\,h$

or $$h = \frac{0^2 - (30)^2}{2(-9.8)} = \mathbf{46\ m}$$

(*iv*) The displacement h is zero when the object strikes the ground. We want to find the corresponding time of flight.

$$h = v_0\,t + \frac{1}{2}g\,t^2$$

Here $h = 0\ ;\ v_0 = +30\ \text{ms}^{-1}\ ;\ g = -9.8\ \text{ms}^{-2}$

$$\therefore \quad 0 = 30\,t - \frac{1}{2} \times 9.8\,t^2$$

or $$0 = 30\,t - 4.9\,t^2$$

or $$0 = t\,(30 - 4.9\,t)$$

$\therefore \quad t = 0$ and $0 = 30 - 4.9\,t$

or $$t = \frac{30}{4.9} = \mathbf{6.1s}$$

Therefore, time of flight is 6.1s. Note that $t = 0$ corresponds to the point of throwing the object upward. Fig. 7.13 shows the graphical solution of the problem.

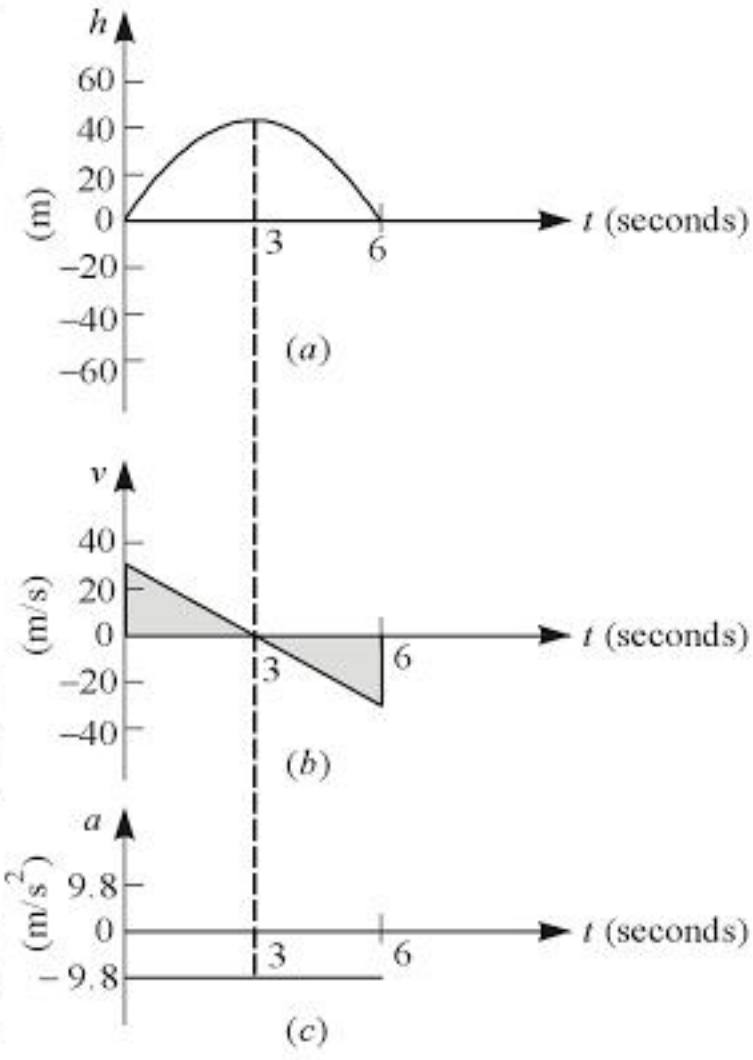


Fig. 7.13

(*a*) The h–t graph of the motion is a parabola. This is not surprising because the position-time graph for a linear motion with constant acceleration is a parabola. Note that the displacement is zero at 6.1s. This is not only indicated on h–t graph but also by the net area on v–t graph. The net area of v–t graph is zero so that net displacement is zero.

(*b*) The $v - t$ graph of the motion is a straight line. It is because the acceleration is constant during the entire motion of the object. Note that as the object moves upward, its velocity decreases uniformly (9.8 ms^{-1} every second). When the object reaches the highest point of motion, its velocity is zero. From now onwards, the object starts its downward journey.

(*c*) The $a - t$ graph of motion is a straight line parallel to the time axis. This implies that acceleration is constant ($= -9.8\ \text{ms}^{-2}$) throughout the journey of the object. Since origin is selected at the ground and g always acts downward, the value of g is negative.

Example 7.11. Water drops fall at regular intervals from the nozzle of a shower on to the floor 3.24 m below. The first drop strikes the floor at the instant the fourth one begins to fall from the nozzle. Find the positions of the drops when a drop just strikes the floor.

Solution. Fig. 7.14 shows the positions of the four drops A, B, C and D. When drop A strikes the floor, the drop D just beigns to fall from the nozzle. The positions of the second and third drops

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are mid-way between them. Let t be the time interval between two successive drops. Referring to Fig. 7.14,

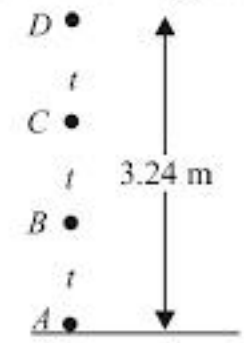


Fig. 7.14

$$CD = \frac{1}{2}gt^2 \; ; \; BD = \frac{1}{2}g\,(2t)^2 = 2gt^2 \; ; \; AD = \frac{1}{2}g\,(3t)^2 = 4.5\,gt^2$$

It is given that $AD = 3.24$ m.

$$\therefore \qquad 4.5\,gt^2 = 3.24$$

or $$gt^2 = 3.24/4.5 = 0.72$$

$$\therefore \qquad CD = \frac{1}{2}gt^2 = \frac{1}{2} \times 0.72 = \mathbf{0.36\ m}$$

$$BD = 2gt^2 = 2 \times 0.72 = \mathbf{1.44\ m} \; ; \qquad AD = \mathbf{3.24\ m}$$

Example 7.12. A parachutist bails out from an aeroplane and after dropping through a distance of 40 m opens the parachute and decelerates at 2 ms^{-2}. If he reaches the ground with a speed of 2 ms^{-1}, how long he is in the air? At what height did he bail out from the plane?

Solution. Time for first 40 m journey.

Now $$v_0 = 0 \; ; \; h = -40 \text{ m} \; ; \; g = -9.8 \text{ m/s}^2 \; ; \; v = ?$$

$$(-v)^2 - v_0^2 = 2gh \qquad \text{or} \qquad v^2 - (0)^2 = 2 \times (-9.8) \times (-40)$$

$$\therefore \qquad v = \sqrt{2 \times 9.8 \times 40} = 28 \text{ m/s}$$

Now $$v = v_0 + gt_1 \qquad \therefore \quad t_1 = \frac{v - v_0}{g} = \frac{-28 - 0}{-9.8} = 2.9 \text{ s}$$

Time for second journey.

$$v_0 = -28 \text{ m/s} \; ; \; v = -2 \text{ m/s} \; ; \; a = +2 \text{ m/s}^2$$

$$v = v_0 + at_2 \qquad \text{or} \qquad -2 = -28 + 2t_2 \quad \therefore \quad t_2 = 13\text{s}$$

Total time t for which the parachute is in air is

$$t = t_1 + t_2 = 2.9 + 13 = \mathbf{15.9\ s}$$

Height from which parachute has fallen during second journey.

$$v^2 - v_0^2 = 2ah \quad \text{or} \quad (-2)^2 - (-28)^2 = 2 \times (+2) \times -h \qquad \therefore \qquad h = 195 \text{ m}$$

$\therefore$ Height at which the parachutist bailed out = 40 + 195 = **235 m**

Example 7.13. A balloon is ascending at the rate of 12 ms^{-1}. When it is at a height of 65 m from the ground, a packet is dropped from it. After how much time and with what velocity does the packet reach the ground? Take g = 10 ms^{-2}.

Solution. $h = v_0\,t + \frac{1}{2}gt^2$

Taking downward motion as positive ; $v_0 = -12$ ms^{-1} ; $g = +10$ ms^{-2}

$$\therefore \qquad 65 = -12 \times t + \frac{1}{2} \times 10 \times t^2$$

or $$5\,t^2 - 12\,t - 65 = 0 \quad \text{or} \quad (5t + 13)\,(t - 5) = 0$$

$$\therefore \qquad t = -13/5 \text{ sec} \; ; \; t = 5 \text{ sec}$$

As the time cannot be negative ; t = **5 sec.**

$$\therefore \qquad v = v_0 + gt = -12 + 10 \times 5 = \mathbf{38\ ms^{-1}}$$

Example 7.14. A bottle dropped from a balloon reaches the ground in 20 s. Determine the height of the balloon if (*i*) it was at rest in the air (*ii*) it was ascending with a speed of 50 ms^{-1}, when the bottle was dropped.

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Solution. (*i*) $h = v_0 t + \frac{1}{2} gt^2$

Taking upward motion as positive, $g = -9.8\ \text{ms}^{-2}$

$\therefore \quad h = 0 \times t - \frac{1}{2} \times 9.8 \times (20)^2 = -1960\ \text{m} \qquad \therefore\ |h| = \mathbf{1960\ m}$

(*ii*) Taking upward motion as positive, $g = -9.8\ \text{ms}^{-2}$

$\therefore \quad h = v_0 t + \frac{1}{2} gt^2 = 50 \times 20 + \frac{1}{2} \times (-9.8) \times (20)^2 = -960\ \text{m}$

$\therefore \quad |h| = \mathbf{960\ m}$

Example 7.15. A stone is dropped from the top of a tower 400 m high and at the same time another stone is projected upward vertically from the ground with a velocity of 100 ms^{-1}. Find when and where the two will meet.

Solution. Suppose the two stones meet after t seconds when the stone from the top of the tower has covered a distance of h. Then the distance of the meeting point from the ground is $(400 - h)$ as shown in Fig. 7.15. Take the downward motion positive.

For downward motion, $g = +9.8\ \text{ms}^{-2}; v_0 = 0.$

$\therefore \quad h = v_0 t + \frac{1}{2} gt^2 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$

or $\quad h = 4.9\ t^2 \qquad \ldots (i)$

For upward motion, $g = -9.8\ \text{ms}^{-2}$; $v_0 = 100\ \text{ms}^{-1}$.

$\therefore \quad 400 - h = v_0 t + \frac{1}{2} gt^2 = 100 \times t + \frac{1}{2}(-9.8)\ t^2$

or $\quad 400 - h = 100\ t - 4.9\ t^2 \qquad \ldots (ii)$

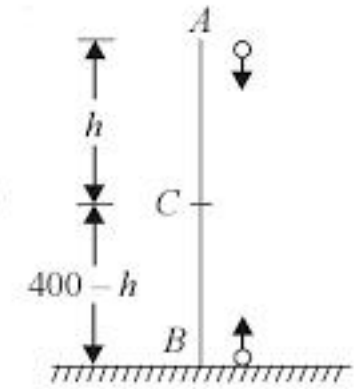


Fig. 7.15

Adding eqs. (*i*) and (*ii*), we have,

$$400 = 100\ t \quad \therefore \quad t = 400/100 = 4\ \text{s}$$

Putting $t = 4$ s in eq. (*i*), $h = 4.9 \times (4)^2 = 78.4$ m.

Therefore, the two stones meet **78.4 m below the top of tower after 4 s.**

Example 7.16. A body is dropped from rest at a height of 150 m and simultaneously another body is dropped from rest from a point 100 m above the ground. What is the difference between their heights after they have fallen for 3 seconds? Take g = 10 ms^{-2}.

Solution. Since initial velocity in the two cases is zero, $h = \frac{1}{2} gt^2$.

Downward distance covered by *first body* in 3 seconds

$$= \frac{1}{2} \times 10 \times (3)^2 = 45\ \text{m}$$

$\therefore$ Height of first body above the ground = 150 – 45 = 105 m

Downward distance covered by *second body* in 3 seconds

$$= \frac{1}{2} \times 10 \times (3)^2 = 45\ \text{m}$$

$\therefore$ Height of second body above the ground = 100 – 45 = 55 m

$\therefore$ Difference in height between the two after 3 seconds = 105 – 55 = **50 m**

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PROBLEMS FOR PRACTICE

1. A train starting from a station is uniformly gaining speed until after 5 minutes it acquires the maximum uniform speed of 80 km/h. Find the average velocity and the distance travelled during variable state of motion. **[40 km/h ; 3.33 km]**

2. A car travels with a uniform velocity of 72 km/h. The driver applies the brakes and the car comes to rest with a uniform retardation in 10 seconds. Find (*i*) the retardation (*ii*) velocity of the car after 3 seconds and (*iii*) the distance car travels after the brakes are applied. **[(*i*) –2 m s^{-2} (*ii*) 14 m s^{-1} (*iii*) 100 m]**

3. A car travels at a uniform velocity of 20 ms^{-1} for 5 seconds. The brakes are then applied and the car comes to rest with uniform retardation in further 8 seconds. Determine (*i*) the distance travelled in first 5 seconds (*ii*) retardation in the last 8 seconds (*iii*) distance travelled by the car after the brakes are applied and (*iv*) total distance travelled. **[(*i*) 100 m (*ii*) –2.5 m s^{-2} (*iii*) 80 m (*iv*) 180 m]**

4. A motorcycle starts from rest and maintains a constant acceleration of 3 ms^{-2} for 6 seconds and then a constant acceleration of 2 ms^{-2} for another 6 seconds. Find the acquired velocity and total distance travelled. **[30 ms^{-1}; 216 m]**

5. A body has a velocity of 5 ms^{-1} and a uniform acceleration of 4 ms^{-2}. Find the distance travelled in the 11th second of its motion. **[47 m]**

6. A body starts from rest and slides down an inclined plane (assumed frictionless) of length 15 m in 15 s. Find its acceleration and its speed at the bottom of the plane. **[0.113 ms^{-2}; 1.995 ms^{-1}]**

7. A body travels 200 cm in the first two seconds and 220 cm in the next four seconds. What will be the velocity at the end of seventh second from the start ? **[10 cm s^{-1}]**

8. An electron travelling with a speed 75×10^{13} ms^{-1} passes through an electric field with an acceleration of 10^{12} ms^{-2}. How long will it take for the electron to double its speed ? **[5×10^{-9} s]**

9. A boy drops a stone into a well. If the stone takes 2.5 s to hit the water, find (*i*) depth of the well (*ii*) the speed with which the stone hits the water.

[(*i*) 30.63 m (*ii*) 24.5 ms^{-1}]

10. A boy standing near the edge of a cliff 125 m above a river throws a stone downward with a speed of 10 ms^{-1}. Find (*i*) with what speed will the stone hit water and (*ii*) how long will it take to descend ? **[(*i*) 50.5 m s^{-1} (*ii*) 4.13 s]**

11. A ball is thrown vertically upward from the ground with a velocity of 39.2 ms^{-1}. Calculate (*i*) the maximum height to which the ball rises and (*ii*) the time taken by the ball to reach the highest point. **[(*i*) 78.4 m (*ii*) 4 s]**

12. A stone is dropped from the top of a building 200 m high and at the same time another stone is projected vertically upward from the ground with a velocity of 50 ms^{-1}. Find where and when the two stones will meet.

[after 4 s at a height of 121.6 m from ground]

13. A ball thrown vertically upward reaches a height of 80 m. Calculate (*i*) the time to reach the highest point (*ii*) the speed of the ball upon arrival on the ground.

[(*i*) 4.04 s *(ii)* 39.59 m s^{-1}]

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7.7. EQUATIONS FOR UNIFORMLY ACCELERATED LINEAR MOTION (CALCULUS METHOD)

In the earlier sections, we derived the equations for uniformly accelerated motion in one dimension by making use of algebra or graphical analysis. Now we shall derive these equations directly from definitions with the help of integral calculus.

(*i*) Velocity-Time relation. The instantaneous acceleration of a moving object is given by ;

$$a = \frac{dv}{dt}$$

or $$dv = a\, dt$$

When $t = 0$; $v = v_0$ and when $t = t$, $v = v$.

We integrate velocity from the initial value v_0 to the final value v, the corresponding limits on the time being 0 and t.

$$\therefore \quad \int_{v_0}^{v} dv = \int_{0}^{t} a\, dt$$

Since acceleration a is constant, it can be taken out of the integral sign.

$$\therefore \quad \int_{v_0}^{v} dv = a\int_{0}^{t} dt$$

or $$|\, v\,|_{v_0}^{v} = a\,|\, t\,|_0^t$$

or $$v - v_0 = at$$

$$\therefore \quad v = v_0 + a\, t \qquad \ldots(i)$$

(*ii*) Position-Time relation. $$v = v_0 + a\, t$$

or $$\frac{dx}{dt} = v_0 + at$$

or $$dx = v_0\, dt + at\, dt$$

When $t = 0$; $x = x_0$ and when $t = t$; $x = x$.

Integrating the above equation for dx through proper limits, we get,

$$\int_{x_0}^{x} dx = \int_{0}^{t} v_0 dt + \int_{0}^{t} a\, t\, dt$$

Since v_0 (= initial velocity) and acceleration a are constant, they can be taken out of the integral sign.

$$\therefore \quad \int_{x_0}^{x} dx = v_0\int_{0}^{t} dt + a\int_{0}^{t} t\, dt$$

or $$|\, x\,|_{x_0}^{x} = v_0\,|\, t\,|_0^t + a\left|\frac{t^2}{2}\right|_0^t$$

$$\therefore \quad x - x_0 = v_0\, t + \frac{1}{2}at^2$$

or $$x = x_0 + v_0\, t + \frac{1}{2}a\, t^2 \qquad \ldots(ii)$$

If $x - x_0 = S$, the displacement of the object in time t, then eq. (*ii*) can be written as :

$$S = v_0\, t + \frac{1}{2}\, a\, t^2$$

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(*iii*) Velocity-Displacement relation.

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt}$$

or $$a = \frac{dv}{dx}\cdot v \qquad \left(\because \frac{dx}{dt} = v\right)$$

or $$a\,dx = v\,dv$$

When $x = x_0$, $v = v_0$ and when $x = x$, $v = v$.

$\therefore$ Integrating above equation with proper limits, we have,

$$\int_{x_0}^{x} a\,dx = \int_{v_0}^{v} v\,dv$$

Since acceleration a is constant, it can be taken out of the integral sign.

$\therefore$ $$a\int_{x_0}^{x} dx = \int_{v_o}^{v} v\,dv \qquad \text{or} \qquad a\,|\,x\,|_{x_0}^{x} = \left|\frac{v^2}{2}\right|_{v_0}^{v}$$

or $$a(x - x_0) = \frac{v^2}{2} - \frac{v_0^2}{2} \qquad \therefore \qquad v^2 - v_0^2 = 2a(x - x_0) \qquad ...(iii)$$

If $x - x_0 = S$, the displacement of the object in time t, then eq. (*iii*) can be written as :

$$v^2 - v_0^{\,2} = 2\,a\,S$$

Note. We can verify the correctness of these equations of uniformly accelerated linear motion by differential calculus.

$$x = x_0 + v_0 t + \frac{1}{2} a\,t^2$$

(*i*) $$v = \frac{dx}{dt} = \frac{d}{dt}\left(x_0 + v_0 t + \frac{1}{2}at^2\right) = v_0 + at$$

$\therefore$ $$v = v_0 + a\,t$$

(*ii*) $$a = \frac{dv}{dt} = \frac{d}{dt}(v_0 + at) = a = \text{constant}$$

The reader may note the important role of mathematics in physics. We derived these equations directly from definitions with the help of integral calculus and then proved their correctness by differential calculus.

CONCEPTUAL QUESTIONS

Q.1. When is an object said to be accelerating?

Ans. Whenever the velocity (magnitude or direction or both) of an object changes, we say that the object is accelerating. However, an object moving at a constant speed in a straight line is not accelerating because its velocity is not changing *i.e.*, magnitude as well as direction of velocity is the same.

Q.2. What do you mean by uniformly accelerated linear motion?

Ans. Uniformly accelerated linear motion means that the magnitude of the acceleration is constant and the motion is in a straight line.

Q.3. Give one most common example of uniformly accelerated linear motion.

Ans. One of the most common and practically important examples of uniformly accelerated linear motion is the vertical movement of an object in the gravitational field of the

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earth. The effect of gravitational attraction is to impart all unsupported bodies an equal downward acceleration of 9.8 ms^{-2} (neglecting the retarding effect of air).

Q.4. We say that acceleration due to gravity is constant only if we ignore air resistance. For what situations can air resistance be ignored?

Ans. The answer to this question depends upon several factors :

(*i*) If you do not require greater accuracy, you can ignore air resistance.

(*ii*) If the body is moving through air at a moderate speed, air resistance can be ignored. It is because the faster an object moves, the more air resistance affects its acceleration.

(*iii*) If the body is heavy and/or streamlined, you can ignore air resistance. It is because shape and surface area of an object greatly affect air resistance. Air resistance is much more important for a light object, such as a ping-pong ball, than for a heavier object of the same size, such as a golf ball.

Q.5. Is the value of g positive or negative?

Ans. The acceleration due to gravity always acts vertically downward whether the body is rising or falling. The value of g will be negative or positive depending upon whether we choose upward direction positive or negative. If we choose upward direction positive, g, being directed downward, is $-9.8\ ms^{-2}$. If we choose downward direction positive, then g is $+9.8\ ms^{-2}$.

Q.6. If a body moves with constant acceleration, what is the average velocity?

Ans. If a body moves with constant acceleration, the velocity-time graph is a straight line: the slope of graph = constant acceleration. Therefore, the average velocity lies midway between the initial velocity (v_0) and the final velocity (v) *i.e.*,

$$v = \frac{v_0 + v}{2}$$

Note that it is true *only* for constant acceleration.

Q.7. What is a freely falling body?

Ans. When an object is under the action of gravity alone, it is called a freely falling body. The presence of other forces, in particular air resistance, may alter dramatically the statement that all objects fall with the same constant acceleration g ($= 9.8\ m\ s^{-2}$).

Q.8. The position of a stone dropped from rest from a cliff is given by $x = 4.9\ t^2$ where x is in metres and t is in seconds. What is the acceleration of the stone?

Ans. $$x = 4.9\ t^2 \quad \text{or} \quad \frac{dx}{dt} = 2(4.9)t = 9.8t$$

or $$\frac{d^2x}{dt^2} = 9.8\ ms^{-2} \quad \text{or} \quad a = 9.8\ ms^{-2}$$

We arrive at an important conclusion that if the displacement of a body varies as the square of elapsed time, the acceleration of the body is constant.

Q.9. In one dimensional motion, can a particle have zero speed at an instant but non-zero acceleration at that instant?

Ans. The answer is yes. For example, when a body starts falling freely, its speed is zero but acceleration is $9.8\ ms^{-2}$.

Q.10. In the position-time ($x - t$) graph shown in Fig. 7.16, what is the acceleration?

Ans. The $x - t$ graph is a straight line. Therefore, slope is constant. This means that velocity of the body is constant. Therefore, acceleration is zero.

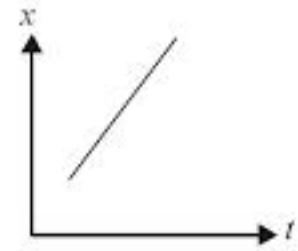


Fig. 7.16

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Q.11. The distance travelled by a body is found to be directly proportional to time. Is the body moving with uniform velocity?

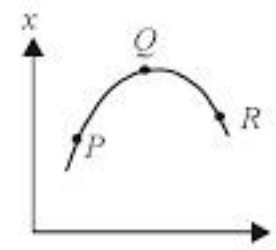


Fig. 7.17

Ans. $s \propto t$ or $s = k\,t$ $\therefore$ $v = \dfrac{ds}{dt} = k$ (constant)

Therefore, the body is moving with uniform velocity.

Q.12. Fig. 7.17 shows $x - t$ graph for a moving body. At what point velocity is zero?

Ans. The velocity at any point on the graph is the slope of the tangent line at that point. The slope of the tangent line at Q is zero. Therefore, velocity is zero at point Q.

VERY SHORT ANSWER QUESTIONS

Q.1. A ball is thrown straight up. What is its velocity and acceleration at the top?

Ans. Velocity at top = 0 ; Acceleration at top = 9.8 ms^{-2} downwards.

Q.2. Can a body have zero velocity and still be accelerating?

Ans. Yes. In case of a body projected vertically upwards, the velocity at the highest point is zero but acceleration is g (= 9.8 ms^{-2}) downwards.

Q.3. A body is thrown vertically upwards with a velocity of 30 m/s. What is its velocity when it hits the ground?

Ans. 30 m/s because the velocity with which the body hits the ground is equal to the velocity with which it is thrown up. The reason for this is that acceleration due to gravity g(= 9.8 ms^{-2}) remains constant during upward or downward journey.

Q.4. Two balls of different masses (one lighter and other heavier) are thrown vertically upward with the same initial speed. Which one will rise to greater height?

Ans. Both the balls will rise to the same height. It is because in the formula $v^2 - v_0^2 = 2gh$, no mass term is involved.

Q.5. What does the area under acceleration-time graph for any time interval represent?

Ans. Area under $a - t$ graph = $\dfrac{1}{2}at = \dfrac{1}{2} \times \dfrac{v - v_0}{t} \times t = \dfrac{v - v_0}{2}$.

Therefore, area under $a - t$ graph during time initerval t represents change in velocity during that time interval.

Q.6. If the acceleration of a particle is constant in magnitude but not in direction, what type of path does the particle follow?

Ans. It will follow a non-linear path.

Q.7. A ball is thrown vertically upwards and takes 5 s to reach the highest point. What is the time for downward journey?

Ans. 5 s because the value of g (= 9.8 ms^{-2} downwards) is constant for upward and downward journey.

Q.8. A man standing on the edge of a cliff throws a stone straight up with initial velocity u and then throws another stone straight down with same initial speed u and from the same position. Find the ratio of the speeds, the two stones would have attained when they hit ground at the base of the cliff.

Ans. When the stone is thrown up with speed u, it will return to the thrower with speed u. Since both stones are falling under gravity with the same initial speed u, they will hit the ground with the same speed. Hence the ratio of the speeds of the two stones will be 1 : 1.

Q.9. A stone is thrown vertically upward from the ground at 30 ms^{-1}. What is the time of flight?

Ans. t = 6.1s (Refer to Example 7.10).

Q.10. The initial and final velocities of a uniformly accelerated body are 20 ms^{-1} and 60 ms^{-1}. What is the average velocity?

Ans. Average velocity = $\dfrac{\text{Initial vel.} + \text{Final vel.}}{2} = \dfrac{20 + 60}{2} = 40\ ms^{-1}$

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Q.11. How will you find the distance covered by a uniformly accelerated body from its $v - t$ graph?

Ans. The distance covered by a body in a given interval of time is equal to the *total area* (positive and negative) under v–t graph during that time interval. Note that negative area (due to negative velocity) is also taken as positive.

Note. The displacement of a body during a given time interval is the *net area* (with due regard to sign to area) under v–t graph during that interval.

Q.12. A car starting from rest has an acceleration of 5 ms^{-2}. How fast will it be after 10 s?

Ans. $v = v_0 + at = 0 + 5 \times 10 = 50\ ms^{-1}$

Q.13. A body is thrown up vertically with velocity u. How high will it rise?

Ans. $v^2 - u^2 = 2gh$ or $0^2 - u^2 = 2(-g)h \quad \therefore\ h = u^2/2g.$

Q.14. Velocity and acceleration are interlinked but is it possible that one is zero while other is not?

Ans. Yes it is possible. A body thrown vertically upward has zero velocity at the highest point but it has acceleration g (= 9.8 ms^{-2} downwards). On the other hand, acceleration is zero for motion with uniform velocity.

Q.15. A body is thrown vertically upward from the surface of earth. What is the direction of acceleration of the body?

Ans. The acceleration is g (= 9.8 ms^{-2}) acting vertically downward whether the body is going upward or coming downward.

Q.16. What is the unique characteristic of uniformly accelerated motion?

Ans. In case of uniformly accelerated motion (*and only for this case*), the average velocity is equal to one-half of sum of initial and final velocities.

Q.17. What is the ratio of the distances travelled by a body falling freely from rest during first, second and third second of its fall?

Ans. $S_{nth} = v_0 + \frac{a}{2}(2n-1) = 0 + \frac{g}{2}(2n-1) = \frac{g}{2}(2n-1)$

$$\therefore\ S_1 : S_2 : S_3 = \frac{g}{2}(2\times1-1) : \frac{g}{2}(2\times2-1) : \frac{g}{2}(2\times3-1) = 1 : 3 : 5$$

Q.18. Can you cite an example where a body subjected to uniform acceleration does not move in a straight line?

Ans. A body subjected to uniform acceleration in one dimensional motion moves in a straight line. However, the projectile is subjected to uniform acceleration and has a parabolic path (motion in a plane).

Q.19. What is the acceleration of a body when its $v - t$ graph is (*i*) perpendicular to time-axis (*ii*) parallel to time-axis?

Ans. (*i*) Inifinity (*ii*) zero.

Q.20. A stone is thrown vertically upward with an initial velocity of 14 ms^{-1}. What is the maximum height reached? (g = 9.8 ms^{-2})

Ans. $h = v_0^2/2g = 14^2/2 \times 9.8 = 10$ m.

SHORT ANSWER QUESTIONS

Q.1. What are the three main characteristics of a freely falling body?

Ans. A body that moves under gravity alone is called a freely falling body.

(*i*) Time taken for upward journey is equal to that for downward journey.

(*ii*) The speed with which body is thrown upward, it hits the ground with the same speed.

(*iii*) The speed of the body at any point in its upward and downward journey is the same.

Q.2. A ball is thrown vertically upwards. Draw its height-time and velocity-time graph.

Ans. The $h - t$ graph for the motion is a parabola and is shown in Fig. 7.18 (*i*). The $v - t$ graph for the motion is a straight line [See Fig. 7.18 (*ii*)] because acceleration is constant during the entire motion of the body.

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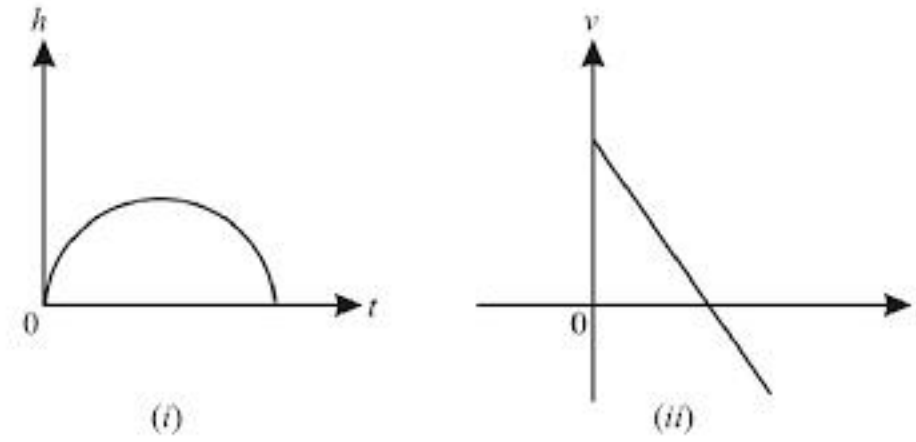


Fig. 7.18

Q.3. A ball is thrown vertically upward from the ground at 30 ms^{-1}. What is the time of flight?

Ans. The displacement $h = 0$ when the ball strikes the ground.

Now $h = v_0 t + \frac{1}{2} g t^2$ or $0 = 30 \times t - \frac{1}{2} \times 9.8\, t^2$ $\therefore$ $t = 6.1$ s

Q.4. Two balls of different masses (one lighter and other heavier) are thrown vertically upwards with the same speed. Which one will pass through the point of projection in their downward journey with greater speed?

Ans. Suppose v_0 is the velocity of projection of the ball and v is the velocity of the ball while passing downward through the point of projection. Since displacement $h = 0$,

$\therefore$ $v^2 - v_0^2 = 2gh$ or $v^2 - v_0^2 = 2g \times 0$ $\therefore$ $v = v_0$

Hence both the balls will have the same speed while passing through the point of projection in their downward journey. Alternatively, there is no mass term in the equation of motion.

Q.5. For a particle in one dimensional motion having uniform acceleration, the instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?

Ans. In one dimensional motion having uniform acceleration, the direction of velocity does not change; only the magnitude of velocity changes (*i.e.* speed). Therefore, the instantaneous speed is always equal to the magnitude of instantaneous velocity of the particle in one dimensional (*i.e.*, in a straight line) motion.

Q.6. What will be the distance moved by a freely falling body from rest in the 5th second of its journey?

Ans. $S_{nth} = v_0 + \frac{a}{2}(2n-1)$. Here $v_0 = 0$; $a = 9.8 ms^{-2}$ and $n = 5$.

$\therefore$ $S_5 = 0 + \frac{9.8}{2}(2 \times 5 - 1) = 4.9\ (9) = 44.1$ m

Q.7. What is the magnitude and direction of acceleration due to gravity?

Ans. A freely falling body always has an acceleration of 9.8 ms^{-2} downward. It does not matter whether the body is rising or falling ; its acceleration is still g (= 9.8 ms^{-2}) downward.

Q.8. The position of a particle moving along a straight line is given by : $x = 2 - 5t + 6t^2$. What is the acceleration of the particle at $t = 2$s ?

Ans. $a = 12$ (Refer to Example 6.22).

Q.9. The acceleration due to gravity is 9.8 ms^{-2}. What does it mean?

Ans. It means that when the motion of a body is under gravity *alone*, its speed decreases at the rate of 9.8 ms^{-1} every second during upward motion. During downward journey, the speed of the body increases at the rate of 9.8 ms^{-1} every second until it hits the ground.

Q.10. A body starts from rest and has a uniform acceleration a. What will be the shape of its (*i*) $x - t$ graph (*ii*) $v - t$ graph (*iii*) $a - t$ graph?

Ans. (*i*) Parabola (*ii*) It will be a straight line passing through the origin (*iii*) It will be a straight line parallel to the time-axis.

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Q.11. A body is dropped from a height h. In how much time will it reach the ground?

Ans. $h = v_0 t + \frac{1}{2}at^2$. Here $v_0 = 0$; $a = g$.

$\therefore\ h = 0 \times t + \frac{1}{2}gt^2$ or $t = \sqrt{\frac{2h}{g}}$

Q.12. The velocity of a particle in ms^{-1} moving along a straight line is given by : $v = 10 + 3t^2$. What is instantaneous acceleration at $t = 4s$?

Ans. $a = \frac{dv}{dt} = \frac{d}{dt}\left(10 + 3t^2\right) = 6t$. Therefore, at $t = 4$s ; $a = 6 \times 4 = 24\ ms^{-2}$.

Q.13. Why does time occur twice in the unit of acceleration?

Ans. $\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}} = \frac{\text{Displacement/Time}}{\text{Time}} = \frac{\text{Displacement}}{(\text{Time})^2}$

Q.14. A food packet is released from a helicopter which is rising steadily at 2 ms^{-1}. What is the velocity of the packet after 2 seconds? (Take $g = 10\ ms^{-2}$).

Ans. $v = v_0 + gt$. Taking upward direction positive, we have, $v_0 = 2\ ms^{-1}$; $g = -10\ ms^{-2}$. Therefore, $v = 2 - 10 \times 2 = -18\ ms^{-1}$. Hence the packet is falling downward with a velocity of $18\ ms^{-1}$ after 2 s.

Q.15. Can you round a curve with zero acceleration?

Ans. No. It is because when you round a curve, the direction of motion is changing from instant to instant. This is possible only by a force and hence by an acceleration.

Q.16. Is the rate of change of acceleration with time important to mechanics? Comment.

Ans. The rate of change of acceleration with time is not important to mechanics. It is because the laws of motion involve acceleration only and to solve problems relating to motion, we do not require rate of change of acceleration with time.

Q.17. A car travelling at 36 km/h takes a U turn in 5 seconds. What is the acceleration of the car?

Ans. Change in velocity in 5 seconds, $\Delta v = 36 - (-36) = 72$ km/h $= 20\ ms^{-1}$

$\therefore$ Acceleration of car, $a = \Delta v/(\Delta t) = 20/5 = 4\ ms^{-2}$.

LONG ANSWER QUESTIONS

Q.1. Explain the terms (*i*) uniform acceleration (*ii*) variable acceleration and (*iii*) average acceleration. **[Refer to Art. 7.1]**

Q.2. Show that in case of uniform acceleration, the average velocity is equal to one-half the sum of initial and final velocities. **[Refer to Art. 7.2]**

Q.3. For uniformly linear accelerated motion, draw and explain (*i*) position-time graph and (*ii*) velocity-time graph. **[Refer to Arts. 7.5 and 7.4]**

Q.4. Derive graphically the following equations for uniformly accelerated linear motion :

(*i*) $v = v_0 + at$ (*ii*) $S = v_0 t + \frac{1}{2}at^2$ (*iii*) $v^2 - v_0^2 = 2aS$ **[Refer to Art. 7.4]**

Q.5. Deduce the above equations of motion by calculus method. **[Refer to Art. 7.7]**

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COMPETITION SUCCESS MATERIAL

Useful Concepts/Information

1. One dimensional motion with uniform acceleration means that magnitude as well as direction of acceleration remains the same.
2. The *equations of kinematics* for a particle moving along x-axis (one dimensional motion) with uniform acceleration 'a' (constant in magnitude and direction) are :

$(i)\ x - x_0 = \left(\frac{v_0 + v}{2}\right)t$

$(ii)\ v = v_0 + at$

$(iii)\ x = x_0 + v_0 t + \frac{1}{2}at^2$

$(iv)\ v^2 - v_0^2 = 2a(x - x_0)$

v_0 = initial velocity of particle
v = final velocity of particle in time t
$\bar{v}$ = average velocity of particle
x_0 = initial displacement of particle from origin *i.e.* at $t = 0$
x = final displacement of the particle from origin in time t
a = constant acceleration

3. The displacement of a particle in the *nth* second during uniformly accelerated motion in one dimension is

$$S_{nth} = v_0 + \frac{a}{2}(2n - 1)$$

4. Objects that move under gravity alone are called *freely falling bodies. A freely falling body always has an acceleration of 9.8 m/s^2 downward.*
5. The motion of a freely falling body (neglecting air resistance) is equivalent to motion in one dimension with constant acceleration. Therefore, kinematic equations for constant acceleration can be applied.
6. While solving problems regarding freely falling bodies, it is very important to adopt one sign convention for displacement. We shall take the vertical displacement h positive upward. Since positive h is upward, the acceleration is negative and given by $a = -g$. *The negative sign simply indicates that the acceleration is downward.* It does not imply that the velocity of particle is increasing or decreasing. With these substitutions, we get the following equations for freely falling bodies :

$(i)\ h = \frac{(v_0 + v)}{2}t$ where $h = x - x_0$ $(ii)\ v = v_0 - g\,t$ $(iii)\ h = v_0 t - \frac{1}{2}gt^2$

$(iv)\ v^2 - v_0^2 = -2gh$ $(v)\ h_{nth} = v_0 - \frac{g}{2}(2n - 1)$

You should note that *negative sign for acceleration is already included in these expressions.*

7. For upward journey, v_0 ,v and h are positive and equations containing 'g' term become:

$(i)\ v = v_0 - gt$ $(ii)\ h = v_0 t - \frac{1}{2}gt^2$ $(iii)\ v^2 - v_0^2 = -2gh$ $(iv)\ h_{nth} = v_0 - \frac{g}{2}(2n - 1)$

For downward journey v_0, v and h are negative and the equations containing 'g' term become:

$$-v = -v_0 - g\,t \quad \text{or} \quad v = v_o + g\,t \quad \ldots (i)$$

$$-h = -v_0\,t - \frac{1}{2}gt^2 \quad \text{or} \quad h = v_o t + \frac{1}{2}g\,t^2 \quad \ldots (ii)$$

$$(-v)^2 - (-v_0)^2 = -2g(-h) \quad \text{or} \quad v^2 - v_0^2 = 2gh \quad \ldots (iii)$$

$$h_{nth} = v_0 + \frac{g}{2}(2n - 1) \quad \ldots (iv)$$

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8. Consider the case of a particle thrown vertically upward from the origin with velocity v_0. In this case, v_0 is positive, $x_0 = 0$ (earth surface). Fig. 7.19 (*i*) shows displacement-time graph and Fig. 7.19 (*ii*) shows velocity-time graph for this motion. The following points are worth noting :

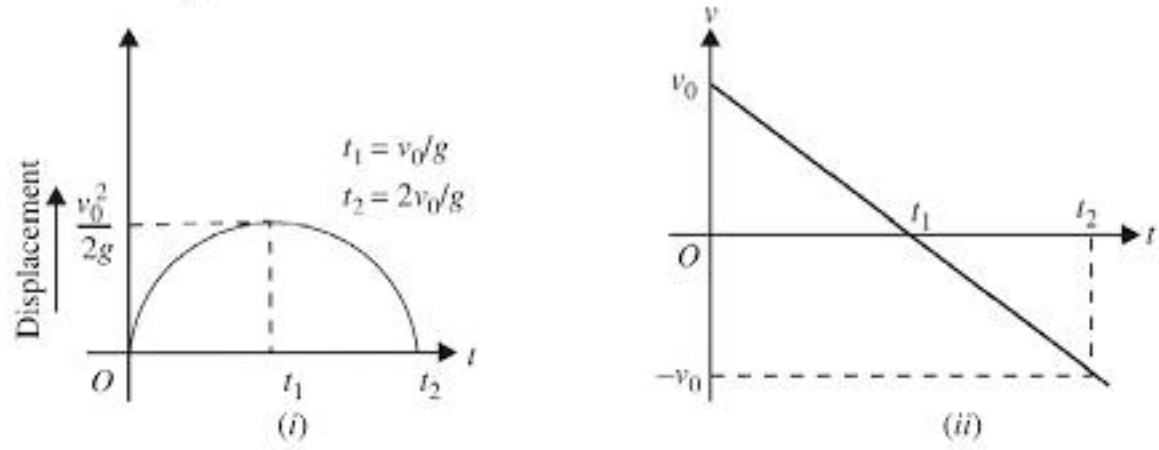


Fig. 7.19

(*i*) The velocity is initially positive but decreases with time and becomes zero at the peak of the path. This will occur at time t_1 given by;

$$0 = v_0 - gt_1 \quad \text{or} \quad t_1 = v_0/g \quad \text{... time for max. vertical displacement}$$

At this time, the vertical displacement is maximum and is given by ;

$$(0)^2 - (v_0)^2 = -2gh_{max} \therefore h_{max} = v_0^2/2g \quad \text{.... max. displacement}$$

(*ii*) At time $t_2 = 2t_1 = 2v_0/g$, the displacement is again zero *i.e.* the particle has returned to the starting point.

$$\therefore \quad t_2 = 2v_0/g \quad \text{... total journey time}$$

(*iii*) At time t_2, the velocity $v = -v_0$ as proved below :

$$v = v_0 - gt_2 = v_0 - \frac{g \times 2v_0}{g} = -v_0$$

Note that there is symmetry in motion. In other words, both the displacement and the magnitude of the velocity repeat themselves in the time interval $t = 0$ to $t = 2v_0/g$.

MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. A body freely falling from rest has a velocity v after it falls through a height h. The distance it has to fall further for its velocity to become double is **[Pb PMT 2001]**

   (*a*) $4h$ (*b*) $6h$

   (*c*) $8h$ (*d*) $10h$

2. The velocity of a bullet is reduced from 200 m/s to 100 m/s while travelling through a wooden block of thickness 10 cm. The retardation, assuming it to be uniform, will be **[AIIMS 2001]**

   (*a*) 10×10^4 m/s^2 (*b*) 1.2×10^4 m/s^2

   (*c*) 13.5×10^4 m/s^2 (*d*) 15×10^4 m/s^2

3. A car moving with a velocity of 40 km/hr can be stopped by applying brakes in 2 m. If the same car is moving with a speed of 80 km / hr, what is the minimum stopping distance? **[AFMC - Pune 2000]**

   (*a*) 16 m (*b*) 12 m

   (*c*) 8 m (*d*) 4 m

4. A particle is thrown vertically upward. Its velocity at half of the height is 10 m/s, then maximum height attained by it is ($g = 10$ m/s^2) **[CBSE PMT 2001]**

   (*a*) 8 m (*b*) 20 m

   (*c*) 10 m (*d*) 16 m

5. A body A starts from rest with an acceleration a_1. After 2 seconds, another body B starts from rest with an acceleration a_2. If they travel equal distances in the fifth second after the start of A, then ratio of $a_1 : a_2$ is equal to **[AIIMS 2001]**

   (*a*) 5 : 9 (*b*) 5 : 7

   (*c*) 9 : 5 (*d*) 9 : 7

6. A stone is thrown up with an initial speed of 4.9 m/s from a bridge in vertically upward

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direction. It falls down in water after 2 sec. The height of the bridge is

[AFMC - Pune 1999]

(*a*) 4.9 m (*b*) 9.8 m

(*c*) 19.8 m (*d*) 24.7 m

7. If a freely falling body in the last second travels a distance equal to the distance travelled in the first three seconds, the time of its travel is

[Pb PMT 1998]

(*a*) 3 sec (*b*) 4 sec

(*c*) 5 sec (*d*) 6 sec

8. A body dropped from a height h strikes the ground with a velocity of 3 m/s. Another body of same mass is dropped from the same height h with an initial velocity of 4 m/s. Final velocity of second mass with which it strikes the ground will be **[AFMC - Pune 1998]**

(*a*) 3 m/s (*b*) 4 m/s

(*c*) 8 m/s (*d*) 5 m/s

9. The acceleration of a particle is increasing linearly with time t as bt. The particle starts from origin with an initial velocity v_0. The distance travelled by the particle in time t will be

[CBSE PMT 1995]

(*a*) $v_0 t + \frac{1}{6}bt^3$ (*b*) $v_0 t + \frac{1}{3}bt^3$

(*c*) $v_0 t + \frac{1}{3}bt^2$ (*d*) $v_0 t + \frac{1}{2}bt^2$

10. If the water falls from a dam into a turbine wheel 19.6 m below, then the velocity of water at the turbine is (Take $g = 9.8$ m/s^2)

[AIIMS 1998]

(*a*) 9.8 m/s (*b*) 19.6 m/s

(*c*) 39.2 m/s (*d*) 98 m/s

11. A car is moving along a straight road with a uniform acceleration. It passes through two points P and Q separated by some distance with velocities 30 km/h and 40 km/h. The velocity of car midway between P and Q is

[CBSE PMT 1993]

(*a*) 33.3 km/h (*b*) $20\sqrt{3}$ km/h

(*c*) $25\sqrt{2}$ km/h (*d*) 35 km/h

12. If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is **[CBSE 2003]**

(*a*) $ut - \frac{1}{2}gt^2$ (*b*) $(u + gt)t$

(*c*) ut (*d*) $\frac{1}{2}gt^2$

13. A man throws balls with the same speed vertically upwards one after the other at an interval of 2 seconds. What should be the speed of the throw so that more than two balls are in the sky at any time? (Given $g = 9.8$ ms^{-2})

[CBSE 2003]

(*a*) Any speed less than 19.6 m/s

(*b*) Only with speed 19.6 m/s

(*c*) More than 19.6 m/s

(*d*) Atleast 9.8 m/s

14. A body moves for a total of nine seconds starting from rest with uniform acceleration and then with uniform retardation, which is twice the value of acceleration and then stops. The duration of uniform acceleration is :

[Manipal 2003]

(*a*) 3 s (*b*) 4.5 s

(*c*) 5 s (*d*) 6 s

15. A body initially at rest is moving with uniform acceleration a. Its velocity after n seconds is v. The displacement of the body in last 2 s is :

[Manipal 2005]

(*a*) $\frac{2v(n-1)}{n}$ (*b*) $\frac{v(n-1)}{n}$

(*c*) $\frac{v(n+1)}{n}$ (*d*) $\frac{2v(n+1)}{n}$

16. When a ball is thrown up vertically, with velocity v_0, it reaches a maximum height of h. If one wishes to triple the maximum height, then the ball should be thrown with a velocity of **[AIIMS 2005]**

(*a*) $3v_0$ (*b*) $9v_0$

(*c*) $\sqrt{3}\,v_0$ (*d*) $\frac{3}{2}v_0$

17. An earthquake generates both transverse (S) and longitudinal (P) sound waves in the earth. The speed of S waves is about 4.5 km/s and that of P waves is about 8.0 km/s. A seismograph records P and S waves from the earthquake. First P wave arrives 4.0 min. before the first S wave. Epicentre of the earthquake is located at a distance of about

[AIIMS 2003]

(*a*) 25 km (*b*) 250 km

(*c*) 2500 km (*d*) 5000 km

18. A ball is thrown vertically upwards. Which of the following plot represents the velocity-time

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graph of the ball during its flight, if the air resistance is not ignored? **[AIIMS 2003]**

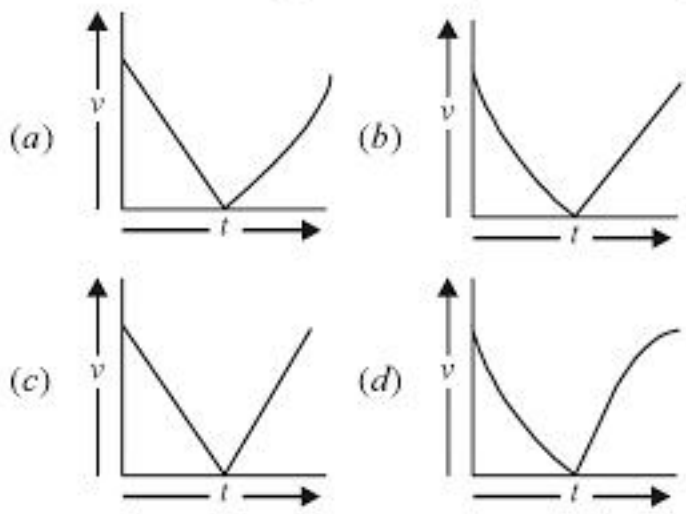


Fig. 7.20

19. A car accelerates from rest at constant rate for first 10 s and covers a distance x. It covers a distance y in next 10 s at the same acceleration. Which of the following is true? **[AFMC 2005]**

(a) $x = 3y$ (b) $y = 3x$
(c) $x = y$ (d) $y = 2x$

20. A coin is dropped in a lift. It takes time t_1 to reach the floor when lift is stationary. It takes time t_2 when lift is moving up with constant acceleration. Then : **[AFMC 2005]**

(a) $t_1 > t_2$ (b) $t_2 > t_1$
(c) $t_1 = t_2$ (d) $t_1 >> t_2$

ANSWERS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. (a)	**2.** (d)	**3.** (c)	**4.** (c)	**5.** (a)
6. (b)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (b)
11. (b)	**12.** (d)	**13.** (c)	**14.** (d)	**15.** (a)
16. (c)	**17.** (c)	**18.** (d)	**19.** (d)	**20.** (a)

HINTS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. $v^2 = v_0^2 + 2gh$ or $v^2 = 0 + 2gh$ $\therefore\ v^2 \propto h$

$\therefore$ $\left(\frac{v_1}{v_2}\right)^2 = \frac{h_1}{h_2}$ or $\left(\frac{v}{2v}\right)^2 = \frac{h}{h_2}$ $\therefore\ h_2 = \mathbf{4h}$

2. Initial velocity, $v_0 = 200$ m/s ; Final velocity, $v = 100$ m/s

Distance travelled, $S = 10$ cm $= 0.1$ m

Now $v^2 - v_0^2 = 2aS$ $\therefore\ a = \frac{v^2 - v_0^2}{2S} = \frac{(100)^2 - (200)^2}{2 \times 0.1} = \mathbf{-15 \times 10^4\ m/s^2}$

The minus sign is due to retardation.

3. $v^2 = v_0^2 + 2as$ or $0 = v_0^2 + 2as$ $\therefore\ v_0^2 \propto s$

$\therefore$ $\left(\frac{v_0}{v_0'}\right)^2 = \frac{s_1}{s_2}$ or $\left(\frac{40}{80}\right)^2 = \frac{2}{s_2}$ $\therefore\ s_2 = \mathbf{8m}$

4. Suppose v_0 is the initial velocity. Let after travelling a distance s, the velocity of the particle be 10 m/s. At maximum height *(i.e* $2s$*)*, $v = 0$.

$\therefore$ $0^2 = v_0^2 - 2g(2s)$ or $v_0^2 = 4gs = 4 \times 10\ s = 40\ s$

Also $(10)^2 = v_0^2 - 2 \times 10s$ or $100 = 40s - 20s$ $(\because v_0^2 = 40\ s)$

$\therefore$ $s = 5$ m

$\therefore$ Maximum height $= 2s = 2 \times 5 = \mathbf{10\ m}$

5. Distance travelled in nth second is

$$S_{nth} = v_0 + \frac{a}{2}(2n - 1)$$

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As per problem, distance covered by body A in the fifth second after start is equal to the distance covered by body B in the third second (5 – 2 = 3 sec) after start.

$$\therefore \quad 0+\frac{a_1}{2}(2\times5-1) = 0+\frac{a_2}{2}(2\times3-1)$$

or $$\frac{9a_1}{2} = \frac{5a_2}{2} \quad \text{or} \quad a_1 : a_2 = \mathbf{5 : 9}$$

6. The time t_1 taken by the stone to reach the highest point from the bridge is given by ;

$$v = v_0 - gt_1 \text{ or } t_1 = \frac{v-v_0}{-g} = \frac{0-4.9}{-9.8} = 0.5 \text{ sec}$$

Therefore, total time of journey from the bridge and back to bridge = 0.5 + 0.5 = 1 sec. After reaching the bridge, the velocity is 4.9 m/s *i.e.* v_0 = 4.9 m/s, time to reach water, t = 2 – 1 = 1 sec.

$$\therefore \quad h = v_0 t + \frac{1}{2}gt^2 = 4.9\times1+\frac{1}{2}\times(9.8)\times(1)^2 = \mathbf{9.8\ m}$$

7. Distance travelled in first 3 seconds

$$= v_0 t + \frac{1}{2}gt^2 = 0\times t+\frac{1}{2}\times9.8\times(3)^2 = 44.1 \text{ m}$$

Let t_1 be the total time of travel. Then,

44.1 = distance travelled in the last second

or $$44.1 = v_0+\frac{1}{2}g(2t_1-1)$$

$$44.1 = 0+\frac{1}{2}\times9.8(2t_1-1)$$

or $$t_1 = \mathbf{5 \text{ sec}}$$

8. For the first body, v_{01} = 0 ; v_1 = 3 m/s

Now $$v_1^2 - v_{01}^2 = 2gh$$

or $$(3)^2 - (0)^2 = 2 \times 9.8 \times h$$

$$\therefore \quad h = 0.46 \text{ m}$$

For the second body, v_{02} = 4 m/s ; v_2 = ?

Now $$v_2^2 - v_{02}^2 = 2gh$$

or $$v_2^2 - (4)^2 = 2 \times 9.8 \times 0.46$$

or $$v_2 = \mathbf{5 \text{ m/s}}$$

9. Acceleration, $a = bt$ or $\frac{dv}{dt} = bt$ or $v = \frac{bt^2}{2} + c$

At $t = 0$, $v = v_0$.

$$\therefore \quad v_0 = \frac{b\times(0)^2}{2}+c$$

$$\therefore \quad v = \frac{bt^2}{2}+v_0 \quad \text{or} \quad \frac{dS}{dt} = \frac{bt^2}{2}+v_0 \quad \text{or} \quad \boldsymbol{S = v_0 t + \frac{bt^3}{6}}$$

10. $v_0 = 0$; h = 19.6 m ; v = ?

$$v^2 - v_0^2 = 2gh \text{ or } v^2-(0)^2 = 2\times9.8\times19.6 \quad \therefore v = \mathbf{19.6 \text{ m/s}}$$

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11. Since the acceleration is uniform, the velocity v of the car midway between P and Q is

$$v = \frac{2v_1v_2}{v_1+v_2} = \frac{2\times30\times40}{30+40} = 34.3 \text{ km/h} = \mathbf{20\sqrt{3}\ km/h}$$

12. Let the body take T seconds to reach maximum height.

$\therefore$ $\quad v = u - gT \quad$ or $\quad 0 = u - gT \quad \therefore \quad T = u/g$

Velocity attained by the body in $(T - t)$ sec. is

$$v' = u - g(T - t) = u - gT + gt$$

$$= u - g \times \frac{u}{g} + gt = gt$$

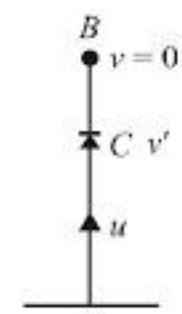


Fig. 7.21

$\therefore$ Distance travelled in the last t sec. is

$$CB = v't - \frac{1}{2}gt^2 = gt^2 - \frac{1}{2}gt^2 = \mathbf{\frac{1}{2}gt^2}$$

13. The time t taken by the ball to reach maximum height is

$$v = v_0 - gt \quad \text{or } 0 = v_0 - gt$$

$\therefore$ $\quad v_0 = gt$

If $t = 2$ s, then $v_0 = 9.8 \times 2 = 19.6$ m/s. If the man throws the ball with a velocity of 19.6 m/s, then after 2 s, it attains a height of 19.6 m. When he throws the second ball, the first ball is at the top (= 19.6 m). When he throws the third ball, the first ball comes to the ground and the second ball will be at the top (= 19.6 m). Therefore, at any time, only two balls will be in air. If he wants to keep more than 2 balls in air, he should throw the ball with a speed greater than 19.6 m/s.

14. Total time, $t = 9$ s ; Acceleration = a ; Retardation = $-2a$

Let t_1 and t_2 be the time for acceleration and retardation respectively ($\therefore\ t_1 + t_2 = 9$ s).

Now $\quad v = v_0 + at_1 \quad$ or $v = 0 + at_1, \quad \therefore \quad t_1 = v/a$

Also $\quad 0 = v - 2at_2 \quad$ or $\quad t_2 = v/2a$

Now $\quad t = t_1 + t_2 \quad$ or $\quad 9 = \frac{v}{a} + \frac{v}{2a} \quad \therefore \quad \frac{v}{a} = 6$

$\therefore$ Duration of acceleration, $t_1 = \frac{v}{a} = \mathbf{6\ s}$

15. Displacement in last 2 seconds $= \frac{1}{2}an^2 - \frac{1}{2}a(n-2)^2 = \frac{1}{2}a\left[n^2 - (n-2)^2\right]$

$$= \frac{1}{2}a\,(4n-4) = 2a\,(n-1)$$

$$\text{Acceleration, } a = \frac{v - v_0}{t} = \frac{v-0}{n} = \frac{v}{n}$$

$\therefore$ Displacement in last 2s $= 2a(n-1) = 2 \times \frac{v}{n}(n-1) = \mathbf{\frac{2v(n-1)}{n}}$

16. $v^2 - v_0^2 = -2gh \quad$ or $\quad (0)^2 - (v_0)^2 = -2\,gh \quad \therefore \quad v_0 \propto \sqrt{h}$

For the first case, $\quad v_0 = v_0\,;\ h = h$

For the second case, $v_0' = ?\ ;\ \ h' = 3h$

$\therefore$ $\quad \frac{v_0'}{v_0} = \sqrt{\frac{h'}{h}} = \sqrt{\frac{3h}{h}} = \sqrt{3} \quad \therefore\ v_0' = \mathbf{\sqrt{3}\,v_0}$

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17. Speed of S waves ; $v_s = 4.5$ km/s ; Speed of P waves, $v_p = 8$ km/s

Difference of time between two waves, $\Delta t = 4$ min. $= 240$ seconds

The relation for the location of epicentre of earthquake (s) is

$$\Delta t = \frac{s}{v_s} - \frac{s}{v_p} \quad \text{or} \quad 240 = \frac{s}{4.5} - \frac{s}{8} \qquad \therefore\ s \quad \textbf{2500 km}$$

18. The velocity of the ball decreases during upward motion and becomes zero at the highest point. During downward motion, the velocity of the ball increases. Since there is air resistance, the velocity–time graph during upward as well as downward motion will not be linear. Remember that air resistance varies as the square of velocity. Therefore, velocity–time graph of the ball during its flight will be represented by **graph (*d*)**.

19. Distance covered in first 10 seconds is

$$x = v_0 t + \frac{1}{2} a t^2 = 0 \times 10 + \frac{1}{2} a \times (10)^2 = 50a$$

Velocity of car after 10 seconds is

$$v = v_0 + at = 0 + a \times 10 = 10a$$

Distance covered in the next 10 seconds is

$$y = vt = 10a \times 10 = 100a = 2x \qquad (\because\ x = 50a)$$

$\therefore$ $\quad y = 2x$

20. In stationary lift, the time taken by the coin to reach the floor is given by ;

$$h = \frac{1}{2} g t^2 \quad \text{or} \quad t = \sqrt{\frac{2h}{g}} \qquad (\because\ v_0 = 0)$$

In the upward moving lift with constant acceleration a, $g' = g + a$.

$$\therefore \qquad t_2 = \sqrt{\frac{2h}{g+a}} \qquad \therefore\ t_2 < t_1$$

MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. A body projected vertically upwards with a velocity u returns to the starting point in 4 seconds. If $g = 10$ m/s^2, the value of u is

[Karnataka CET 1999]

(a) 5 m/s (b) 10 m/s

(c) 15 m/s (d) 20 m/s

2. The ratios of the distances covered by a freely falling particle, starting from rest in the first, second, third ...nth seconds of its motion

(a) form the series corresponding to the squares of the first n natural numbers

(b) form an A.P.

(c) form a series corresponding to the difference of the squares of the successive natural numbers

(d) both choices (b) and (c) are correct

[Haryana CET 2001]

3. A particle is thrown vertically upwards. The graph between its speed v and time t is given by (neglecting air resistance) the figure

[Roorkee 1998]

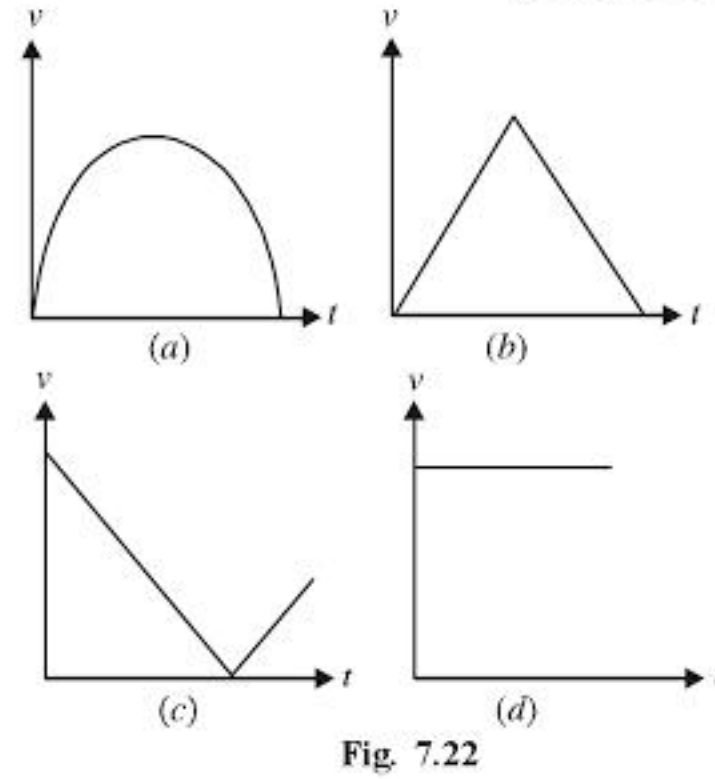


Fig. 7.22

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4. A man in balloon rises vertically with an acceleration of 4.9 m/s^2 releases a ball 2 seconds after the balloon rises from the ground. The greatest height above the ground reached by the ball (g = 9.8 m/s^2) is **[Haryana CET 2000]**

(a) 9.8 m (b) 24.5 m

(c) 14.7 m (d) 19.6 m

5. A body falling from rest describes distances S_1, S_2 and S_3 in the first, second and third seconds of its fall. The ratio of $S_1 : S_2 : S_3$ is **[Karnataka CET 1997]**

(a) 1 : 2 : 3 (b) 1 : 3 : 5

(c) 1 : 1 : 1 (d) 1 : 4 : 9

6. A body released from a great height falls freely toward earth. Another body is released from the same height exactly one second later. The separation between both the bodies two seconds after the release of the second body is **[Karnataka CET 1997]**

(a) 4.9 m (b) 9.8 m

(c) 19.6 m (d) 24.5 m

7. A ball is dropped on to the floor from a height of 10 m. It rebounds to a height of 2.5 m. If the ball is in contact with the floor for 0.01 second, what is the average acceleration during contact? **[BHU 1997]**

(a) 1400 m/s^2 (b) 2100 m/s^2

(c) 700 m/s^2 (d) 2800 m/s^2

8. A ball P is dropped vertically and another ball Q is thrown horizontally from the same height and at the same time. If air resistance is neglected, then, **[BHU 1994]**

(a) ball P reaches the ground first

(b) ball Q reaches the ground first

(c) both the balls reach the ground at the same time

(d) respective masses of the two balls will decide the time

9. A stone falls from a balloon descending at a uniform rate of 12 m/s. The displacement of the stone from the point of release after 10 seconds is **[Haryana CET 1995]**

(a) 725 m (b) 610 m

(c) 510 m (d) 490 m

10. A body is projected vertically up with a velocity $\vec{v}$ and after some time it returns to the point from which it was projected. The average velocity and average speed of the body for the total flight are

(a) $\vec{v}$ and 0 (b) 0 and $v/2$

(c) 0 and 0 (d) 0 and $\vec{v}$

11. A particle moving with a uniform acceleration travels 24 m and 64 m in the first two consecutive intervals of 4 sec each. Its initial velocity is **[MP PET 1995]**

(a) 1 m/sec (b) 10 m/sec

(c) 5 m/sec (d) 2 m/sec

12. The velocity-time graph of a body moving in a straight line is shown in Fig. 7.23. The displacement and distance travelled by the body in 6 seconds are respectively. **[MP PET 1994]**

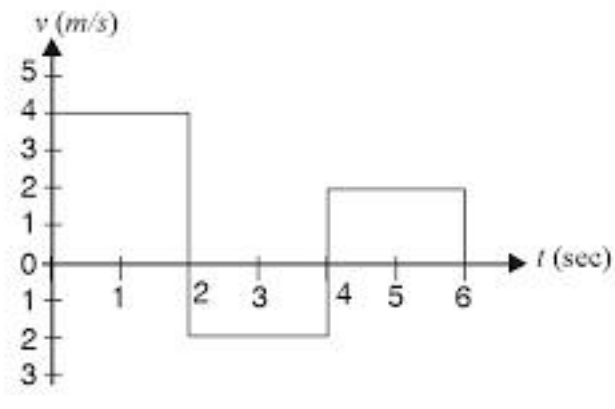


Fig. 7.23

(a) 8 m, 16 m (b) 16 m, 8 m

(c) 16 m, 16 m (d) 8 m, 8 m

13. Two bodies of different masses are dropped from heights of 16 m and 25 m respectively. The ratio of time taken by them to reach the ground is **[Pb CET 1991]**

(a) 25/16 (b) 5/4

(c) 4/5 (d) 16/25

14. A wooden block of mass 10 g is dropped from the top of a cliff 100 m high. Simultaneously a bullet of mass 10 g is fired from the foot of the cliff upward with a velocity of 100 m/s. The bullet and the wooden block will meet each other after a time (in second) **[EAMCET 1991]**

(a) 7 (b) 10

(c) 0.5 (d) 1

15. For the velocity-time graph shown in Fig. 7.24, the distance covered by the body in the last two seconds of its motion is what fraction of

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the total distance covered in all the seven seconds ? **[MPPET 1998]**

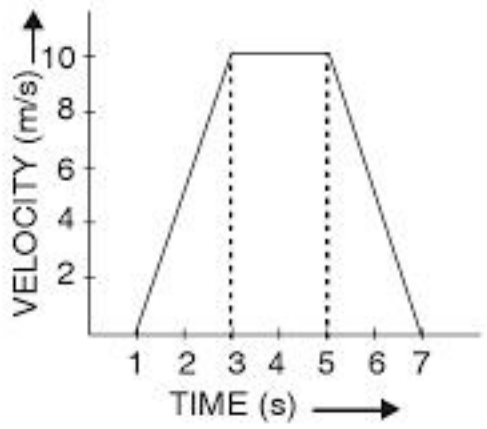


Fig. 7.24

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{4}$

(*c*) $\frac{1}{3}$ (*d*) $\frac{2}{3}$

16. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance travelled is $15S$, then : **[AIEEE 2005]**

(*a*) $S = ft$ (*b*) $S = \frac{1}{6}ft^2$

(*c*) $S = \frac{1}{72}ft^2$ (*d*) $S = \frac{1}{4}ft^2$

17. Three balls A, B and C are thrown from the top of a tower with the same speed. A is thrown straight up, B is thrown straight down and C is thrown horizontally. They hit the ground with speeds v_A, v_B and v_C respectively. Then: **[DCE 2006]**

(*a*) $v_A = v_B = v_C$ (*b*) $v_B > v_C > v_A$

(*c*) $v_A = v_B > v_C$ (*d*) $v_A > v_B = v_C$

18. A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in $T/3$ seconds? **[AIEEE 2004]**

(*a*) $h/9$ metre from the ground

(*b*) $7h/9$ metre from the ground

(*c*) $8h/9$ metre from the ground

(*d*) $17h/18$ metre from the ground

19. A small block slides, without friction, down an inclined plane starting from rest. Let S_n be the distance travelled from $t = (n-1)$ to $t = n$. Then $\frac{S_n}{S_{n+1}}$ is **[IIT 2004]**

(*a*) $\frac{2n-1}{2n}$ (*b*) $\frac{2n+1}{2n-1}$

(*c*) $\frac{2n-1}{2n+1}$ (*d*) $\frac{2n}{2n+1}$

20. The velocity–displacement graph of a particle moving along a straight line is shown in Fig. 7.25. The most suitable acceleration-displacement graph will be **[IIT 2005]**

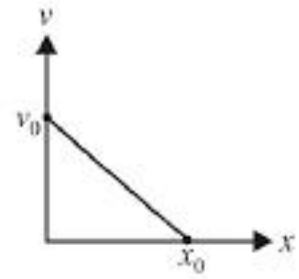


Fig. 7.25

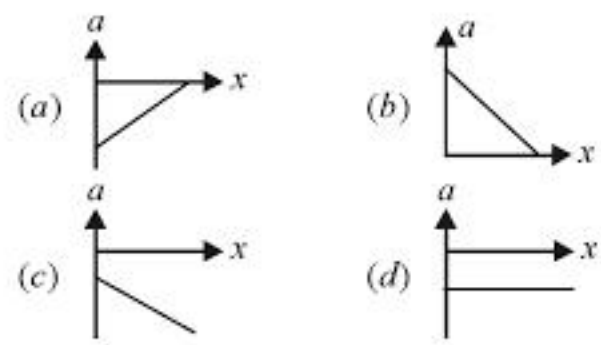


Fig. 7.26

ANSWERS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. (*d*)	**2.** (*d*)	**3.** (*c*)	**4.** (*c*)	**5.** (*b*)
6. (*d*)	**7.** (*b*)	**8.** (*c*)	**9.** (*b*)	**10.** (*b*)
11. (*a*)	**12.** (*a*)	**13.** (*c*)	**14.** (*d*)	**15.** (*b*)
16. (*c*)	**17.** (*c*)	**18.** (*c*)	**19.** (*c*)	**20.** (*a*)

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HINTS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. Time of ascent = Time of descent $= 4/2 = 2$ sec.

For upward journey, we have, $v = v_0 - g\,t$

or $0 = v_0 - 10 \times 2$

$\therefore$ $v_0 = \mathbf{20\ m/s}$

2. The distance covered by the particle in the nth second (with $v_0 = 0$) is

$$S_n = \frac{1}{2}g[n^2 - (n-1)^2] = \frac{1}{2}g(2n-1)$$

$\therefore$ $$S_1 = \frac{1}{2}g;\ S_2 = \frac{3}{2}g;\ S_3 = \frac{5}{2}g.$$

Therefore, **choices (*b*) and (*c*) are correct.**

3. For upward motion, $v = v_0 - gt$. Therefore, velocity decreases linearly with time. For downward motion, $v = v_0 + gt = 0 + gt = gt$. Again for downward motion, the velocity increases linearly with time.

4. Velocity v acquired by the ball in 2 seconds is

$$v = v_0 + at = 0 + 4.9 \times 2 = 9.8 \text{ m/s}$$

$\therefore$ $$s = 0 + \frac{1}{2} \times (4.9) \times (2)^2 = 9.8 \text{ m}$$

When the ball is released, it moves under gravity alone. Its velocity decreases till it reaches the highest point. Let it rise to a height h after its release from balloon.

$\therefore$ $(0)^2 - (9.8)^2 = -2 \times 9.8 \times h$ $\therefore$ $h = 4.9$ m

Greatest height of ball above ground

$= 9.8 + 4.9 = \mathbf{14.7\ m}$

5. Initial velocity, $v_0 = 0$ $\therefore$ $S = \frac{1}{2}gt^2$

$$S_1 = \frac{1}{2}g \times (1)^2 = \frac{1}{2}g$$

$$S_1 + S_2 = \frac{1}{2}g \times (2)^2 = 2g\ ; \qquad S_1 + S_2 + S_3 = \frac{1}{2}g \times (3)^2 = 4.5g$$

$\therefore$ $$S_1 = \frac{1}{2}g\ ;\ S_2 = 2g - \frac{1}{2}g = 1.5g\ ;\quad S_3 = 4.5g - 2g = 2.5\ g$$

$\therefore$ $$S_1 : S_2 : S_3 = \frac{1}{2}g : 1.5\ g : 2.5\ g = \mathbf{1 : 3 : 5}$$

6. Since initial velocity in the two cases is zero, $S = \frac{1}{2}gt^2$.

Distance covered by first body in 3 sec $= \frac{1}{2} \times 9.8 \times (3)^2 = 44.1$ m

Distance covered by second body in 2 sec $= \frac{1}{2} \times 9.8 \times (2)^2 = 19.6$ m

Separation of two bodies 2 seconds after the release of second body

$= 44.1 - 19.6 = \mathbf{24.5\ m}$

7. $v_1 = \sqrt{2gh_1} = \sqrt{2 \times 10 \times 10} = \sqrt{200}$ m/s ; $\quad v_2 = -\sqrt{2gh_2} = -\sqrt{2 \times 10 \times 2.5} = -\sqrt{50}$ m/s

Average acceleration, $\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_1 - v_2}{\Delta t} = \frac{\sqrt{200} + \sqrt{50}}{0.01} = \mathbf{2100\ m/s^2}$

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8. The balls will reach the ground due to gravity. Since the initial vertical velocity of the balls is zero, the **two balls will reach the ground at the same time.**

9. $v_0 = 12$ m/s ; $g = 9.8$ m/s^2 ; $t = 10$ sec.

$$s = v_0 t + \frac{1}{2} g t^2 = 12 \times 10 + \frac{1}{2} \times 9.8 \times (10)^2 = \mathbf{610\ m}$$

10. Displacement for total flight is zero. Therefore, average velocity is **zero.** The speed of the body changes from v to zero during upward journey and from zero to v during downward journey. Therefore, average speed $= (v + 0)/2 =$ **$v/2$.**

11. Let v_0 and a be the initial velocity and uniform acceleration respectively.

$$\therefore \qquad 24 = v_0 \times 4 + \frac{1}{2} a \times 4^2 \quad \text{or} \quad 4v_0 + 8a = 24 \qquad \ldots (i)$$

$$\text{Also} \qquad (24 + 64) = v_0 \times 8 + \frac{1}{2} a \times 8^2 \text{ or } 8v_0 + 32a = 88 \qquad \ldots (ii)$$

Solving eqs. (i) and (ii), $v_0 =$ **1 m/s.**

12. Displacement is equal to the *net* area under velocity – time graph.

$$\therefore \qquad \text{Displacement} = 4 \times 2 - 2 \times 2 + 2 \times 2 = \mathbf{8m}$$

Distance travelled is equal to the total area under velocity – time graph.

$$\therefore \quad \text{Distance travelled} = 4 \times 2 + 2 \times 2 + 2 \times 2 = \mathbf{16m}$$

13.

$$h = v_0 t + \frac{1}{2} g t^2 \text{ or } h = \frac{1}{2} g t^2 \quad (\because v_0 = 0) \qquad \therefore \ t \propto \sqrt{h}$$

$$\therefore \qquad \frac{t_1}{t_2} = \sqrt{\frac{h_1}{h_2}} = \sqrt{\frac{16}{25}} = \mathbf{\frac{4}{5}}$$

14. Let the block and bullet meet after time t at a distance x vertically below the top of the cliff.

For the downward motion of the block, we have, $x = 0 + \frac{1}{2} g t^2$.

For the upward motion of the bullet, we have,

$$100 - x = 100\, t - \frac{1}{2} g t^2$$

$$\text{or} \qquad 100 - x = 100t - x \qquad (\because \frac{1}{2} gt^2 = x)$$

$$\therefore \qquad t = \mathbf{1\ sec}$$

15. Total distance covered in 7 seconds is equal to the area under velocity–time graph for $t = 7$ sec.

$$\text{Total distance covered} = \frac{1}{2} \times 2 \times 10 + 2 \times 10 + \frac{1}{2} \times 2 \times 10 = 40 \text{ m}$$

Distance covered in the last 2 seconds

$$= \frac{1}{2} \times 2 \times 10 = 10 \text{ m}$$

$$\text{Fraction of distance} = 10/40 = \mathbf{1/4}$$

16. Fig. 7.27 shows the v–t graph for the motion of the car. The car accelerates for time t_1 and attains the velocity v. It moves at velocity v for time t. The car then decelerates for time t_2 to come to stop.

$$v^2 - v_0^2 = 2fS \quad \text{or} \quad v^2 - 0^2 = 2fS \qquad \therefore \ v = \sqrt{2fS}$$

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Total distance travelled $= \frac{1}{2}vt_1 + vt + \frac{1}{2}vt_2$

or $15S = \frac{1}{2}vt_1 + vt + \frac{1}{2}vt_2$ ... (*i*)

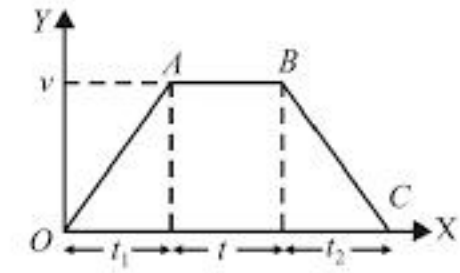


Fig. 7.27

Now $S = \frac{1}{2}ft_1^2 \quad \therefore \quad t_1 = \sqrt{\frac{2S}{f}}$

Also $0 = v - \frac{f}{2}t_2 \quad$ or $\quad t_2 = \frac{2v}{f} = \frac{2 \times \sqrt{2fS}}{f} = \sqrt{\frac{8S}{f}}$

Putting the values of v, t_1 and t_2 is eq. (*i*), we get,

$$\boldsymbol{S = \frac{1}{72}ft^2}$$

17. The three balls A, B and C are thrown up, vertically downward and horizontally respectively from the top of a tower (point T) with the same speed v as shown in Fig. 7.28. When ball A returns to point T, it has velocity v *i.e.*, the same as that of ball B at point T. Therefore, balls A and B hit the ground with the same velocity *i.e.*, $v_A = v_B$.

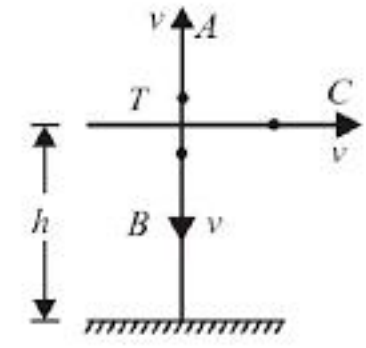


Fig. 7.28

Now $v_A = v_B = \sqrt{v^2 + 2gh}$

However, for ball C, the vertical velocity at point T is zero so that it will hit the ground with velocity v_C given by :

$v_C^2 - v_0^2 = 2gh \quad$ or $\quad v_C^2 - 0^2 = 2gh$

$\therefore \quad v_C = \sqrt{2gh} \qquad \therefore \quad \boldsymbol{v_A = v_B > v_C}$

18. $h = v_0T + \frac{1}{2}gT^2 \quad$ or $\quad h = 0 \times T + \frac{1}{2}gT^2 \qquad \therefore \; T = \sqrt{\frac{2h}{g}}$

At time $= T/3$, $h = 0 + \frac{1}{2}g\left(\frac{T}{3}\right)^2 = \frac{1}{2}g\frac{T^2}{9} = \frac{1}{2}g\frac{2h}{g \times 9} = \frac{h}{9}$ m

$\therefore$ Position of ball from ground $= h - \frac{h}{9} = \boldsymbol{\frac{8h}{9}}$ **m**

19. $S_n = v_0 + \frac{a}{2}(2n - 1) = \frac{a}{2}(2n - 1) \quad (\because \; v_0 = 0)$

$S_{n+1} = \frac{a}{2}[2(n + 1) - 1] = \frac{a}{2}[2n + 1] \quad (\because \; v_0 = 0)$

$\therefore \qquad \frac{S_n}{S_{n+1}} = \boldsymbol{\frac{2n-1}{2n+1}}$

20. From the given $v - x$ graph, we have,

Intercept $= v_0$ and slope $= -v_0/x_0$

$\therefore$ The equation of the given line is

$$v = -\frac{v_0}{x_0}x + v_0 \quad (y = mx + c)$$

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$$\therefore \qquad \frac{dv}{dt} = -\frac{v_0}{x_0}\frac{dx}{dt} \quad \text{or} \quad a = -\frac{v_0}{x_0}v$$

$$\therefore \qquad a = -\frac{v_0}{x_0}\left(-\frac{v_0}{x_0}x + v_0\right) = \frac{v_0^2}{x_0^2}x - \frac{v_0^2}{x_0}$$

Therefore, the $a - x$ graph is a straight line with slope $= v_0^2/x_0^2$ (positive) and intercept is v_0^2/x_0^2 (negative). The slope and intercept reveal that **graph (*a*)** is the most suitable representation.

NUMERICAL PROBLEMS FOR COMPETITIVE EXAMINATIONS

1. **Brakes are applied to a train travelling at 72 km/h. After passing over 200 m; its velocity is reduced to 36 km/h. At the same rate of retardation, how much further it will go before it is brought to rest ?**

Hint. For the first part of motion :

$$v_0 = 72 \text{ km/h} = 20 \text{ ms}^{-1}; \quad v = 36 \text{ km/h} = 10 \text{ ms}^{-1}; \quad S = 200 \text{ m}$$

Now $\quad v^2 - v_0^2 = 2aS \quad \therefore \quad a = \dfrac{v^2 - v_0^2}{2S} = \dfrac{(10)^2 - (20)^2}{2 \times 200} = -0.75 \text{ ms}^{-2}$

For the second part of motion : $v_0 = 10 \text{ ms}^{-1}$; $v = 0$; $a = -0.75 \text{ ms}^{-2}$; $S = ?$

Again $\quad v^2 - v_0^2 = 2aS \quad \therefore \quad S = \dfrac{v^2 - v_0^2}{2a} = \dfrac{(0)^2 - (10)^2}{2 \times (-0.75)} = \mathbf{66.67 m}$

2. **A body with uniform acceleration covers 76 m in the 5th second and 116 m in the 10th second. Find the distance travelled in the 15th second. What is the distance travelled in next 5 seconds ?**

Hint. The distance travelled in the nth second is given by ;

$$S_{nth} = v_0 + \frac{a}{2}(2n - 1)$$

$$\therefore \qquad S_{5th} = v_0 + \frac{a}{2}(2 \times 5 - 1) \quad \text{or} \quad 76 = v_0 + \frac{9a}{2} \qquad ..(i)$$

Also $\quad S_{10th} = v_0 + \dfrac{a}{2}(2 \times 10 - 1) \quad \text{or} \quad 116 = v_0 + \dfrac{19a}{2} \qquad ...(ii)$

Solving eqs. (*i*) and (*ii*), $v_0 = 40 \text{ ms}^{-1}$; $a = 8 \text{ ms}^{-2}$

$\therefore$ Distance covered in the 15th second is

$$S_{15th} = v_0 + \frac{a}{2}(2n - 1) = 40 + \frac{8}{2}(2 \times 15 - 1) = \mathbf{156 \text{ m}}$$

Distance covered in 10 seconds is

$$S = v_0 t + \frac{1}{2}at^2 = 40 \times 10 + \frac{1}{2} \times 8 \times (10)^2 = 800 \text{ m}$$

Distance covered in 15 seconds is

$$S' = v_0 t' + \frac{1}{2}at'^2 = 40 \times 15 + \frac{1}{2} \times 8 \times (15)^2 = 1500 \text{ m}$$

$\therefore$ Distance covered in next 5 seconds

$$= S' - S = 1500 - 800 = \mathbf{700 \text{ m}}$$

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3. A bus starts to move with acceleration of 1 ms^{-2}. A man who is 48 m behind the bus runs to catch it with a constant velocity of 10 ms^{-1}. In how much time he will catch the bus?

Hint. Suppose the man catches the bus after t sec. The distance covered by the bus in time t sec is

$$S = v_0 t + \frac{1}{2} a t^2 = 0 \times t + \frac{1}{2} \times (1) \times t^2$$

or $$S = \frac{1}{2} t^2$$

$$\text{For man, } 10 = \frac{S + 48}{t}$$

or $$S + 48 = 10\,t \quad \text{or} \quad \frac{1}{2} t^2 + 48 = 10t \qquad \left(\because S = \frac{1}{2} t^2\right)$$

or $$t^2 - 20t + 96 = 0 \quad \text{or} \quad (t - 8)\,(t - 12) = 0$$

$\therefore$ $$t = \mathbf{8 \text{ sec}} \text{ or } \mathbf{12 \text{ sec}}$$

4. A ball thrown up is caught by the thrower after 4 seconds. How high did it go and with what velocity was it thrown ? How far below its highest point was it 3 seconds after start?

Hint. The time for upward journey is equal to the time for the downward journey. Therefore, time taken by the ball to reach the highest point = 2 sec. At the highest point $v = 0$.

$\therefore$ $$v = v_0 + g\,t = v_0 - 9.8 \times 2$$

or $$0 = v_0 - 9.8 \times 2 \quad \therefore \quad v_0 = \mathbf{19.6 \text{ ms}^{-1}}$$

Now $$v^2 - v_0^2 = 2\,g\,h \quad \therefore \quad h = \frac{v^2 - v_0^2}{2g} = \frac{(0)^2 - (19.6)^2}{2 \times (-9.8)} = \mathbf{19.6 \text{ m}}$$

The ball takes 2 seconds to reach the highest point. Therefore, to find height below the highest point, $t = 3 - 2 = 1$ s.

$\therefore$ $$h = v_0 t + \frac{1}{2} g t^2 = 0 \times t + \frac{1}{2} \times 9.8 \times (1)^2 = \mathbf{4.9 \text{ m}}$$

5. A rifle bullet loses 1/20 of its velocity in passing through a plank. What is the least number of planks required to stop the bullet ?

Hint. Let

S = thickness of the plank

a = uniform retardation of the plank

v_0 = initial velocity of the bullet

The velocity of the bullet after passing through a plank = $v_0 - \frac{v_0}{20} = \frac{19 v_0}{20}$

Now $$v^2 - v_0^2 = 2\,a\,S \quad \text{or} \quad \left(\frac{19 v_0}{20}\right)^2 - v_0^2 = 2aS \quad \text{or} \quad -\frac{39 v_0^2}{400} = 2aS \qquad \ldots(i)$$

Suppose n number of planks are further required to stop the bullet completely. Then,

$$0 - \left(\frac{19 v_0}{20}\right)^2 = 2\,a(nS) \quad \text{or} \quad -\frac{(19)^2 v_0^2}{400\,n} = 2\,aS \qquad \ldots(ii)$$

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From eqs. (*i*) and (*ii*), we have, $-\frac{(19)^2 v_0^2}{400\,n} = -\frac{39 v_0^2}{400}$

$\therefore \quad n = \frac{(19)^2}{39} \quad 10$

$\therefore$ Minimum number of planks required = 10 + 1 = **11**

6. **A stone after falling freely under the influence of gravity for 1 second strikes a glass pane held horizontally. In breaking through the glass pane, the stone loses half the velocity. How far will it travel in next two seconds?**

Hint. Let v ms^{-1} be the final velocity when the stone strikes the glass pane. For this part of motion, $v_0 = 0$; $t = 1$ s ; $g = 9.8$ ms^{-2}.

$\therefore \quad v = v_0 + gt = 0 + 9.8 \times 1 = 9.8$ ms^{-1}

After striking the glass pane, the stone loses half the velocity. For this part of motion, $v_0 = 4.9$ ms^{-1} ; $g = 9.8$ ms^{-2} ; $t = 2$ s ; $h = ?$

$\therefore \quad h = v_0 t + \frac{1}{2} g t^2 = 4.9 \times 2 + \frac{1}{2} \times 9.8 \times (2)^2 = \mathbf{29.4\ m}$

7. **A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, find (*i*) the maximum velocity reached (*ii*) the total distance travelled. [IIT]**

Hint. Suppose the car accelerates for t_1 seconds. Then it decelerates for $(t - t_1)$ seconds.

(*i*) **Maximum velocity reached.** For accelerated motion of the car, $v_0 = 0$; $a = \alpha$; time = t_1 and $v = v_m$ (maximum velocity).

$\therefore \quad v_m = v_0 + a t_1$ or $v_m = \alpha t_1$

The velocity αt_1 becomes zero after time $(t - t_1)$.

$\therefore \quad v = v_0 + at$ or $0 = \alpha t_1 - \beta (t - t_1) \quad \therefore \quad t_1 = \frac{\beta t}{\alpha + \beta}$

$\therefore$ Maximum velocity, $v_m = \alpha t_1 = \alpha \times \frac{\beta t}{\alpha + \beta} = \mathbf{\frac{\alpha \beta}{\alpha + \beta} t}$

(*ii*) **Total distance travelled.**

Let S_1 = distance covered by car during accelerated motion

S_2 = distance covered by car during decelerated motion

Total distance covered, $S = S_1 + S_2$

Now $v^2 - v_0^2 = 2\,a\,S$

Here $v = v_m$; $v_0 = 0$; $a = \alpha$; $S = S_1$

$\therefore \quad v_m^2 - (0)^2 = 2\,\alpha\,S_1 \quad \therefore \quad S_1 = \frac{v_m^2}{2\alpha}$

Again, $v^2 - v_0^2 = 2aS$

Here $v = 0$; $v = v_m$; $a = -\beta$; $S = S_2$

$\therefore \quad (0)^2 - (v_m)^2 = 2 \times (-\beta)\, S_2$ or $S_2 = \frac{v_m^2}{2\beta}$

Total distance covered, $S = S_1 + S_2 = \frac{v_m^2}{2\alpha} + \frac{v_m^2}{2\beta} = \frac{v_m^2}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

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$$= \frac{v_m^2(\alpha+\beta)}{2\alpha\beta} \qquad ...(i)$$

Putting the value of v_m in eq. (*i*), we get,

$$S = \left(\frac{\alpha\beta}{\alpha+\beta}t\right)^2 \times \frac{\alpha+\beta}{2\alpha\beta} = \frac{\boldsymbol{\alpha\beta t^2}}{\boldsymbol{2(\alpha+\beta)}}$$

8. A solid ball of density half that of water falls freely under gravity from a height of 19.6 m and then enters water. Upto what depth will the ball go ? How much time will it take to come again to the water surface ? Neglect air resistance and viscosity effects of water. **[*Roorkee*]**

Hint. Let V m^3 be the volume of the ball.

$$\text{Density of ball, } \rho = \frac{\text{density of water}}{2} = \frac{1000}{2} = 500 \text{ kg/m}^3$$

$$\text{Mass of ball, } m = V\rho = 500\ V \text{ kg}$$

The velocity v of the ball when it hits the water surface is given by ;

$$v^2 = v_0^2 + 2gh = 2gh \quad \therefore \quad v = \sqrt{2gh} = \sqrt{2\times9.8\times19.6} = 19.6 \text{ ms}^{-1} \qquad (\because\ v_0 = 0)$$

$$\text{Upthrust of water} = V\rho_w g = V \times 1000 \times g = 1000\ Vg$$

Effective weight of the ball when in water

$$= mg - 1000\ Vg = 500\ Vg - 1000\ Vg = -500\ Vg$$

Acceleration a of the ball is given by ;

$$ma = -500\ Vg \quad \text{or} \quad 500\ V \times a = -500\ Vg \ (\because\ m = 500\ V)$$

$$\therefore \qquad a = -g$$

The depth h upto which the ball will go in water can be found from the relation :

$$v^2 - v_0^2 = 2ah$$

Here $v = 0$; $v_0 = 19.6 \text{ ms}^{-1}$; $a = -g$

$$\therefore \qquad h = \frac{v^2 - v_0^2}{2a} = \frac{(0)^2 - (19.6)^2}{2\times(-9.8)} = \mathbf{19.6\ m}$$

The time t' taken by the ball to come again to the water surface can be found by using the relation :

$$h = v_0 t' + \frac{1}{2}at'^2$$

Here $h = -19.6$ m ; $v_0 = 0$; $a = -9.8 \text{ ms}^{-2}$

$$\therefore \qquad -19.6 = 0 \times t' + \frac{1}{2}(-9.8)\ t'^2$$

$$\therefore \qquad t'^2 = \frac{2\times19.6}{9.8} = 4 \quad \text{or} \quad t' = \sqrt{4} = \mathbf{2\ sec}$$

9. A rocket is fired vertically and ascends at a constant vertical acceleration. In 5 seconds, it attains an upward velocity of 150 ms^{-1} . After 5 seconds, its engine stops functioning and the rocket continues to move as a free particle and finally returns to earth. Plot the velocity-time graph of the rocket. From the graph, find (*i*) acceleration of the rocket during fuel burning (*ii*) the height reached by the rocket during the period of fuel burning (*iii*) the maximum height reached by the rocket.Take $g = 10 \text{ ms}^{-2}$.

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Hint. Figure 7.29 shows the velocity-time graph of the rocket. During the part OA of the graph, the rocket is under constant acceleration and its velocity is increasing. At point A, the velocity of the rocket is maximum (= 150 ms^{-1}). At point A, the engine stops functioning and the velocity of the rocket starts decreasing but the rocket is still rising. This part of motion is represented by graph AB. At point B, the rocket has attained maximum height and its velocity is zero. Now the rocket starts downward journey represented by graph BC. Time taken during the part AB of the graph can be found using the equation :

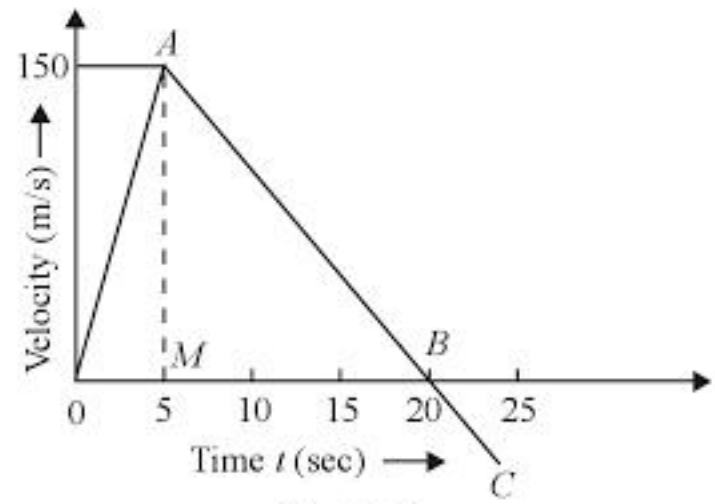


Fig. 7.29

$$v = v_0 + a\,t$$

Here $v = 0$; $v_0 = 150\ ms^{-1}$; $a = -g = -10\ ms^{-2}$

$\therefore \quad 0 = 150 - 10\,t \quad$ or $t = 150/10 = 15$ sec.

Thus, the total time for the upward journey of the rocket = 5 + 15 = 20 seconds.

(*i*) Acceleration of the rocket during fuel burning

$$= \text{Slope of line } OA = \frac{AM}{OM} = \frac{150}{5} = \mathbf{30\ ms^{-2}}$$

(*ii*) Height reached by the rocket during fuel burning

$$= \text{Area of } \Delta OAM = \frac{1}{2} \times OM \times AM \quad = \frac{1}{2} \times 5 \times 150 = \mathbf{375\ m}$$

(*iii*) Maximum height reached by rocket

$$= \text{Area of } \Delta OAB = \frac{1}{2} \times OB \times AM \quad = \frac{1}{2} \times 20 \times 150 = \mathbf{1500\ m}$$

10. The velocity-time graph for a car is shown in Fig. 7.30. Draw acceleration-time graph from it.

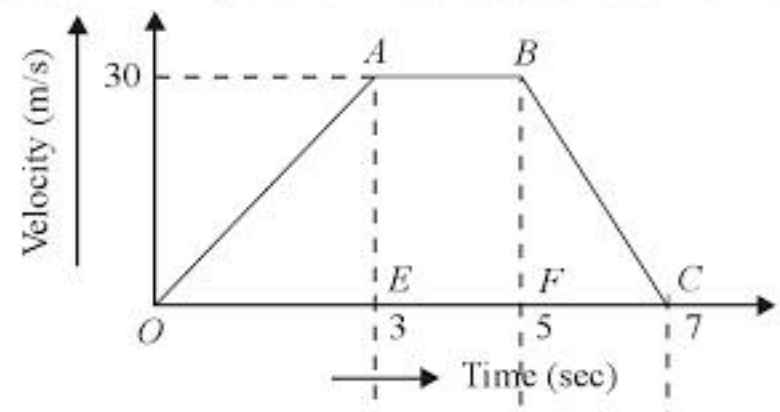


Fig. 7.30

Hint. (*i*) Referring to velocity-time graph [Fig. 7.30], during the part OA, the velocity is increasing uniformly. In other words, the car is under constant acceleration. This constant acceleration is equal to slope of the line OA.

$\therefore$ Constant acceleration during OA

$$= \frac{AE}{OE} = \frac{30}{3} = 10\ ms^{-2}$$

The acceleration-time graph for this part of motion is PQ as shown in Fig. 7.31.

(*ii*) Referring to velocity-time graph, during the part AB, the velocity is constant so that acceleration is zero. The accelertion-time graph for this part of motion is RS [See Fig. 7.31].

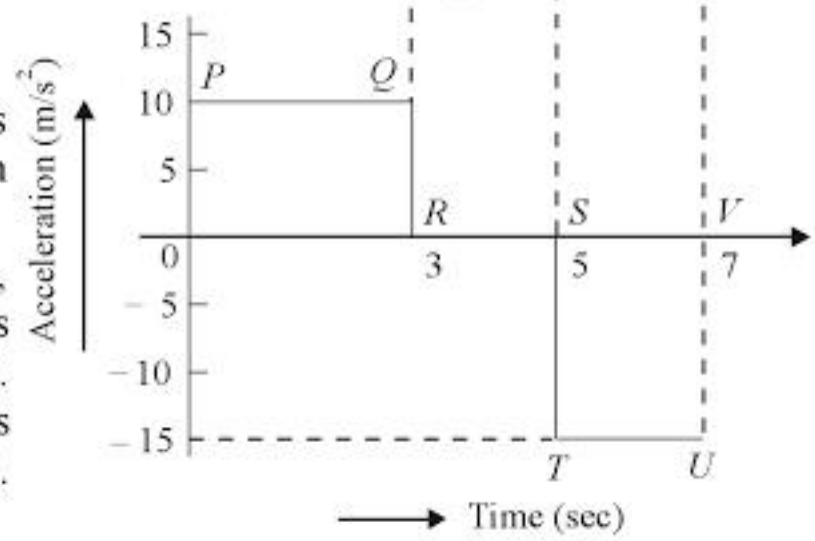


Fig. 7.31

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(*iii*) Referring to velocity-time graph, during the part BC, the velocity is decreasing uniformly. In other words, the acceleration is constant and negative. It is given by the slope of line BC in Fig. 7.30.

Constant retardation during part $BC = -\frac{BF}{FC} = -\frac{30}{2} = -15\,\text{ms}^{-2}$

The acceleration-time graph for this part of motion is TU as shown in Fig. 7.31.

11. A ball falls freely on a perfectly elastic plate from a height of 3.0 m. At instant $t = 0$, the velocity of the ball is zero. Draw the velocity-time graph for the motion of the ball.

Hint. The time t taken by the ball in falling from rest through 3.0 m is given by ;

$$h = \frac{1}{2} g t^2 \quad \text{or} \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 3.0}{9.8}} = 0.78 \text{ sec}$$

The velocity v of the ball just before striking the plate is

$$v = \sqrt{2gh} = \sqrt{2\times 9.8\times 3.0} = 7.67\,\text{ms}^{-1}$$

Figure 7.32 shows the velocity-time graph for the motion of the ball. At $t = 0$, the velocity of the ball is zero. As it falls towards the plate under gravity, its velocity goes on increasing till it becomes maximum ($= 7.67\ \text{ms}^{-1}$) after 0.78 sec. On striking the plate, it rebounds with the same velocity because the plate is perfectly elastic. Now the motion of the ball is against gravity so that its velocity goes on decreasing till it becomes zero after 0.78 sec. The ball again starts downward journey.

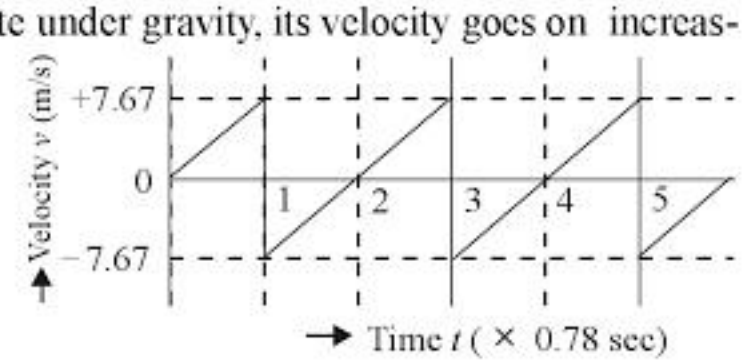


Fig. 7.32

12. From the top of a tower 98 m high, a stone is projected vertically upward with a velocity of 39.2 ms^{-1}. On its return, it misses the tower and finally strikes the ground. Calculate (*i*) the greatest elevation (*ii*) the velocity with which it strikes the ground (*iii*) the time it takes to reach the ground.

Hint. Take the upward motion as positive.

(*i*) Let h be the height from the top of the tower to which the stone rises.

Now $v^2 - v_0^2 = 2gh$

Here $v = 0$; $v_0 = 39.2\ \text{ms}^{-1}$; $g = -9.8\ \text{ms}^{-2}$

$$\therefore \quad h = \frac{v^2 - v_0^2}{2g} = \frac{(0)^2 - (39.2)^2}{2\times(-9.8)} = 78.4 \text{ m}$$

$\therefore$ Greatest elevation $= 78.4 + 98 =$ **176.4 m**

(*ii*) The velocity v with which it strikes the ground is given by ;

$$v^2 - v_0^2 = 2gh$$

Here $v_0 = 0$; $g = -9.8\ \text{m s}^{-2}$; $h = -176.4$ m

$$\therefore \quad v^2 - 0^2 = 2 \times (-9.8) \times (-176.4)$$

or $$v = \sqrt{2\times(-9.8)\times(-176.4)} = \mathbf{58.8\ ms^{-1}}$$

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(*iii*) The total time t taken to reach the ground is given by ;

$$v = v_0 + g\,t$$

Here $v = -58.8\ \text{ms}^{-1}$; $v_0 = 39.2\ \text{ms}^{-1}$; $g = -9.8\ \text{ms}^{-2}$

$$\therefore \qquad t = \frac{v - v_0}{g} = \frac{(-58.8) - (39.2)}{-9.8} = \mathbf{10\ s}$$

13. In figure 7.33, ball 1 is released from rest at the top of a smooth inclined plane and at the same instant ball 2 is projected up the plane from the foot of the plane with such velocity that they meet half-way up the plane. Determine (i) the velocity of projection and (ii) the velocity of each ball when they meet.

Hint. (*i*) In the common time t, distance travelled by ball 1 is given by ;

$$\frac{l}{2} = 0 \times t + \frac{1}{2}(g \sin\theta)t^2 \qquad \ldots(i)$$

The distance travelled by ball 2 in time t is

$$\frac{l}{2} = v_2 t + \frac{1}{2}(-g \sin\theta)t^2 \qquad \ldots(ii)$$

Adding eqs. (*i*) and (*ii*), we have,

$$\frac{l}{2} + \frac{l}{2} = v_2 t \quad \text{or} \quad t = \frac{l}{v_2}$$

Putting $t = l/v_2$ in eq. (*i*), we have,

$$\frac{l}{2} = \frac{1}{2}(g \sin\theta)\frac{l^2}{v_2^2}$$

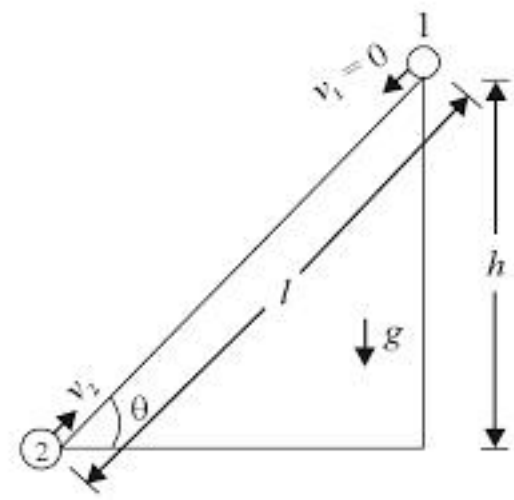


Fig. 7.33

or $\qquad v_2^2 = g l \sin\theta$

$$\therefore \qquad v_2 = \sqrt{g l \sin\theta} = \sqrt{\boldsymbol{gh}} \qquad (\because l \sin\theta = h)$$

(*ii*) $\qquad v_1'^2 = 0^2 + 2(g \sin\theta)\frac{l}{2} \quad \text{or} \quad v_1' = \sqrt{g\ l \sin\theta} = \sqrt{\boldsymbol{gh}}$

Now $\qquad v_2'^2 = v_2^2 + 2(-g \sin\theta)\frac{l}{2} = g l \sin\theta - g l \sin\theta = \mathbf{0}$

14. A bullet is fired from a rifle with a velocity of 750 ms^{-1}. If the length of the barrel is 0. 6 m, find (*i*) the average velocity of the bullet while being accelerated (*ii*) time taken by the bullet to travel the length of the barrel (*iii*) the average acceleration.

Hint. (*i*) Initial velocity of bullet, $v_0 = 0$; Final velocity of bullet, $v = 750\ \text{ms}^{-1}$

$$\therefore \qquad \text{Average velocity} = \frac{v_0 + v}{2} = \frac{0 + 750}{2} = \mathbf{375\ ms^{-1}}$$

(*ii*) If t is the time taken by the bullet to cross the barrel ($S = 0.6$ m), then,

$$S = \left(\frac{v_0 + v}{2}\right)t = 375 \times t$$

$$\therefore \qquad t = \frac{S}{375} = \frac{0.6}{375} = \mathbf{0.0016\ s}$$

(*iii*) If a is the average acceleration, then,

$$v = v_0 + a\,t$$

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or $$a = \frac{v - v_0}{t} = \frac{750 - 0}{0.0016} = \mathbf{468750\ ms^{-2}}$$

15. A ball after having fallen from rest under the influence of gravity for 6 s crashes through a horizontal glass plate, thereby losing two-third of its velocity. If it then reaches the ground in 2 s, find the height of the plate above the ground.

Hint. The velocity v of the ball just before striking the glass plate is given by ;

$$v = v_0 + g\,t$$

Here $v_0 = 0$; $g = -9.8\ ms^{-2}$; $t = 6$ s

$\therefore$ $$v = 0 + (-9.8) \times 6 = -58.8\ ms^{-1}$$

The velocity of the ball after passing through the glass plate, $v_1 = v/3 = -58.8/3 = -19.6\ ms^{-1}$. The height h of the glass plate above the ground is given by ;

$$h = v_1 t + \frac{1}{2} g\,t^2$$

Here $v_1 = -19.6\ ms^{-1}$; $t = 2$ s ; $g = -9.8\ ms^{-2}$; $h = -h$

$\therefore$ $$-h = -19.6 \times 2 + \frac{1}{2} \times (-9.8) \times (2)^2$$

or $$h = \mathbf{58.8\ m}$$

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N.C.E.R.T. TEXTBOOK EXERCISES

NCERT CHAPTER 3 : MOTION IN A STRAIGHT LINE

Q.1. In which of the following examples of motion can the body be considered approximately a point object: (*a*) a railway carriage moving without jerks between two stations. (*b*) a monkey sitting on top of a man cycling smoothly on a circular track. (*c*) a spinning cricket ball that turns sharply on hitting the ground. (*d*) a tumbling beaker that has slipped off the edge of a table?

Ans. When the size of an object is very small compared to the scale of observation, the object can be considered as a point object.

(*a*) The size of railway carriage is very small compared to the distance between the two stations. Therefore, the carriage can be considered a point object.

(*b*) The size of the monkey is very small compared to the distance covered by the cyclist. Therefore, monkey can be considered a point object.

(*c*) The size of the spinning cricket ball is comparable to the distance through which the ball may turn as it hits the ground. Therefore, cricket ball *cannot* be considered a point object.

(*d*) The size of the beaker is not negligible as compared to the height of the table. Therefore, in this case, the beaker *cannot* be considered a point object.

Q.2. The position-time ($x - t$) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. 3.01. Choose the correct enteries in the brackets below: (*a*) (A/B) lives closer to the school than (B/A), (*b*) (A/B) starts from the school earlier than (B/A), (*c*) (A/B) walks faster than (B/A), (*d*) A and B reach home at the (same/different) time, (*e*) (A/B) overtakes (B/A) on the road (once/twice).

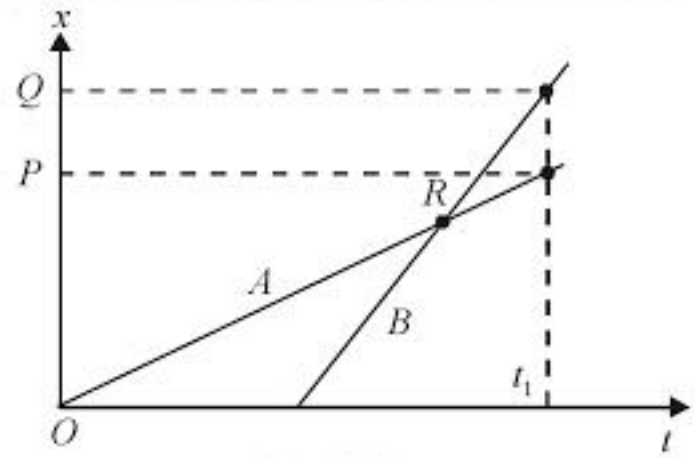


Fig. 3.01

Ans. (*a*) From the graph, it is clear that P is closer to O than Q. Therefore, child A lives closer to the school than the child B.

(*b*) It is clear that child A starts at $t = 0$, while the child B starts at later time.

(*c*) The $x - t$ graph for child B is steeper than that of child A. In other words, slope of x–t graph for child B is greater than that of child A. Since the slope of x–t graph is equal to the speed, child B walks faster than child A.

(*d*) It is clear from the graph that both children reach their homes at t_1, *i.e.*, they reach home at the same time.

(*e*) The x–t graphs for the children intersect only once at R. Therefore, B overtakes A on the road only once.

Q.3. A woman starts from her home at 9.00 A.M., walks with a speed of 5 km/h on straight road upto her office 2.5 km away, stays at the office upto 5:00 P.M. and returns home by an auto with a speed of 25 km/h. Choose suitable scale and plot the $x - t$ graph of her motion.

Ans. Time taken to reach office $= \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{2.5\text{ km}}{5\text{ km/h}} = 0.5\text{ hr}$

Time taken to return from office

$= \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{2.5\text{ km}}{25\text{ km/h}} = 0.1\text{ hr} = 6\text{ minutes}$

Therefore, the woman reaches the office at 9.30 A.M. and returns home at 5.06 P.M.

The $x - t$ graph of the motion of the woman is shown in Fig. 3.02.

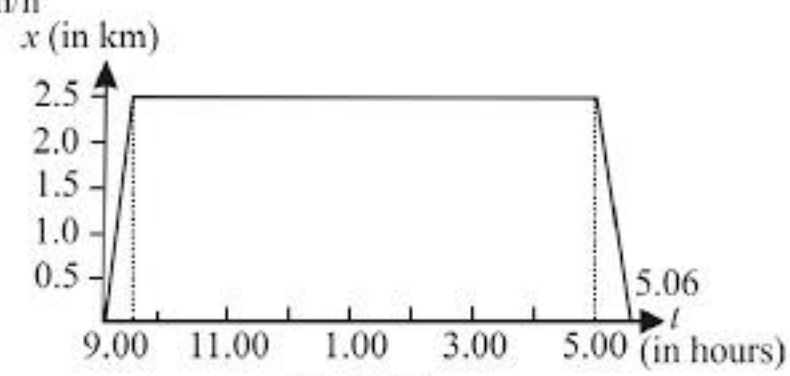


Fig. 3.02

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Q.4. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward followed again by 5 steps forward and 3 steps backward and so on. Each step is 1 m long and requires 1 s. Plot the $x - t$ graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.

Ans. Fig. 3.03 shows the $x - t$ graph of the motion of the drunkard. It is clear from the graph that the required time = **37 s**.

We can also find the required time mathematically. Assigning the forward motion positive and backward motion negative,

$$x = \underbrace{5\text{m} + (-3\text{m})}_{\text{2 m, 8 steps}} + \underbrace{5\text{m} + (-3\text{m})}_{\text{2 m, 8 steps}} + \underbrace{5\text{m} + (-3\text{m})}_{\text{2 m, 8 steps}} + \underbrace{5\text{m} + (-3\text{m})}_{\text{2 m, 8 steps}} + \underbrace{5\text{m}}_{\text{5 steps}}$$

$\therefore$ Total number of steps = 8 + 8 + 8 + 8 + 5 = 37 steps

Since time taken for one step is 1 s, the total time taken to fall in pit 13 m away from the start = 37 × 1 = **37 s**.

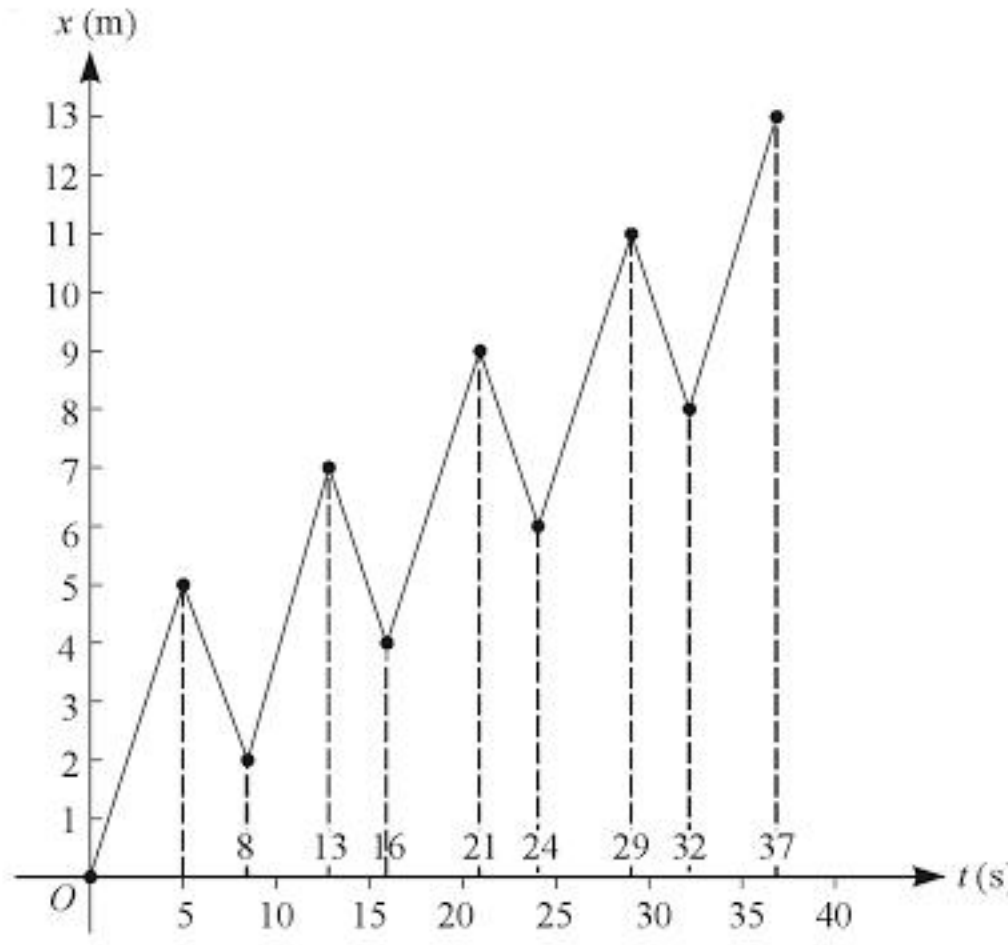


Fig. 3.03

Q.5. A jet aeroplane travelling at the speed of 500 km/h ejects its products of combustion at the speed of 1500 km/h relative to jet plane. What is the speed of the latter w.r.t. an observer on the ground ?

Ans. Considering the positive direction of x-axis to be the direction in which the jet plane moves, we have,

v_{JE} = Relative speed of jet plane *w.r.t.* earth = + 500 km/h

v_{CJ} = Relative speed of combustion products *w.r.t.* jet = – 1500 km/h

$\therefore$ Relative speed of combustion products *w.r.t.* earth is

$$v_{CE} = v_{CJ} + v_{JE} = (-1500) + (+500) = \mathbf{-1000\ km/h}$$

Q.6. A car moving along a straight highway with a speed of 126 km/h is brought to stop within a distance of 200 m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop ?

Ans. $v_0 = 126$ km/h $= 35$ m s^{-1} ; $v = 0$; $S = 200$ m ; $a = ?$; $t = ?$

$$\text{Average speed of car, } \bar{v} = \frac{v_0 + v}{2} = \frac{35 + 0}{2} = 17.5 \text{ ms}^{-1}$$

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$$S = \bar{v}\ t \quad \therefore \quad t = \frac{S}{\bar{v}} = \frac{200}{17.5} = 11.43 \text{ s}$$

Now $v = v_0 + a\ t$

$$\therefore \quad a = \frac{v - v_0}{t} = \frac{0 - 35}{11.43} = -3.06 \text{ ms}^{-2}$$

Q.7. Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km/h in the same direction with A ahead of B. The driver of B decides to overtake A and accelerates by 1 m s^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between the trains?

Ans. Let x metres be the original distance between the trains. Now for each train v_0 = 72 km/h = 20 ms^{-1}.

For train A: $v_0 = 20$ m s^{-1} ; $a = 0$; $t = 50$ s

$\therefore$ Distance travelled by A, $S_A = v_0\ t = 20 \times 50 = 1000$ m

For train B: $v_0 = 20$ m s^{-1} ; $a = 1$ m s^{-2} ; $t = 50$ s

$\therefore$ Distance travelled by B, $S_B = v_0\ t + \frac{1}{2}\ a\ t^2 = 20 \times 50 + \frac{1}{2} \times 1 \times (50)^2 = 2250$ m

If L_A and L_B are the lengths of the trains, then the statement of the problem suggests that

$$S_B = x + S_A + L_A + L_B$$

or $$2250 = x + 1000 + 400 + 400$$

$\therefore$ $$x = 450 \text{ m}$$

Q.8. On a two-lane road, a car A is travelling with a speed of 36 km/h. Two cars B and C approach car A in opposite directions with a speed of 54 km/h each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

Ans. Fig 3.04 shows the conditions of the problem.

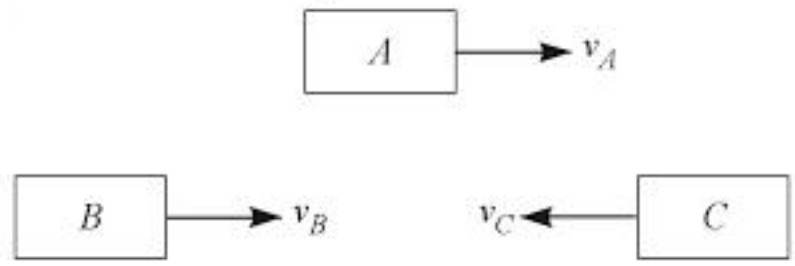


Fig. 3.04

$v_A = +36$ km/h $= +10$ ms^{-1} ; $v_B = +54$ km/h $= +15$ ms^{-1} ; $v_C = -15$ ms^{-1}

Relative velocity of car C *w.r.t.* car $A = v_C - v_A = (-15) - (+10) = -25$ m s^{-1}

Time required by car C to cover 1 km to reach car $A = \frac{1000}{25} = 40$ s

In order to avoid accident, the car B should be accelerated such that it overtakes car A in 40 s. Let the minimum required acceleration be 'a'. Relative velocity of car B *w.r.t.* car $A = v_B - v_A = 15 - 10 = 5$ ms^{-1}.

Now $v_0 = 5$ ms^{-1} ; $t = 40$ s ; $S = 1000$ m ; $a = ?$

Now, $$S = v_0 t + \frac{1}{2}\ at^2$$

or $$1000 = 5 \times 40 + \frac{1}{2}\ a \times (40)^2$$

or $$800\ a = 800 \quad \therefore \quad a = 1 \text{ ms}^{-2}$$

Q.9. Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of 20 km/h in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion and every 6 min

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in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road ?

Ans. Let the constant speed with which the buses are plying on the road be v km/h.

$\therefore$ Speed of bus going from A to B *w.r.t.* the cyclist $= (v - 20)$ km/h

Speed of the bus going from B to A *w.r.t.* the cyclist $= (v + 20)$ km/h

The distance covered by the bus in time $T = v\,T$.

$$\therefore \quad \text{Time} = \frac{\text{Distance}}{\text{Velocity}}$$

$$\text{or} \quad \frac{18}{60} = \frac{vT}{v-20} \qquad \ldots(i)$$

$$\text{and} \quad \frac{6}{60} = \frac{vT}{v+20} \qquad \ldots(ii)$$

Dividing eq. (i) by eq. (ii), we get, $\dfrac{v+20}{v-20} = 3 \qquad \therefore \; v = \mathbf{40\ km/h}$

Putting $v = 40$ km/h in eq. (i), we get,

$$T = \frac{18(v-20)}{v} = \frac{18(40-20)}{40} = \mathbf{9\ min}$$

Q.10. A player throws a ball upwards with an initial speed of 29.4 ms^{-1}. (*a*) What is the direction of acceleration during the upward motion of the ball? (*b*) What are the velocity and acceleration of the ball at the highest point of its motion? (*c*) Choose the $x = 0$ m and $t = 0$ s to be the location and time of the ball at its highest point, vertically downward to be the positive direction of X-axis, give the signs of position, velocity and acceleration of the ball during its upward and downward motion. (*d*) To what height does the ball rise and after how long does the ball return to the player's hands? [Take $g = 9.8\ ms^{-2}$ and neglect air resistance].

Ans. (*a*) The ball moves under the action of gravity. Therefore, the direction of acceleration due to gravity is always vertically downwards.

(*b*) At the highest point, the velocity of the ball is zero and acceleration due to gravity is 9.8 ms^{-2} vertically downwards.

(*c*) When the highest point is chosen as the location for $x = 0$ and $t = 0$ and vertically downward direction to be the positive direction of X-axis, we have,

During upward motion. The signs of position and velocity are negative and the sign of acceleration is positive.

During downward motion. The signs of position, velocity and acceleration are positive.

(*d*) For the upward motion, $v_0 = -29.4\ ms^{-1}$ and $g = +9.8\ ms^{-2}$.

$$\therefore \quad v^2 - v_0^2 = 2gh \quad \text{or} \quad h = \frac{-v_0^2}{2g} = \frac{-(-29.4)^2}{2\times 9.8} = \mathbf{-44.1\ m}$$

The negative sign shows that the distance is covered in the upward direction.

If t is the time taken to reach the highest point, then,

$$v = v_0 + gt \quad \text{or} \quad 0 = -29.4 + 9.8t \quad \therefore \; t = 3\text{s}$$

Now for vertical motion under gravity, time of ascent = time of descent.

$\therefore$ Time taken by the ball to return to player's hands $= 3 + 3 = \mathbf{6\ s}$

Q.11. Read each statement below carefully and state with reasons and examples, if it is true or false; A particle in one-dimensional motion (*a*) with zero speed at an instant may have non-zero acceleration at that instant (*b*) with zero speed may have non-zero velocity (*c*) with constant speed must have zero acceleration (*d*) with positive value of acceleration must be speeding up.

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Ans. (*a*) The statement is true. When a body is thrown vertically upward from the ground, then at the highest point, the body has zero speed but acceleration of 9.8 ms^{-2} (= *g*) is acting vertically downwards.

(*b*) It is false. It is because the speed of a body is equal to magnitude of its velocity. Therefore, when speed is zero, velocity is also zero.

(*c*) It is true. When a body moves in a straight line, its direction is unchanged. If the speed of the body is constant, change in velocity is zero and hence acceleration is zero.

(*d*) The velocity of body moving in a straight line is given by: $v = v_0 + at$. The statement given in the problem is not correct if v_0 is negative at $t = 0$. However, the given statement is true if both v_0 and a are positive.

Q.12. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Plot the speed-time graph of its motion between $t = 0$ to 12 s. ($g = 10$ m/s^2)

Ans. Since the ball is dropped from a height of 90 m, we have, $v_0 = 0$; $g = 10$ m/s^2 and $h = 90$ m.

Time taken by the ball to reach floor is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 90}{10}} = 4.24 \text{ s}$$

Velocity of the ball at floor is

$$v = \sqrt{2gh} = \sqrt{2\times 10\times 90} = 30\sqrt{2} \text{ m/s}$$

Rebound velocity of ball is

$$v' = \left(\frac{9}{10}\right)v = \left(\frac{9}{10}\right)\times 30\sqrt{2} = 27\sqrt{2} \text{ m/s}$$

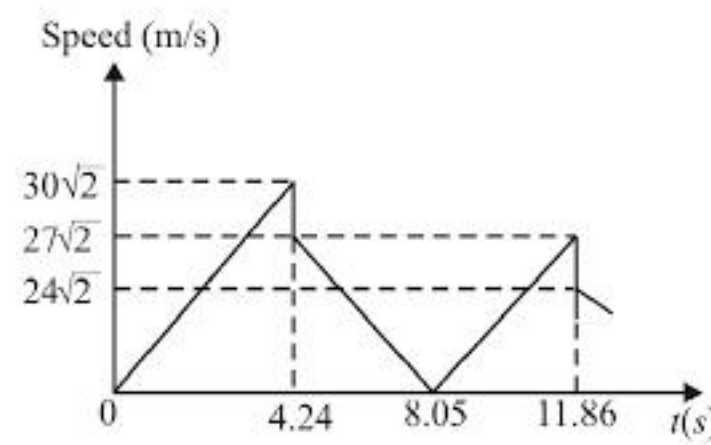


Fig. 3.05

Time taken by the ball to reach the highest point is

$$t' = \frac{v'}{g} = \frac{27\sqrt{2}}{10} = 3.81 \text{ s}$$

Total time = $t + t' = 4.24 + 3.81 = 8.05$ s

The ball takes further 3.81 s to reach the floor again where its velocity before striking the floor will be $27\sqrt{2}$ m/s.

Rebound velocity of the ball is $v'' = \frac{9}{10}\times 27\sqrt{2} = 24.3\sqrt{2}$ m/s

The ball will start upward motion after time

$$= 8.05 + 3.81 = 11.86 \text{ s}$$

Fig. 3.05 shows the speed-time graph of motion of the ball between $t = 0$ and $t = 12$ s.

Q.13. Explain clearly, with examples, the distinction between :

(*a*) magnitude of displacement (sometimes called distance) over an interval of time and the total length of the path covered by a particle over the same interval.

(*b*) magnitude of average velocity over an interval of time and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval]. Show in both (*a*) and (*b*) that the second quantity is either greater than or equal to first. When is the equality sign true? [For simplicity, consider one-dimensional motion only].

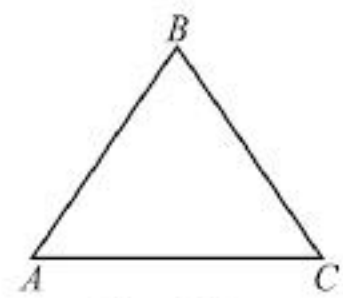


Fig. 3.06

Ans. (*a*) Suppose a particle moves from *A* to *B* and *B* to *C* in time *t* as shown in Fig. 3.06. Then magnitude of displacement of the particle = *AC* whereas the total length of the path covered by the particle = *AB* + *BC*. Note that the total path length (= *AB* + *BC*) is greater than the magnitude of displacement (= *AC*). If the particle moves in a straight line, then magnitude of displacement is equal to the total path length covered by the particle in a given time.

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(*b*) Referring to Fig. 3.06,

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{AC}{t}$$

$$\text{Average speed} = \frac{\text{Total path length}}{\text{Time}} = \frac{AB + BC}{t}$$

Since $(AB + BC) > AC$, the average speed is greater than the average velocity. If the particle moves in a straight line, displacement = total path length in a given time so that average speed will be equal to the average velocity. Thus in either case (*a*) or (*b*) the second quantity is either greater than or equal to first quantity. Equality sign is true when displacement is equal to distance.

Q.14. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km/h. What is the (*a*) magnitude of average velocity and (*b*) average speed of the man over the interval of time (*i*) 0 – 30 minutes (*ii*) 0 – 50 minutes (*iii*) 0 – 40 minutes?

Ans. (*i*) **Time interval 0–30 minutes**. While going to the market, the man walks at a speed of 5 km/h.

$\therefore$ Displacement in 30 min = 5/2 = 2.5 km

$$\text{Average velocity} = \frac{\text{Net displacement}}{\text{Time}} = \frac{2.5\text{km}}{0.5\text{h}} = \mathbf{5\ km/h}$$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{2.5\text{km}}{0.5\text{h}} \quad \mathbf{5\ km/h}$$

(*ii*) **Time interval 0 – 50 minutes.** While going to the market the man walks at a speed of 5 km/h. Since the market is 2.5 km from his home, he takes 30 minutes to reach the market. While coming back from the market, he walks at a speed of 7.5 km/h so that time taken to reach home = 2.5/7.5 = 1/3 hour = (1/3) × 60 = 20 minutes. This means that in 50 minutes the man walks back home. Therefore, net displacement = 0.

$\therefore$ $$\text{Average velocity} = \frac{\text{Net displacement}}{\text{Time}} = \frac{0}{50\,\text{min}} = \mathbf{0}$$

Now in 50 min, the total distance travelled = 2.5 + 2.5 = 5 km and total time taken = 50/60 hr = 5/6 hr.

$\therefore$ $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{5}{5/6} = \mathbf{6\ km/h}$$

(*iii*) **Time interval 0–40 minutes.** In 30 minutes, the man covers a distance of 2.5 km in going from home to the market. While returning home, he covers a distance of 1.25 km in 10 minutes ($\because$ return speed is 7.5 km/h).

$\therefore$ Net displacement during time interval 0 – 40 min
= 2.5 – 1.25 = 1.25 km

$$\text{Average velocity} = \frac{\text{Net displacement}}{\text{Total time}} = \frac{1.25}{40/60} = \mathbf{1.875\ km/h}$$

Total distance travelled in 40 min = 2.5+ 1.25= 3.75 km

$\therefore$ $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time}} = \frac{3.75}{40/60} = \mathbf{5.625\ km/h}$$

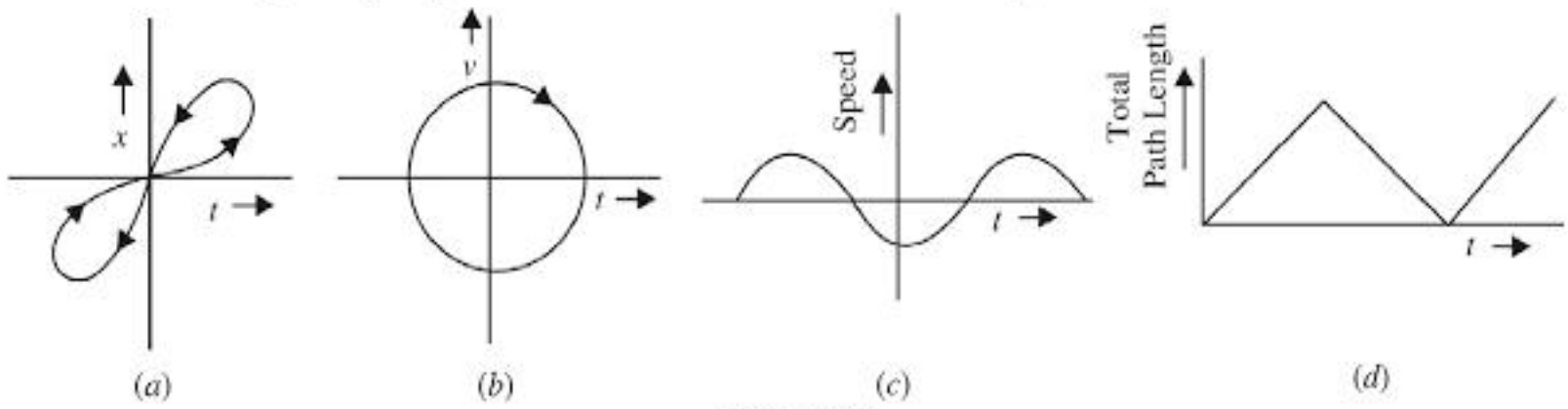
Q.15. In Q.13 and Q.14, we have carefully distinguished between average speed and magnitude of average velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?

Ans. Instantaneous velocity, $v = \frac{dx}{dt}$

In such a small interval of time (dt), the direction of motion of the particle is not supposed to change so that displacement is effectively equal to the distance. Therefore, instantaneous speed is equal to the magnitude of instantaneous velocity.

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Q.16. Look at the graphs (*a*) to (*d*) [Fig. 3.07] carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.



(*a*) (*b*) (*c*) (*d*)

Fig. 3.07

Ans. (*a*) The graph shown in Fig. 3.07 (*a*) does not represent one-dimensional motion. It is because the particle has two positions at the same time which is not possible in one-dimensional motion.

(*b*) The graph shown in Fig. 3.07 (*b*) does not represent one-dimensional motion. It is because a particle cannot have positive and negative velocity at the same time in one-dimensional motion.

(*c*) The graph shown in Fig. 3.07 (*c*) does not represent one-dimensional motion because the particle speed can never be negative.

(*d*) The graph shown in Fig. 3.07 (*d*) does not represent one-dimensional motion. It is because the graph shows that total path length decreases after certain time. However, the total path length of a moving particle can never decrease with time.

Q.17. Fig. 3.08 shows $x - t$ graph of one dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not, suggest a suitable physical context for this graph.

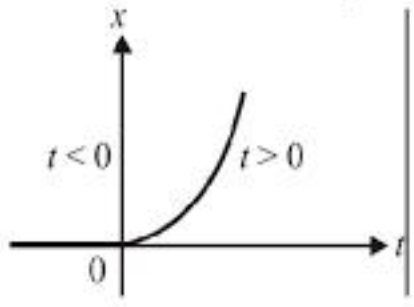


Fig. 3.08

Ans. No, because the $x - t$ graph does not represent the path taken by the particle. From the graph, at $t = 0$, $x = 0$. This graph represents the motion of a freely falling body.

Q.18. A police van moving on a highway with a speed of 30 km/h fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km/h. If the muzzle speed of bullet is 150 ms^{-1}, with what speed does the bullet hit thief's car?

Ans. Speed of police van, $v_{PE} = 30 \text{ km/h} = \frac{25}{3} \text{ ms}^{-1}$

Speed of bullet, $v_{BE} = 150 \text{ ms}^{-1}$

Speed of thief's car, $v_{TE} = 192 \text{ km/h} = \frac{160}{3} \text{ ms}^{-1}$

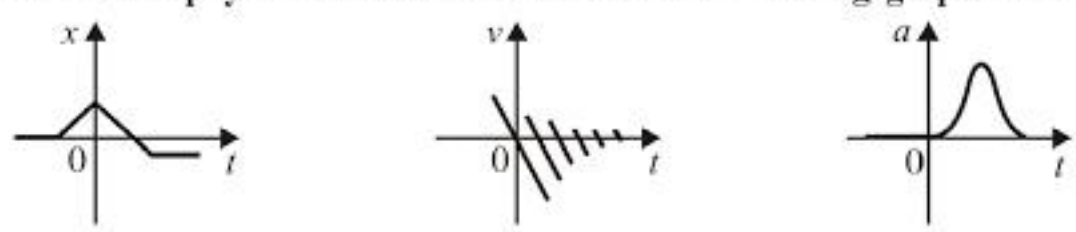
When the bullet is fired from police van, the net speed of bullet is the sum of v_{PE} and v_{BE}.

$$\therefore \quad \text{Net speed of bullet} = v_{PE} + v_{BE} = \frac{25}{3} + 150 = \frac{475}{3} \text{ ms}^{-1}$$

The bullet will hit the thief's car with relative speed of the bullet *w.r.t.* thief's car. Therefore, the speed with which the bullet hits thief's car is

$$v = \frac{475}{3} - \frac{160}{3} = \frac{315}{3} = \mathbf{105 \text{ ms}^{-1}}$$

Q.19. Suggest a suitable physical situation for each of the following graphs in Fig. 3.09.



(*a*) (*b*) (*c*)

Fig. 3.09

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Ans. (*a*) The $x - t$ graph in Fig. 3.09(*a*) shows that initially $x = 0$ (*i.e.*, the body is at rest). Then x increases with time, attains a constant value and again reduces to zero with time. Then x increases in the opposite direction till it again attains a constant value (*i.e.*, body comes to rest). Such a physical situation arises when a ball resting on a smooth floor is kicked. The ball rebounds from a wall with reduced speed and moves to the opposite wall which stops it.

(*b*) The $v - t$ graph in Fig. 3.09(*b*) shows that the velocity of the body changes sign again and again with passage of time and each time magnitude of velocity is reduced. Such a physical situation arises when a ball is thrown up with some initial velocity and returns back as freely falling body. On striking the ground, it rebounds with reduced speed each time it strikes the ground.

(*c*) The $a - t$ graph in Fig. 3.09(*c*) shows that initially the body moves with uniform velocity (*i.e.*, $a = 0$). Its acceleration increases for a short time and then falls to zero and thereafter, the body moves with a constant velocity. Such a physical situation arises when a cricket ball moving with a uniform speed is hit with a bat for a short interval of time.

Q.20. Figure 3.010 gives the $x - t$ plot of a particle executing one-dimensional simple harmonic motion. (You will learn about this motion in more detail in chapter 25). Give the signs of position, velocity and acceleration variables of the particle at $t = 0.3$ s, 1.2 s, –1.2 s.

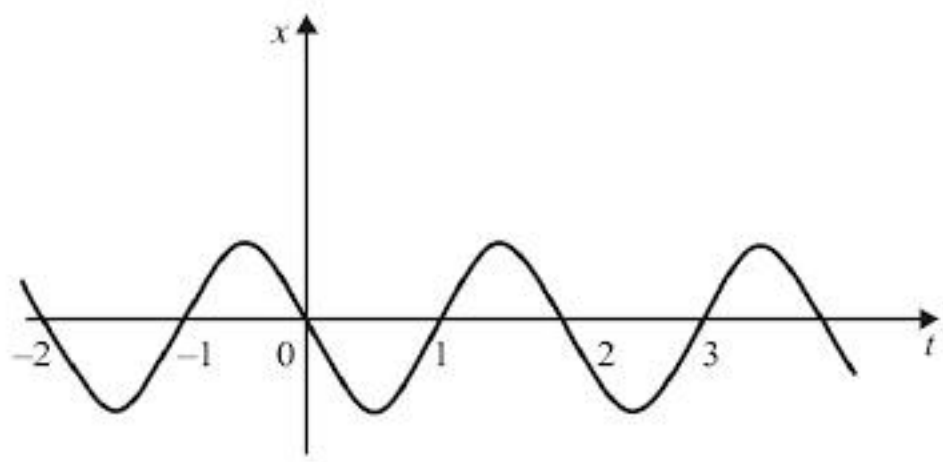


Fig. 3.010

Ans. In S.H.M., acceleration $a = -\omega^2 x$ where ω (*i.e.*, angular frequency) is constant.

(*i*) **At $t = 0.3$ s.** It is clear that x is negative. The slope of $x - t$ graph is negative so that velocity is also negative. Since $a = -\omega^2 x$ and x is negative, acceleration a is positive.

(*ii*) **At $t = 1.2$ s.** It is clear that x is positive. The slope of $x - t$ graph is positive so that velocity is also positive. Since $a = -\omega^2 x$ and x is positive, acceleration a is negative.

(*iii*) **At $t = -1.2$ s.** It is clear that x is negative. The slope of $x - t$ graph is negative. Since both x and t are negative, velocity is positive. The acceleration a is also positive.

Q.21. Fig. 3.011 shows the $x - t$ graph of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval the average speed is greatest and in which it is the least? Give the sign of average speed for each interval.

Ans. The average speed in a small interval of time is equal to the slope of $x - t$ graph during that interval of time. It is clear from Fig. 3.011 that average speed is greatest in the time interval 3 because slope of $x - t$ graph is greatest in this time interval. The average speed is least in interval 2 because slope is least in this interval. The average speed is positive in time intervals 1 and 2 because slope of $x - t$ graph is positive for these time intervals. However, average speed in time interval 3 is negative because slope of $x - t$ graph is negative.

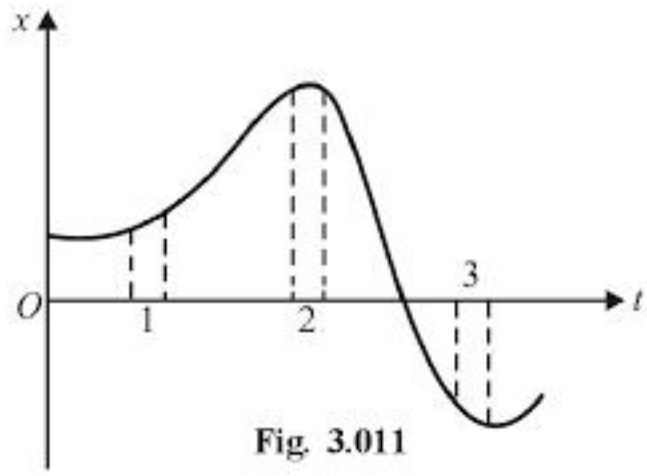


Fig. 3.011

Q.22. Fig. 3.012 gives the speed-time graph of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude? In which interval is the average speed greatest? Choosing the positive direction

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as the constant direction of motion, give the signs of 'u' and 'a' in the three intervals. What are the accelerations at the points A, B, C and D?

Ans. The average acceleration in a small interval of time is equal to the slope of velocity-time graph during that interval of time. It is clear from Fig. 3.012 that slope of $v - t$ graph is maximum in interval 2 (as compared to intervals 1 and 3). Therefore, average acceleration is maximum in time interval 2. The average speed is greatest in time interval 3 because point D is at maximum on speed axis.

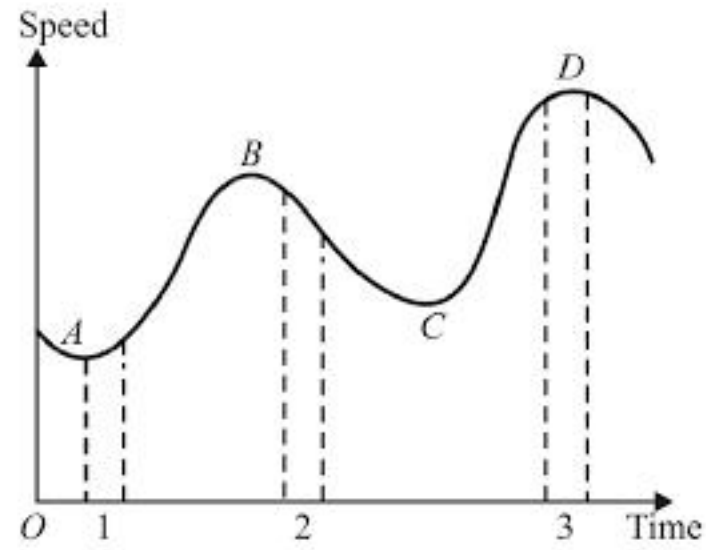


Fig. 3.012

In interval 1, the slope of velocity-time graph is positive. Therefore, acceleration as well as speed is positive. In interval 2, the slope of velocity-time graph is negative. Therefore, acceleration is negative but speed is positive. In interval 3, the velocity-time graph is parallel to the time axis. Therefore, acceleration is zero but speed is positive. At points A, B, C and D, the velocity-time graph is parallel to time axis. Therefore, acceleration is zero at these four points.

Q.23. A three wheeler starts from rest, accelerates uniformly with 1 ms^{-2} on a straight road for 10 s and then moves with uniform velocity. Plot a graph between the distance covered by the vehicle during the nth second (n = 1, 2, 3, ...) versus n. What do you expect the plot to be during accelerated motion: a straight line or a parabola?

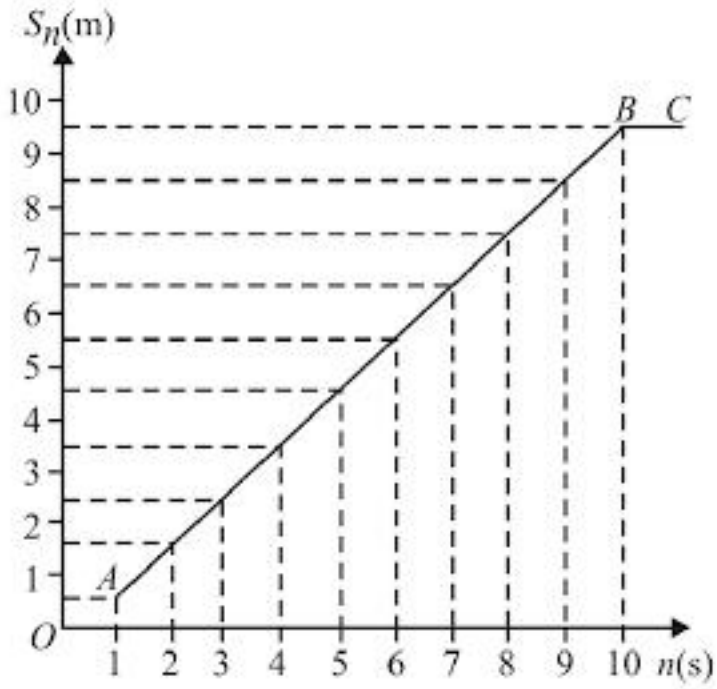


Fig. 3.013

Ans. Here $v_0 = 0$; $a = 1$ m/s^2

Distance covered in the nth second is

$$S_{nth} = v_0 + \frac{a}{2}(2n-1)$$

or $$S_{nth} = 0 + \frac{1}{2}(2n-1) = 0.5(2n-1) \quad ...(i)$$

Putting n = 1, 2, 3, ... in eq. (i), we get the following table of S_{nth} versus n.

n	1	2	3	4	5	6	7	8	9	10
S_{nth}	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5

When we draw the graph between S_{nth} and n [See Fig. 3.013], we get straight line AB because $S_{nth} \propto n$. After 10 s, the three wheeler moves with uniform velocity so that graph is a straight line BC parallel to time axis.

Q.24. A boy standing on a stationary lift (open from above) throws a ball upwards with maximum initial speed he can, equal to 49 m/s. How much time does the ball take to return to his hands ? If the lift starts moving up with a uniform speed of 5 m/s and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ?

Ans. Let the time taken by the ball to reach the highest point be t. Then,

Now $v_0 = 49$ m/s ; $v = 0$; $g = -9.8$ m/s^2

$v = v_0 + gt$ or $0 = 49 - 9.8t$ $\therefore$ $t = 49/9.8 = 5$ s

Since time taken for upward journey is equal to the time taken for downward journey,

$\therefore$ Total time taken $= 2t = 2 \times 5 =$ **10 s**

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When the lift starts moving upward with uniform speed of 5 ms^{-1}, there is no change of relative velocity of ball *w.r.t.* boy. Therefore, the total time taken will remain the same *i.e.*, **10 s.**

Q.25. On a long horizontal moving belt (Fig. 3.014), a child runs to and fro with a speed of 9 km/h (with respect to belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km/h. For an observer on stationary platform outside, what is the (*a*) speed of the child running in the direction of motion of the belt (*b*) speed of the child running opposite to the direction of motion of the belt and (*c*) time taken by the child in cases (*a*) and (*b*)? Which of the answers alter, if motion is viewed by one of the parents?

Ans. Fig. 3.014 shows the conditions of the problem. Considering that positive direction of X-axis is from left to right, we have,

v_{BE} = Speed of belt *w.r.t.* earth = 4 km/h

v_{CB} = Speed of child *w.r.t.* belt = 9 km/h

(*a*) **When child runs in the direction of motion of belt.** In this case, v_{BE} = +4 km/h and v_{CB} = +9 km/h.

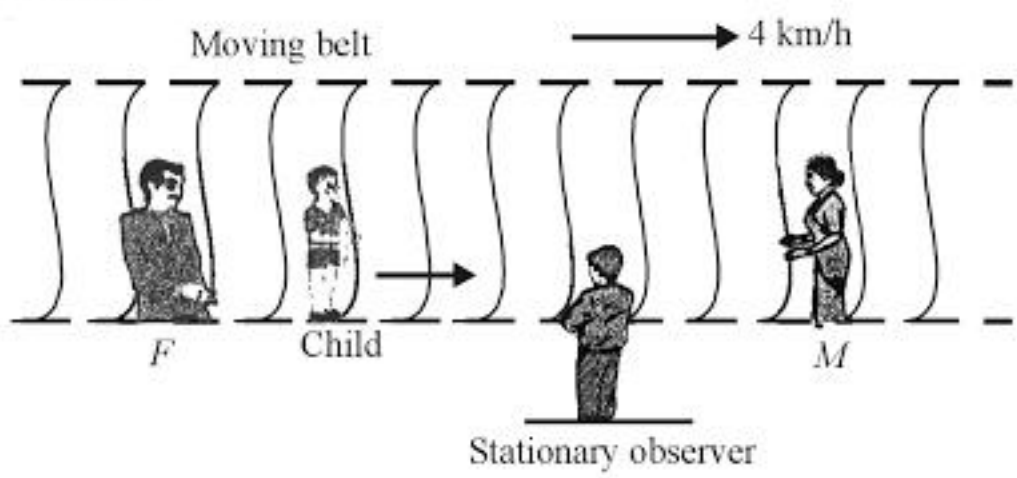


Fig. 3.014

Speed of child *w.r.t.* earth, $v_{CE} = v_{CB} + v_{BE} = (+9) + (+4) =$ **13 km/h**

(*b*) **When child runs opposite to the direction of belt.** In this case, v_{BE} = +4 km/h ; v_{CB} = –9 km/h.

Speed of child *w.r.t.* earth, $v_{CE} = v_{CB} + v_{BE} = (-9) + (+4) =$ **–5 km/h**

The negative sign shows that the child appears to be moving from right to left with a speed of 5 km/h.

(*c*) **Time taken.** Distance between parents = 50 m

Since parents and child are in the same frame of reference (*i.e.*, moving belt), the speed of the child as observed by the stationary observer will be 9 km/h (= 2.5 m/s) for the either direction of motion of the child.

$\therefore$ Time taken in cases (*a*) and (*b*) = 50/2.5 = **20 s**

If motion is viewed by one of the parents, the speed of the child is 9 km/h in either direction. Therefore, answers (*a*) and (*b*) will alter. However, answer (*c*) remains unaltered.

Q.26. Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms^{-1} and 30 ms^{-1}. Verify that the graph shown in Fig. 3.015 correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take $g = 10\ ms^{-2}$. Give the equations for the linear and curved parts of the plot.

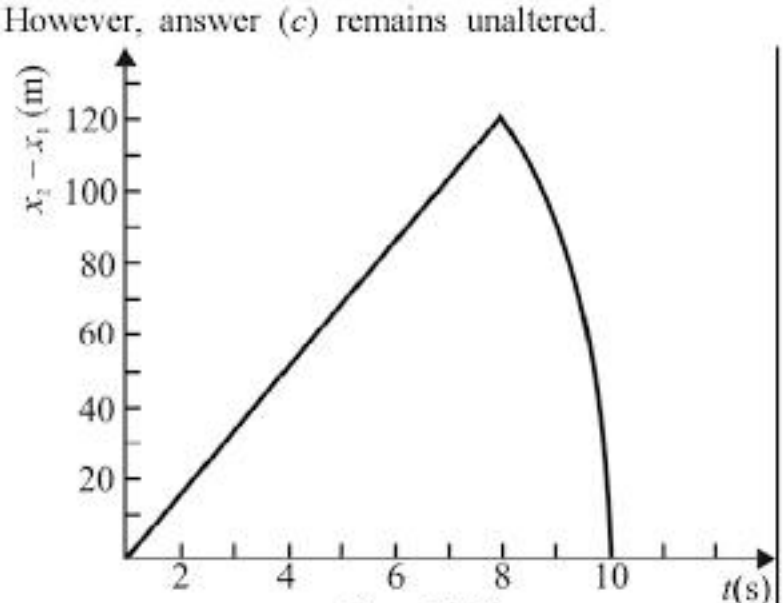


Fig. 3.015

Ans. For a uniformly accelerated motion, the position x of a body in time t is given by ;

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$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

Motion of first stone. $x_0 = 200$ m ; $v_0 = 15$ ms^{-1} ; $a = g = -10$ ms^{-2}

If x_1 is the position (height above the ground) of the first stone after time t, then,

$$x_1 = 200 + 15t + \frac{1}{2}(-10)t^2 \quad \text{or} \quad x_1 = 200 + 15t - 5t^2 \qquad ...(i)$$

The time after which the first stone hits the ground can be found by putting $x_1 = 0$ in eq. (*i*).

$$0 = 200 + 15t - 5t^2 \quad \text{or} \quad t^2 - 3t - 40 = 0 \quad \therefore \; t = 8 \text{ s}$$

Motion of second stone. $x_0 = 200$ m ; $v_0 = 30$ ms^{-1} ; $a = g = -10$ ms^{-2}

If x_2 is the position (height above the ground) of the second stone after time t, then,

$$x_2 = 200 + 30t + \frac{1}{2}(-10)t^2 \quad \text{or} \quad x_2 = 200 + 30t - 5t^2 \qquad ...(ii)$$

The time after which the second stone hits the ground can be found by putting $x_2 = 0$ in eq. (*ii*) *i.e.*,

$$0 = 200 + 30t - 5t^2 \quad \text{or} \quad t^2 - 6t - 40 = 0 \quad \therefore \; t = 10 \text{ s}$$

From eqs. (*i*) and (*ii*), $x_2 - x_1 = 15t$ ...(*iii*)

Conclusions. The above calculations verify the correctness of the graph in Fig. 3.015 as shown below.

(*i*) Eq. (*iii*) (*i.e.*, $x_2 - x_1 = 15t$) describes the relative position of second stone *w.r.t.* first with time till the first stone hits the ground. Since first stone reaches the ground after 8 seconds, it means that upto 8 s $(x_2 - x_1) \propto t$ or graph between $(x_2 - x_1)$ and t is a straight line.

(*ii*) After 8 s, only the second stone will be in motion for 2 s. Therefore, from 8 s to 10 s, the motion of the second stone will be governed by the equation $x_2 = 200 + 30t - 5t^2$. For this reason graph is curved from 8 s to 10 s.

Q.27. The speed-time graph of a particle moving along a fixed direction is shown in Fig. 3.016. Obtain the distance travelled by the particle between (*a*) $t = 0$ to 10 s (*b*) $t = 2$ to 6 s. What is the average speed of the particle over the intervals (*a*) and (*b*)?

Ans. (*a*) The distance travelled by the particle from 0 to 10 s

= Area of ΔOAB from 0 to 10 s

$$= \frac{1}{2} \times 10 \times 12 = \mathbf{60 \text{ m}}$$

Average speed = 60/10 = **6 ms^{-1}**

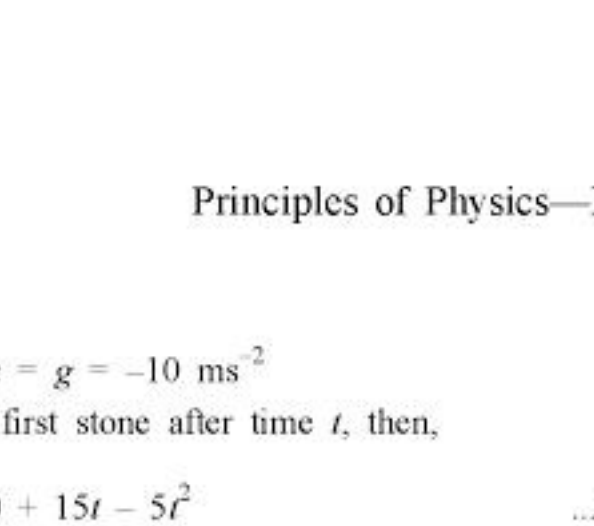


Fig. 3.016

(*b*) From time $t = 0$ s to $t = 5$ s, the particle moves with uniform acceleration a_1 given by ;

$$a_1 = \frac{v - v_0}{t} = \frac{12 - 0}{5} = 2.4 \text{ m/s}$$

Velocity of particle after 2 s is

$$v_1 = v_0 + a_1 \times 2 = 0 + 2.4 \times 2 = 4.8 \text{ m/s}$$

Distance travelled by the particle from $t = 2$ s to $t = 5$ s is

$$S_1 = v_1 t_1 + \frac{1}{2} a_1 t_1^2 \qquad \text{(Here } t_1 = 5 - 2 = 3\text{s)}$$

$$= 4.8 \times 3 + \frac{1}{2}(2.4) \times 3^2 = 25.2 \text{ m}$$

From time $t = 5$ s to $t = 10$ s, the particle moves with uniform retardation given by ;

$$a_2 = \frac{0 - 12}{5} = -2.4 \text{ m/s}^2$$

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For the motion of the particle from $t = 5$ s to $t = 6$ s, we have,

$$v_2 = 12 \text{ ms}^{-1},\ a_2 = -2.4 \text{ m/s}^2;\ t_2 = 1 \text{ s}$$

$\therefore$ Distance travelled by the particle from $t = 5$ s to $t = 6$ s is

$$S_2 = v_2 t_2 + \frac{1}{2} a_2 t_2^2$$

$$= 12 \times 1 + \frac{1}{2}(-2.4) \times 1^2 = 10.8 \text{ m}$$

$\therefore$ Total distance travelled, $S = S_1 + S_2 = 25.2 + 10.8 = \mathbf{36\ m}$

$$\text{Average speed} = \frac{36}{6-2} = \frac{36}{4} = \mathbf{9\ ms^{-1}}$$

Q.28. The velocity-time graph of a particle in one dimensional motion is shown in Fig. 3.017. Which of the following formulae are correct for describing the motion of the particle over the time interval t_1 and t_2.

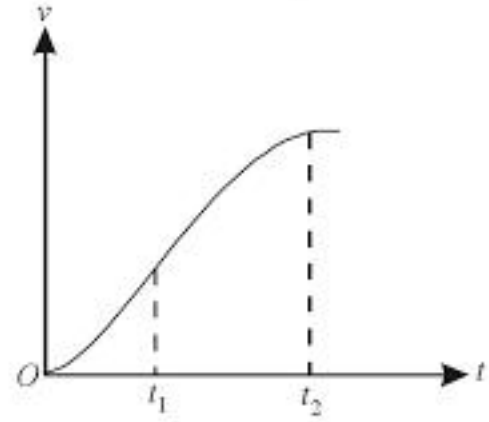


Fig. 3.017

(*i*) $x(t_2) = x(t_1) + v(t_1)\,(t_2 - t_1) + \frac{1}{2} a (t_2 - t_1)^2$

(*ii*) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(*iii*) $v_{av} = \{r(t_2) - r(t_1)\}/(t_2 - t_1)$

(*iv*) $a_{av} = \{v(t_2) - v(t_1)\}/(t_2 - t_1)$

(*v*) $x(t_2) = x(t_1) + v_{av}(t_2 - t_1) + \frac{1}{2} a_{av} (t_2 - t_1)^2$

(*vi*) $x(t_2) - x(t_1)$ = area under the $v - t$ curve bounded by the time axis and dotted line shown

Ans. If we look at the $v - t$ graph over the time interval t_1 and t_2, we find that slope is neither constant nor uniform. Therefore, relations (*i*), (*ii*) and (*v*) are not correct. However, relations (*iii*), (*iv*) and (*vi*) are correct.

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Unit III

KINEMATICS

(Description of motion in two and three dimensions)

- **Vectors**
- **Motion in a Plane**
- **N.C.E.R.T. Textbook Exercises**

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8

Vectors

INTRODUCTION

A particle confined to a straight line (one dimensional motion) can move in only two directions *viz.* either upward or downward from the selected origin or towards right or left from the origin. We can take the motion of the particle to be positive in one of these directions and negative in the other direction. Therefore, in one dimensional motion, the vector quantities like displacements, velocities and accelerations can have either positive or negative values. We can use simple arithmetic to add or subtract vector quantities in this case. However, the situation is not that simple in two or three dimensional motion. For example, in case of two dimensional motion, the particle moves in a plane –say XY plane. The position of the particle is described by two numbers *viz.* x and y coordinates. Suppose a particle is at point P_1 (x_1, y_1) at any instant and after some time it is at another point P_2 (x_2, y_2). The vectors $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$ (O is the origin of the coordinate system) are not along the same line as was the case with one dimensional motion. To carry out the addition or subtraction of vectors $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$, we require special methods. This is called *vector algebra*. Therefore, in order to study two dimensional motion (*i.e.,* motion in a plane) and three dimensional motion (*i.e.,* motion in space), we need the help of vector algebra. In this chapter, we shall discuss the various properties of vectors.

8.1. VECTOR

The physical quantities in physics are divided into two groups *viz. scalars* and *vectors.* Those physical quantities which have magnitude only are called scalars *e.g.,* mass, length, temperature, speed *etc.* The physical quantities which have magnitude and direction are called vectors *e.g.,* displacement, velocity, acceleration *etc.* If a physical quantity has both magnitude and direction but does not obey the laws of vector addition, it will not be called a vector. For example, electric current in a wire has both magnitude and direction but it does not obey the laws of vector addition. Therefore, electric current is not a vector.

A physical quantity that has both magnitude and direction and also obeys the laws of vector addition is called a vector.

A vector is represented *graphically* by a straight line with an arrowhead as shown in Fig. 8.1. The length of line (OA) represents the magnitude of the vector (on suitable scale) and the arrowhead indicates its direction.

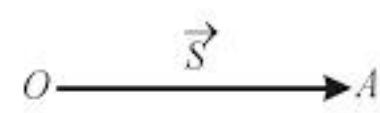


Fig. 8.1

Thus in Fig. 8.1, displacement vector $(\vec{S})$ is represented graphically. The starting point (O) of the arrowed line is called **tail of the vector** and the end (A) of the arrowed line is called **head** or **tip of the vector**.

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In *writing*, a vector is represented by a single letter with an arrow on it. Thus in Fig. 8.1, displacement vector will be written as $\vec{S}$. The magnitude of the vector is represented by the same letter used for the vector without arrow. For example, the magnitude of displacement vector $\vec{S}$ is represented as S or $|\vec{S}|$.

Discussion. The following points are worth noting :

(*i*) The magnitude of a vector is a scalar and is always positive.

(*ii*) The magnitude of a vector is also called *modulus* of the vector and is represented by enclosing the vector symbol between two vertical lines. For example, the modulus of displacement vector $\vec{S}$ will be represented as $|\vec{S}|$.

(*iii*) Vectors can be added, subtracted and multiplied. *However, division of a vector by another vector is not a valid operation in vector algebra.* It is because the division of a vector by a direction is not possible.

8.2. SOME DEFINITIONS IN VECTOR ALGEBRA

(*i*) Equal vectors. *The two vectors are said to be equal if they have the same magnitude and the same direction.*

In Fig. 8.2, the two vectors $\vec{A}$ and $\vec{B}$ are equal vectors because they have the same magnitude and the same direction. For two vectors to be equal, it is not necessary that their tails should have the same starting location. Equal vectors may have different locations but they must have the same magnitude and the same direction. Thus in Fig. 8.2(*i*), the two equal vectors $\vec{A}$ and $\vec{B}$ have the same starting location (O). However, in Fig. 8.2(*ii*), the two equal vectors $\vec{A}$ and $\vec{B}$ have different starting locations (O). *This property of equal vectors is important because we can shift a vector from one location to another in a diagram provided its magnitude and direction are not changed.*

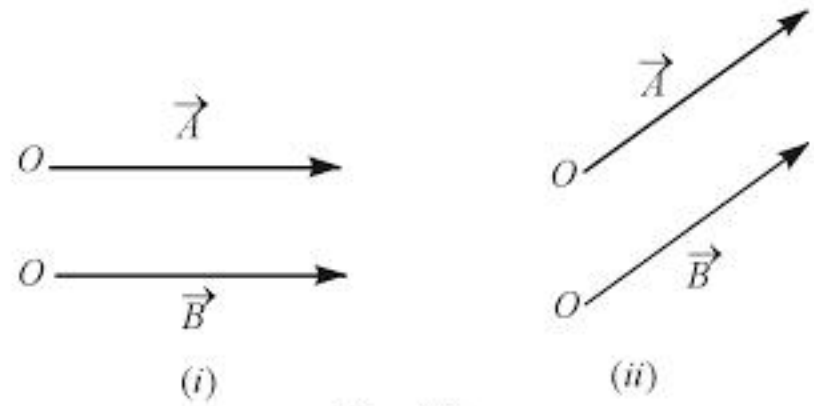


Fig. 8.2

(*ii*) Negative of a vector. *A vector is said to be negative vector of a given vector if its magnitude is the same as that of the given vector but its direction is opposite.*

The negative vector of $\vec{A}$ is represented as $-\vec{A}$ as shown in Fig. 8.3. Note that vectors $\vec{A}$ and $-\vec{A}$ have the same magnitude but opposite directions. In other words, the vectors have the same length but opposite directions. In order to find the negative of a vector, we merely reverse its arrowhead. Clearly, the addition of a vector to its own negative vector will result in zero magnitude *i.e.*,

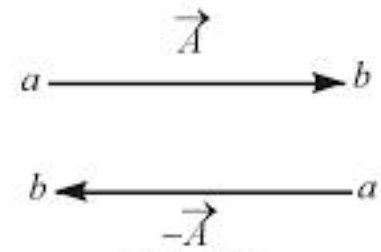


Fig. 8.3

$$\vec{A} + (-\vec{A}) = 0$$

This also applies to ordinary numbers. For example, the negative of 3 is –3 and their sum $= 3 + (-3) = 0$.

(*iii*) Unit vector. In vector algebra, it is often useful to represent a vector by its magnitude and a *unit vector*. A unit vector is a dimensionless vector that has a magnitude of 1 and has the same direction as that of the given vector.

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A unit vector of the given vector is a vector of unit magnitude and has the same direction as that of the given vector.

For example, any vector $\vec{A}$ can be written as :

$$\vec{A} = A\hat{A}$$

Here A is the magnitude of $\vec{A}$ and $\hat{A}$ is the unit vector whose magnitude is 1 and direction is the same as that of $\vec{A}$. Note that *unit vector* $\hat{A}$ *is read as 'A cap' or 'A hat'.*

A unit vector is used to specify a given direction. Unit vectors have no other physical significance. Vectors can be conveniently written in terms of unit vectors which point along the chosen coordinate axes. In rectangular coordinate system, these unit vectors are called $\hat{i}$, $\hat{j}$ and $\hat{k}$; they point respectively along the positive X, Y and Z axes as shown in Fig. 8.4. Since unit vectors have magnitude of 1, they may be used to give direction to vectors. For example, if a force (vector) of 6 N is acting along the $+X$ direction, we can represent the force as $6\hat{i}$ N. Similarly, if an object has a velocity (vector) of $-5\hat{j}$ ms^{-1}, it means that magnitude of velocity is 5 ms^{-1} and its direction is along the –ve Y direction. Note that the only purpose of unit vectors is to indicate the direction of a vector.

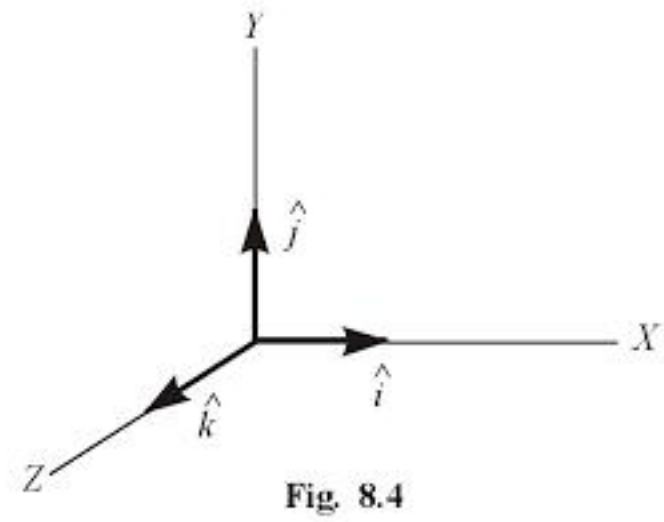


Fig. 8.4

(*iv*) Zero vector. *A vector that has zero magnitude is called zero vector or null vector.*

A zero vector or null vector is represented by $\vec{0}$ (arrow over the number zero). There is no need to specify the direction of a zero vector as its length is zero. The result of addition of a vector to its own negative vector is a zero vector *i.e.*, $\vec{A} + (-\vec{A}) = \vec{0}$. Similarly, when a vector is multiplied by zero, the result is a zero vector *i.e.*, $0(\vec{A}) = \vec{0}$.

Examples. (*a*) The velocity vector of a stationary object is a zero vector.

(*b*) The acceleration vector of an object moving with uniform velocity is a zero vector.

(*c*) The position vector of the origin of the coordinate axes is a zero vector.

8.3. POSITION VECTOR AND DISPLACEMENT VECTOR IN A PLANE

Consider the motion of an object in the XY plane with origin at O. Suppose at any time t_1, the object is at point $P_1(x_1, y_1)$ and at time t_2, it is at point $P_2(x_2, y_2)$.

1. Position vector. *The* ***position vector*** *of an object is the vector from the origin O of the coordinate system to the position of the object.*

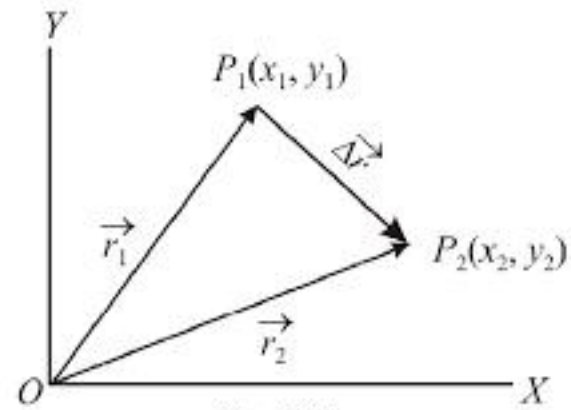


Fig. 8.5

Thus in Fig. 8.5, the position vector of the object at point P_1 is $\overrightarrow{OP_1}$ $(=\vec{r}_1)$ while the position vector of the object at point P_2 is $\overrightarrow{OP_2}$ $(=\vec{r}_2)$. The position vector of an object provides two important informations :

(*i*) *It gives straight line distance of the object from the origin O (starting point).*

(*ii*) *It gives the direction of the object w.r.t. origin.*

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2. Displacement vector. Displacement is a vector quantity and is called displacement vector. We know that the displacement of an object is the straight line distance (shortest distance) between the initial position (P_1) and the final position (P_2) of the object. Thus in Fig. 8.5, $\overrightarrow{P_1P_2}$ $(= \overrightarrow{\Delta r})$ is the displacement vector. The straight line distance P_1P_2 is the magnitude of the displacement vector and the direction of the displacement vector is from P_1 to P_2.

Hence displacement vector is a vector that points from object's initial position to its final position and whose magnitude is equal to the straight line distance between the two points.

$$\text{Position vector, } \overrightarrow{OP_1} = x_1\hat{i} + y_1\hat{j}$$

$$\text{Position vector, } \overrightarrow{OP_2} = x_2\hat{i} + y_2\hat{j}$$

$$\therefore \quad \text{Displacement vector, } \overrightarrow{P_1P_2} = \text{Final position} - \text{Initial position}$$

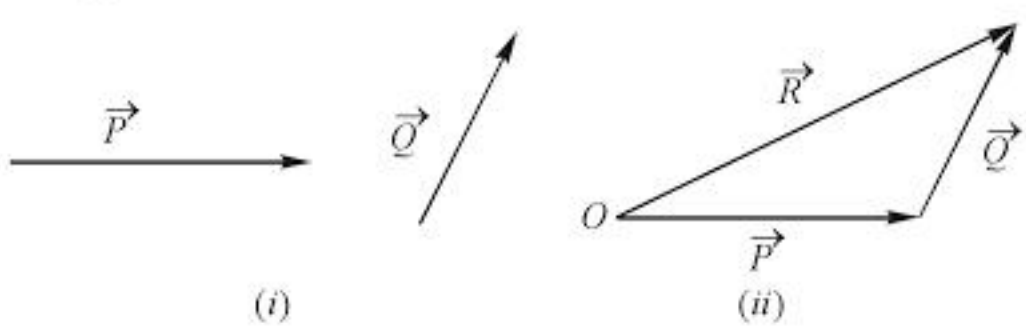
$$= \overrightarrow{OP_2} - \overrightarrow{OP_1}$$

$$= (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$$

$$\therefore \quad \overrightarrow{P_1P_2} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

8.4. ADDITION OF VECTORS

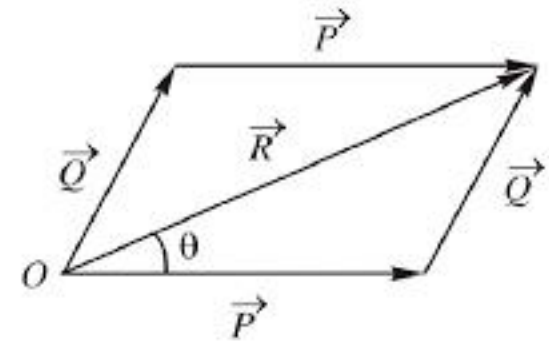
Vector addition can be done most easily by a graphical method. Suppose we want to add two vectors $\vec{P}$ and $\vec{Q}$ (suppose these are displacement vectors) as shown in Fig. 8.6(*i*). To add them graphically, we place the tail of vector $\vec{Q}$ at the head (or tip) of vector $\vec{P}$ as shown in Fig. 8.6(*ii*). Note that we can move a vector from one place to another but we cannot change the magnitude and direction of the vector while moving it. Having moved vector $\vec{Q}$, we draw another vector $\vec{R}$ from that tail of vector $\vec{P}$ to the head of vector $\vec{Q}$. This vector $\vec{R}$ represents the net displacement (or vector sum of $\vec{P}$ and $\vec{Q}$) and is called the **resultant** of vectors $\vec{P}$ and $\vec{Q}$. We can write the resultant in the form of mathematical equation as :



$$\vec{R} = \vec{P} + \vec{Q}$$

Hence the ***resultant vector*** *of two or more vectors is the single vector which produces the same effect as is produced by the individual vectors together.* The advantage of resultant vector is that we can replace a number of vectors by a single vector. This makes mathematical analysis a simple affair.

The method of vector addition described above is called *triangle law of vector addition.* An alternative graphical method for adding two vectors $\vec{P}$ and $\vec{Q}$ is shown in Fig. 8.7. In this construction, the tails of the vectors $\vec{P}$ and $\vec{Q}$ are together and the resultant vector $\vec{R}$ is the diagonal of the parallelogram formed with the vectors $\vec{P}$ and $\vec{Q}$ as the adjacent sides. This is called *parallelogram law of vector addition.*



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(*i*) Triangle law of vector addition. *If two vectors acting simultaneously at a point are represented in magnitude and direction by the two sides of a triangle taken in the same order, then third or closing side of the triangle taken in the opposite order represents their resultant in magnitude and direction.*

The triangle law of vector addition is illustrated in Fig. 8.6(*ii*). Here vectors $\vec{P}$ and $\vec{Q}$ are represented in magnitude and direction by the two sides of a triangle taken in the same order. The third or closing side of the triangle taken in the opposite order represents their resultant $\vec{R}$ in magnitude and direction.

$$\therefore \qquad \vec{R} = \vec{P} + \vec{Q}$$

(*ii*) Parallelogram law of vector addition. *If two vectors acting simultaneously at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then the diagonal of the parallelogram passing through that point represents their resultant in magnitude and direction.*

The parallelogram law of vector addition is shown in Fig. 8.7. Here vectors $\vec{P}$ and $\vec{Q}$ are represented in magnitude and direction by the adjacent sides of the parallelogram. Then diagonal of the parallelogram passing through point O represents their resultant $\vec{R}$ in magnitude and direction.

8.5. ADDITION OF MORE THAN TWO VECTORS GRAPHICALLY

The *tail-to-tip method can be used to add any number of vectors graphically. As stated earlier, in this method the vectors are placed tip to tail one at a time *i.e.*, tail of the second vector is placed on the tip of the first vector, tail of the third vector is placed on the tip of the second vector and so on. The magnitudes and directions of the vectors should not be changed as they are moved about. The resultant vector $\vec{R}$ is represented in magnitude and direction from the tail of the first vector to the tip of the last vector. In other words, $\vec{R}$ is the closing side of the figure (of vectors) taken in the opposite order.

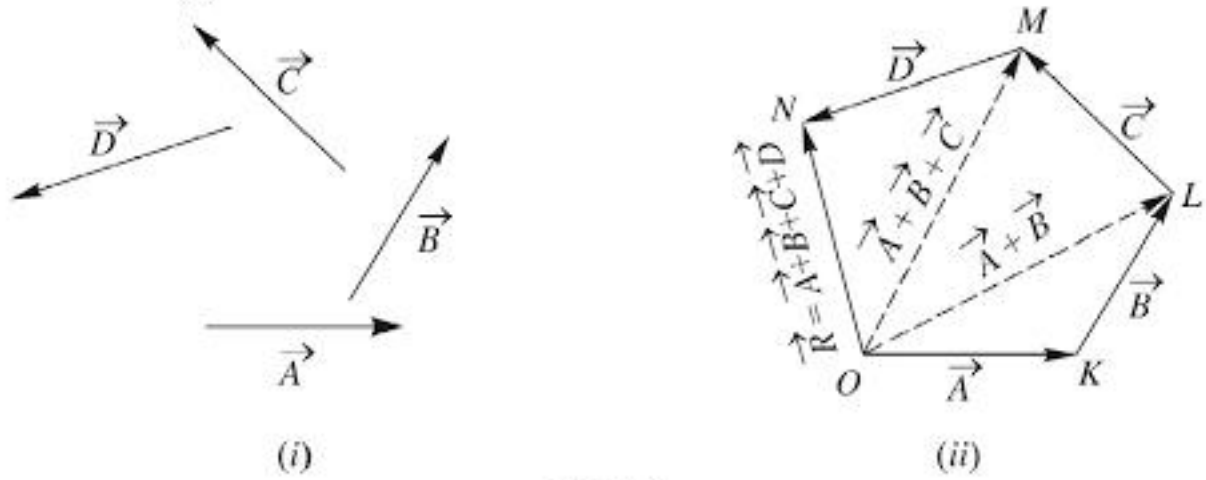


Fig. 8.8

Let us illustrate the tail-to-tip method of vector addition considering four vectors $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ [See Fig. 8.8 (*i*)]. Applying tail-to-tip method, these vectors are represented in magnitude and direction by the sides $\overrightarrow{OK}$, $\overrightarrow{KL}$, $\overrightarrow{LM}$ and $\overrightarrow{MN}$ of the regular polygon respectively as shown in Fig. 8.8 (*ii*). Then, the resultant $\vec{R}$ is represented in magnitude and direction from the tail of the first vector to the tip of the last vector. In other words, the closing side of the polygon taken in the opposite order represents the resultant $\vec{R}$ in magnitude and direction *i.e.*, $\vec{R} = \overrightarrow{ON}$. This provides

* Note that tail-to-tip method has been explained in Art. 8.4.

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us a useful rule for adding more than two vectors graphically. This is called **polygon law of vector addition**. It may be stated as under:

If a number of vectors acting simultaneously at a point are represented in magnitude and direction by the sides of a polygon taken in the same order, then closing side of the polygon taken in opposite order represents the resultant in magnitude and direction.

Proof. A little thought will show that polygon law of vector addition is an extension of triangle law of vector addition. According to triangle law of vector addition, $\overrightarrow{OL}$ is the resultant of $\vec{A}$ and $\vec{B}$ *i.e.*,

$$\overrightarrow{OL} = \vec{A} + \vec{B}$$

Likewise, $\overrightarrow{OM}$ is the resultant of $\overrightarrow{OL}$ and $\vec{C}$ *i.e.*,

$$\overrightarrow{OM} = \overrightarrow{OL} + \vec{C} = \vec{A} + \vec{B} + \vec{C}$$

$$\therefore \quad \overrightarrow{OM} = \vec{A} + \vec{C} + \vec{B}$$

Again $\overrightarrow{ON}$ is the resultant of $\overrightarrow{OM}$ and $\vec{D}$.

$$\therefore \quad \overrightarrow{ON} = \overrightarrow{OM} + \vec{D} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

$$\therefore \quad \vec{R} = \overrightarrow{ON} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

8.6. ANALYTICAL METHOD OF VECTOR ADDITION

The graphical method of adding vectors takes considerable time if done accurately. We can find the magnitude and direction of the resultant (or sum) of two vectors mathematically by using triangle or parallelogram law of vector addition.

1. Triangle law of vector addition. Let two vectors $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by the sides $\overrightarrow{OA}$ and $\overrightarrow{AC}$ of a triangle OAC taken in the same order. Then according to triangle law of vector addition, the third or closing side of the triangle taken in the opposite order represents the resultant $\vec{R}\ (=\overrightarrow{OC})$ in magnitude and direction. Let $\vec{R}$ make an angle α with $\vec{P}$ *i.e.*, [See Fig. 8.9] $\angle AOC = \alpha$.

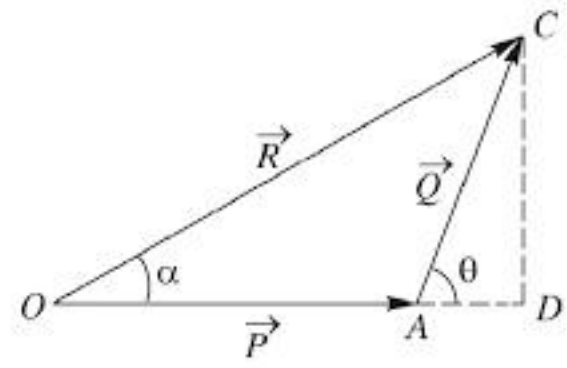


Fig. 8.9

(i) Magnitude of Resultant. From C, draw CD perpendicular to OA produced. Suppose $\angle CAD = \theta$. Note that θ is the angle between the two vectors. In the right angled triangle ODC,

$$(OC)^2 = (OD)^2 + (CD)^2 = (OA + AD)^2 + (CD)^2$$

$$\therefore \quad R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$$

[In right angled triangle ACD, $AD = Q\cos\theta$ and $CD = Q\sin\theta$]

$$= P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$$

$$= P^2 + Q^2(\sin^2\theta + \cos^2\theta) + 2PQ\cos\theta$$

$$= P^2 + Q^2 + 2PQ\cos\theta \qquad (\because \ \sin^2\theta + \cos^2\theta = 1)$$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \ \ldots (i)$$

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(*ii*) Direction of resultant. From right angled triangle ODC,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\therefore \quad \tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} \qquad \ldots(ii)$$

Thus, the magnitude of the resultant is given by eq. (*i*) and the direction of $\vec{R}$ with $\vec{P}$ is given by eq. (*ii*).

2. Parallelogram law of vector addition. Let two vectors $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by the adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OB}$ of the parallelogam $OACB$ (See Fig. 8.10). Suppose the angle between the vectors is θ *i.e.*, $\angle AOB = \theta$. According to parallelogram law of vector addition, the diagonal $\overrightarrow{OC}$ represents the resultant $\vec{R}\ (= \overrightarrow{OC})$ in magnitude and direction. Suppose $\vec{R}$ makes angle α with $\vec{P}$ *i.e.*, $\angle AOC = \alpha$.

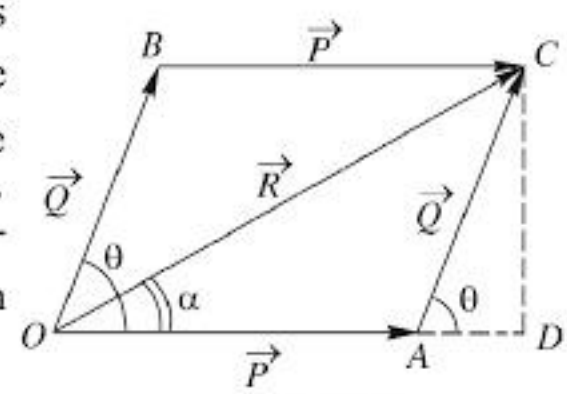


Fig. 8.10

(*i*) Magnitude of resultant. From C, draw CD perpendicular on OA produced. From geometry, $\angle DAC = \theta$. In right angled triangle ODC,

$$(OC)^2 = (OD)^2 + (CD)^2 = (OA + AD)^2 + (CD)^2$$

$$\therefore \quad R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2 \qquad [\because\ AD = Q\cos\theta\ ;\ CD = Q\sin\theta]$$

$$= P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$$

$$= P^2 + Q^2(\sin^2\theta + \cos^2\theta) + 2PQ\cos\theta$$

$$= P^2 + Q^2 + 2PQ\cos\theta$$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad \ldots(iii)$$

(*ii*) Direction of resultant. From right angled triangle ODC,

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\therefore \quad \tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} \qquad \ldots(iv)$$

Thus, the magnitude of the resultant is given by eq. (*iii*) and the direction of $\vec{R}$ with $\vec{P}$ is given by eq. (*iv*).

Different cases.

(*a*) ***When the vectors act along the same direction*** *i.e.*, $\theta = 0°$.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 0°} = \sqrt{(P + Q)^2} \quad \therefore \quad R = P + Q$$

$$\tan\alpha = \frac{Q\sin 0°}{P + Q\cos 0°} = \frac{Q(0)}{P + Q(1)} = 0 \quad \therefore \quad \alpha = 0°$$

Therefore, the magnitude of the resultant of two vectors in the same direction is equal to the sum of the magnitudes of two vectors and the direction of the resultant is along the direction of $\vec{P}$ *and* $\vec{Q}$.

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(*b*) ***When the vectors are at right angles*** *i.e.*, $\theta = 90°$.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90°} = \sqrt{P^2 + Q^2}$$

$$\tan\alpha = \frac{Q\sin 90°}{P + Q\cos 90°} = \frac{Q(1)}{P + Q(0)} = \frac{Q}{P} \quad \therefore \quad \tan\alpha = \frac{Q}{P}$$

(*c*) ***When the vectors act along opposite directions*** *i.e.*, $\theta = 180°$.

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180°} = \sqrt{(P - Q)^2}$$

$\therefore \quad R = P - Q$

$$\tan\alpha = \frac{Q\sin 180°}{P + Q\cos 180°} = \frac{Q(0)}{P + Q(-1)} = 0$$

$\therefore \quad \alpha = 0°$ or $180°$

Thus the magnitude of the resultant of two vectors acting in the opposite directions is equal to the difference in the magnitudes of the vectors. The resultant acts in the direction of the larger vector.

8.7. SOME PROPERTIES OF VECTORS

We now discuss some useful properties of vectors.

(*i*) Multiplication of a vector by a real number. *If we multiply a vector* $\vec{A}$ *by a real positive number n, we get the vector* $n\vec{A}$ *which has the same direction as* $\vec{A}$ *and magnitude nA i.e.,*

$$n\vec{A} = n(\vec{A})$$

Thus the magnitude of the vector becomes n times while its direction remains unchanged. For example, if we multiply vector $\vec{A}$ by 2, we get another vector $\vec{B}$ which has the same direction as that of $\vec{A}$ but twice the magnitude *i.e.*

$$\vec{B} = 2\vec{A}$$

If a vector is multiplied by a negative real number (i.e., –n), the magnitude of the vector becomes nA but direction is opposite to that of $\vec{A}$ *i.e.,*

$$-n(\vec{A}) = -n\vec{A}$$

For example, if we multiply vector $\vec{A}$ by –2, we get another vector $\vec{B}$ parallel to $\vec{A}$ but in opposite direction and of magnitude $2A$ *i.e.,*

$$\vec{B} = -2\vec{A}$$

(*ii*) Multiplication of a vector by a scalar. *If we multiply a vector* $\vec{A}$ *by a scalar S, the result is another vector with direction of* $\vec{A}$ *but magnitude SA. If S is negative,* $S\vec{A}$ *has a direction opposite to that of* $\vec{A}$.

If the quantities S and $\vec{A}$ both happen to have units, then vector $S\vec{A}$ is expressed in units indicated for such a product. For example, if $\vec{A}$ = 10 m/s north and S = 2 s, then $S\vec{A}$ = 10 m/s × 2 s = 20 m north.

(*iii*) Vector addition obeys commutative law. *According to this law, the resultant of the vectors remains the same in whatever order they may be added.*

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In order to find the vector sum of $\vec{A}$ and $\vec{B}$, we can add (vectorially) either $\vec{A}$ to $\vec{B}$ or $\vec{B}$ to $\vec{A}$ (See Fig. 8.11). Vectors $\vec{A} + \vec{B}$ and $\vec{B} + \vec{A}$ are the same because they are parallel and have the same magnitude. Their order in the addition does not matter.

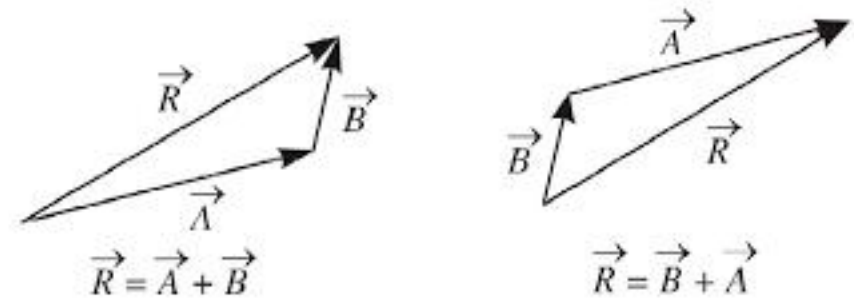


Fig. 8.11

$$\therefore \quad \vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

It is interesting to note that commutative law also holds for ordinary algebra because $3 + 2 = 2 + 3$.

(*iv*) Vector addition obeys associative law. *According to this law, the resultant of the vectors remains the same in whatever grouping they may be added.*

Suppose we are to find the vector sum of the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$. Then we can add $\vec{C}$ to the vector sum of $\vec{A} + \vec{B}$ or add $\vec{A}$ to the vector sum of $\vec{B} + \vec{C}$. Therefore, as shown in Fig. 8.12 :

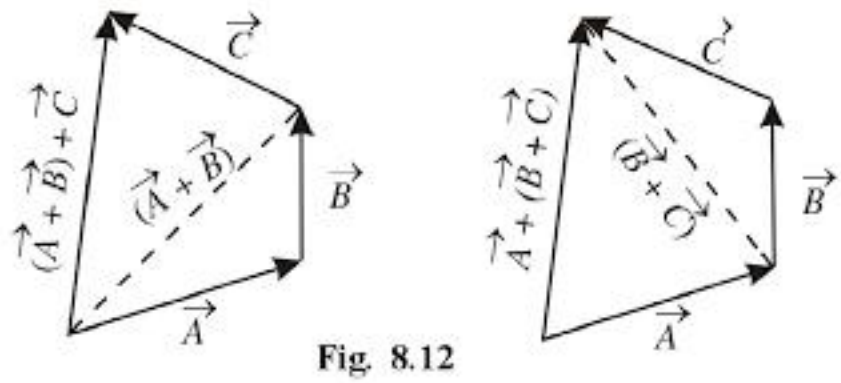


Fig. 8.12

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

This is quite logical. For the addition of any number of vectors, we can merely draw them in head-to-tail fashion. The sum of all vectors is then just a vector from the tail of the first vector to the head of the last vector. It is interesting to note that associative law also holds good for ordinary algebra because $(2 + 3) + 4 = 2 + (3 + 4)$.

Example 8.1. The angle between two vectors of equal magnitude is 120°. Prove that the magnitude of their resultant is equal to either of them.

Solution. $P = Q \; ; \; \theta = 120°$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{P^2 + P^2 + 2P^2\cos 120°}$$

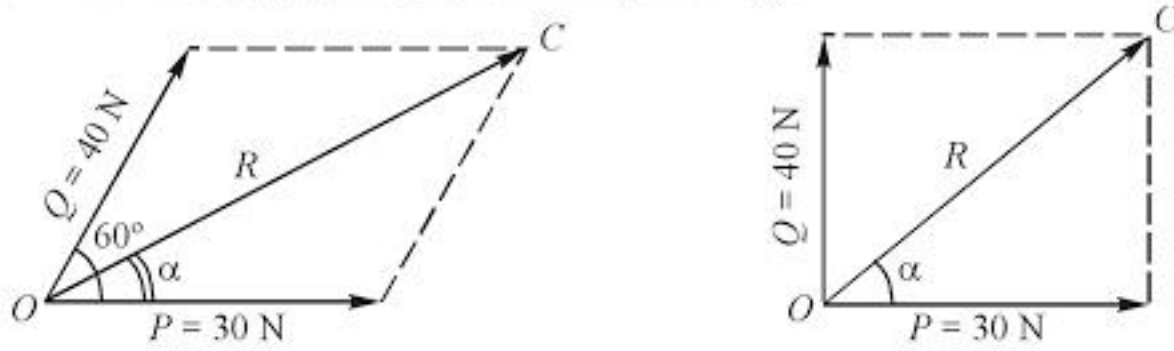
$$= \sqrt{2P^2 + 2P^2\left(\frac{-1}{2}\right)} = \sqrt{2P^2 - P^2} = P$$

$$\therefore \quad R = P$$

Example 8.2. Two forces of 30 N and 40 N are inclined to each other at an angle of 60°. Find their resultant. What will be the angle if the forces are inclined at right angles to each other?

Solution. $P = 30\ N \; ; \; Q = 40\ N$

When $\theta = 60°$. The situation is shown in Fig. 8.13 (*i*).



(*i*) Fig. 8.13 (*ii*)

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$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{(30)^2 + (40)^2 + 2\times 30\times 40\cos 60^\circ} = \mathbf{60.83\ N}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{40\sin 60^\circ}{30 + 40\cos 60^\circ} = 0.69 \quad \therefore \quad \theta = \tan^{-1} 0.69 = 34.6^\circ$$

Therefore, the resultant has a magnitude of 60.83 N and makes an angle of 34.6° with 30 N force.

When θ = 90°. The situation is shown in Fig. 8.13 (*ii*).

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{P^2 + Q^2 + 2PQ\cos 90^\circ}$$

$$= \sqrt{P^2 + Q^2} = \sqrt{(30)^2 + (40)^2} = 50\ \text{N}$$

$$\tan\alpha = \frac{Q}{P} = \frac{40}{30} = 1.33 \quad \therefore \quad \alpha = \tan^{-1} 1.33 = \mathbf{53.06^\circ}$$

Therefore, the resultant has a magnitude of 50 N and makes an angle of 53.06° with 30 N force.

Example 8.3. The resultant of two equal forces acting at right angles to each other is 1414 N. Find the magnitude of each force.

Solution. $P = Q\ ;\ R = 1414\ \text{N}\ ;\ \theta = 90^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{P^2 + P^2 + 2P^2\cos 90^\circ} = \sqrt{2}\,P$$

$$\therefore \quad P = \frac{R}{\sqrt{2}} = \frac{1414}{\sqrt{2}} = 1000\ \text{N} \quad \therefore \quad P = Q = \mathbf{1000\ N}$$

Example 8.4. The magnitudes of two vectors are equal and the angle between them is θ. Show that their resultant divides angle θ equally.

Solution. $P = Q\ ;\ \theta = \theta$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{P\sin\theta}{P + P\cos\theta} = \frac{\sin\theta}{1 + \cos\theta}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1 + \left(2\cos^2\frac{\theta}{2} - 1\right)} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2} \quad \therefore \quad \alpha = \frac{\theta}{2}$$

We can also find the magnitude of the resultant.

$$R^2 = P^2 + P^2 + 2P^2\cos\theta = 2P^2 + 2P^2\cos\theta = 2P^2(1 + \cos\theta)$$

$$= 2P^2\left[1 + \left(2\cos^2\frac{\theta}{2} - 1\right)\right] = 4P^2\cos^2\frac{\theta}{2}$$

$$\therefore \quad R = 2P\cos\frac{\theta}{2}$$

These results are very important.

Example 8.5. The sum of the magnitudes of two forces acting at a point is 18 N and the magnitude of the resultant is 12 N. If the resultant is at 90° with the force of smaller magnitude, what are the magnitudes of forces?

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Solution. Fig. 8.14 shows the conditions of the problem. Here $\vec{P}$ and $\vec{Q}$ are the two forces and $\vec{R}$ is their resultant. It is assumed that $\vec{P}$ is the smaller force.

It is given that $P + Q = 18$ N and $R = 12$ N. Referring to Fig. 8.14, we have,

$$R^2 = Q^2 - P^2 \quad \text{or} \quad R^2 = (Q + P)(Q - P)$$

$$\therefore \quad 144 = 18(Q - P) \quad \text{or} \quad Q - P = 8 \text{ N}$$

Now $P + Q = 18$ N and $Q - P = 8$ N. Therefore, $Q =$ **13 N** and $P =$ **5 N.**

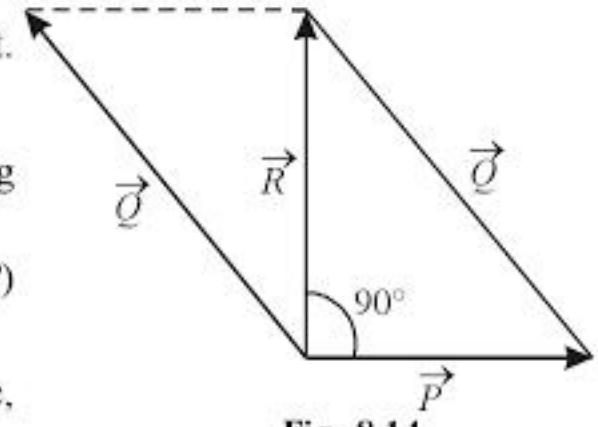


Fig. 8.14

Example 8.6. A particle has a displacement of 12 m toward east and 5 m toward north and then 6 m vertically upward. Find the magnitude of the resultant displacement.

Solution. Fig. 8.15 shows the conditions of the problem. $\overrightarrow{OB}$ is the resultant of $\overrightarrow{OA}$ and $\overrightarrow{AB}$.

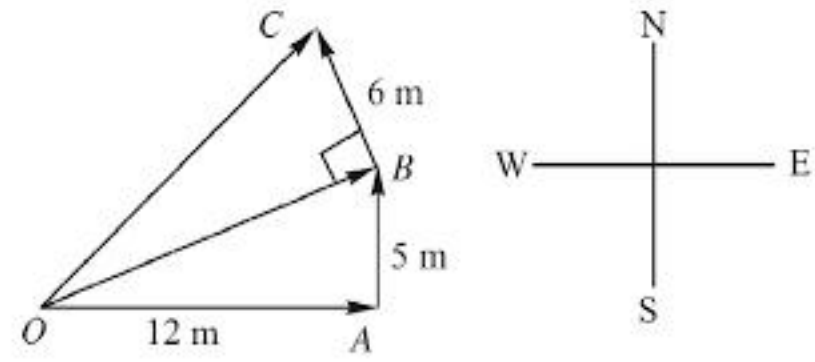


Fig. 8.15

$$\therefore \quad OB = \sqrt{(OA)^2 + (AB)^2} = \sqrt{(12)^2 + (5)^2} = 13 \text{ m}$$

Again $\overrightarrow{OC}$ is the resultant of $\overrightarrow{OB}$ and $\overrightarrow{BC}$.

$$\therefore \quad OC = \sqrt{(OB)^2 + (BC)^2} = \sqrt{(13)^2 + (6)^2} = \sqrt{205} = \mathbf{14.32 \text{ m}}$$

Example 8.7. A person moves 30 m north, then 20 m east and finally $30\sqrt{2}$ m south-west. What is the displacement from the original position?

Solution. Figure 8.16 shows the conditions of the problem. Resultant displacement is given by ;

$$\vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$$

Let $\hat{i}$ and $\hat{j}$ be the unit vectors along east (E) and north (N) respectively. Then,

$$\vec{S}_1 = 30\hat{j} \; ; \; \vec{S}_2 = 20\hat{i} \; ;$$

$$\vec{S}_3 = -30\sqrt{2}(\cos 45°\hat{i} + \sin 45°\hat{j}) = -30\hat{i} - 30\hat{j}$$

$$\therefore \quad \vec{S} = (30\hat{j}) + (20\hat{i}) + (-30\hat{i} - 30\hat{j}) = -10\hat{i}$$

Therefore, the resultant displacement is **10 m west.**

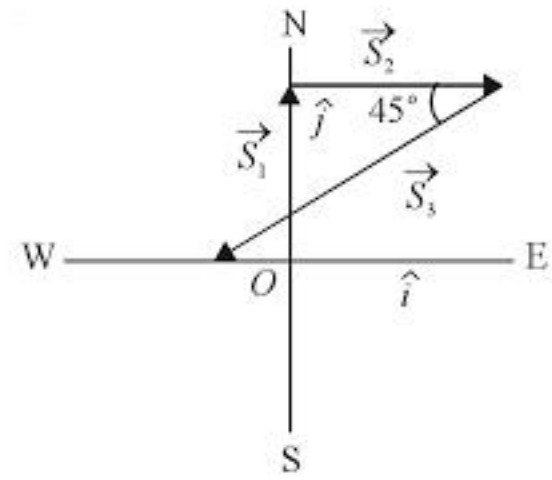


Fig. 8.16

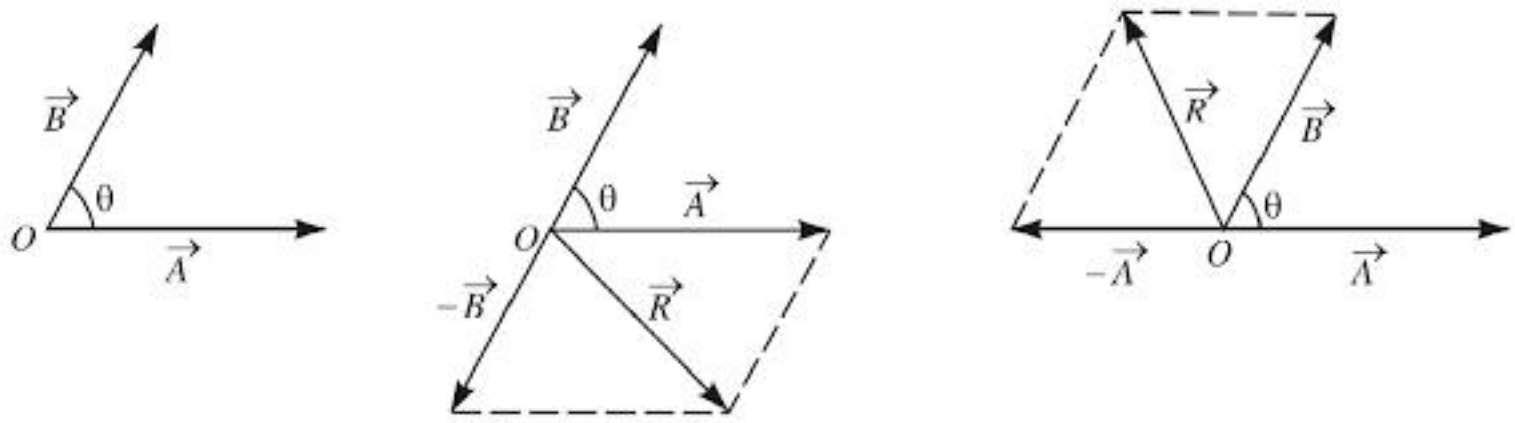
8.8. SUBTRACTION OF VECTORS

In order to subtract vector $\vec{B}$ from vector $\vec{A}$, first reverse the direction of $\vec{B}$, thus producing $-\vec{B}$. Then add $\vec{A}$ and $(-\vec{B})$ by parallelogram law of vector addition.

$$\therefore \quad \vec{R} = \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$$

Consider two vectors $\vec{A}$ and $\vec{B}$ of the same kind and inclined to each other at an angle θ as shown in Fig. 8.17. (*i*).

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(*i*) (*ii*) (*iii*)

Fig. 8.17

(*i*) In order to find $\vec{A}-\vec{B}$, reverse the direction of $\vec{B}$, thus producing $-\vec{B}$ as shown in Fig. 8.17 (*ii*). Then find the sum of $\vec{A}$ and $-\vec{B}$ by parallelogram law of vector addition. The required difference is $\vec{R}$ and is shown in Fig. 8.17 (*ii*).

$$\therefore \quad \vec{R} = \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$$

(*ii*) In order to find $\vec{B}-\vec{A}$, reverse the direction of $\vec{A}$, thus producing $-\vec{A}$ as shown in Fig. 8.17(*iii*). Then find the sum of $\vec{B}$ and $-\vec{A}$. The required difference is $\vec{R}$ and is shown in Fig. 8.17 (*iii*).

$$\therefore \quad \vec{R} = \vec{B} + (-\vec{A}) = \vec{B} - \vec{A}$$

8.9. RELATIVE VELOCITY IN TWO DIMENSIONS

Two bodies A and B moving in a plane (two dimensional motion) may not have their motions along the same line. Consequently, their velocities $\vec{v}_A$ and $\vec{v}_B$ will be inclined at some angle as shown in Fig. 8.18.

(*i*) In order to find the relative velocity of body A *w.r.t.* to body B $(\vec{v}_{AB})$, we superimpose a velocity $-\vec{v}_B$ on both the bodies. As a result, the body B is brought to rest. Then the resultant velocity of the body A is $\vec{v}_A - \vec{v}_B$ and is equal to $\vec{v}_{AB}$.

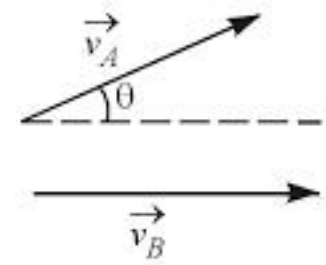


Fig. 8.18

$$\therefore \quad \vec{v}_{AB} = \vec{v}_A - \vec{v}_B = \vec{v}_A + (-\vec{v}_B)$$

Therefore, in order to find $\vec{v}_{AB}$, reverse the direction of $\vec{v}_B$ and find its sum with $\vec{v}_A$ by parallelogram law of vector addition.

(*ii*) Similarly, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

Again parallelogram law of vector addition can be used to find the sum of $\vec{v}_B$ and $-\vec{v}_A$.

8.10. MAGNITUDE AND DIRECTION OF RELATIVE VELOCITY

Consider two bodies A and B moving with velocities $\vec{v}_A$ and $\vec{v}_B$ respectively inclined at an angle θ.

(*i*) Relative velocity of *A w.r.t. B*. As discussed above, relative velocity of A *w.r.t.* B is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = \vec{v}_A + (-\vec{v}_B)$$

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In order to find $\vec{v}_{AB}$, we find the sum of $\vec{v}_A$ and $-\vec{v}_B$ by parallelogram law of vector addition. This is illustrated in Fig. 8.19 (*i*) where the vector sum of $\vec{v}_A$ and $-\vec{v}_B$ is $\vec{v}_{AB}(=\overrightarrow{OC})$.

Magnitude of $\vec{v}_{AB}$ is

$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_A v_B \cos(180° - \theta)}$$

or $$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

The angle α which $\vec{v}_{AB}$ makes with $-\vec{v}_B$ is

$$\tan\alpha = \frac{v_A \sin(180° - \theta)}{v_B + v_A \cos(180° - \theta)} \quad \text{or} \quad \tan\alpha = \frac{v_A \sin\theta}{v_B - v_A \cos\theta}$$

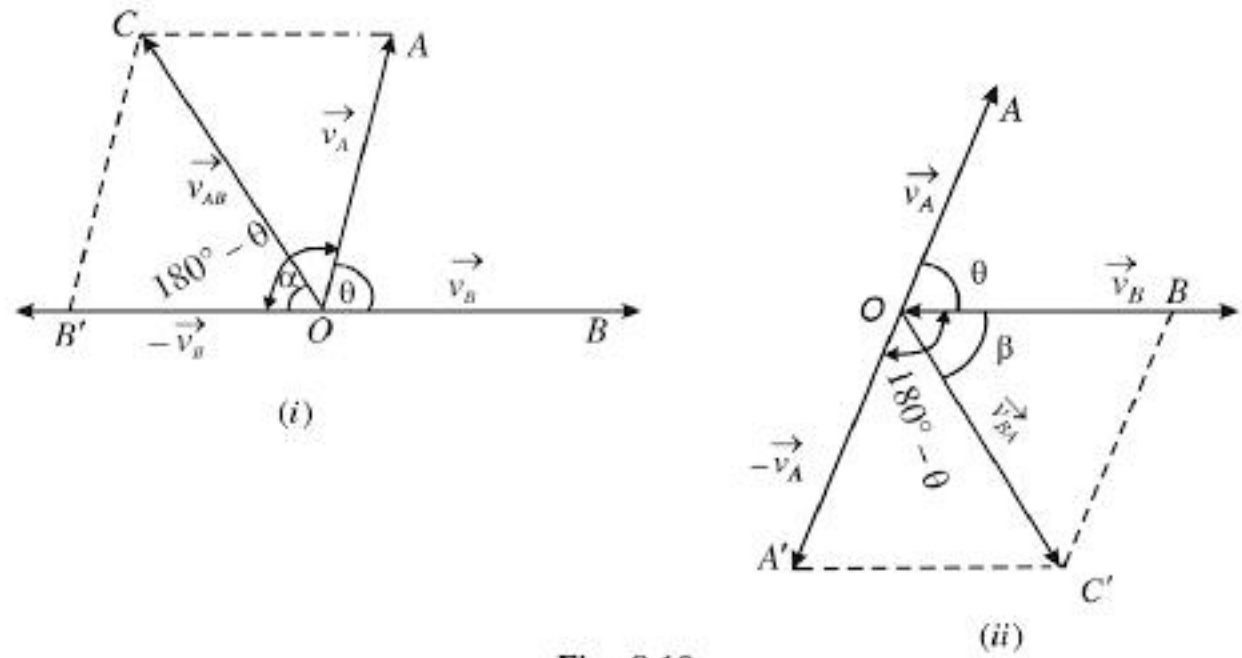


Fig. 8.19

(*ii*) Relative velocity of *B w.r.t. A*. The relative velocity of B *w.r.t.* A is

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = \vec{v}_B + (-\vec{v}_A)$$

In order to find $\vec{v}_{BA}$, we find the sum of $\vec{v}_B$ and $-\vec{v}_A$ by parallelogram law of vector addition. This is illustrated in Fig. 8.19 (*ii*) where the vector sum of $\vec{v}_B$ and $-\vec{v}_A$ is $\vec{v}_{BA}$ $(=\overrightarrow{OC'})$.

Magnitude of $\vec{v}_{BA}$ is

$$v_{BA} = \sqrt{v_A^2 + v_B^2 + 2v_A v_B \cos(180° - \theta)}$$

or $$v_{BA} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

The angle β which $\vec{v}_{BA}$ makes with $\vec{v}_B$ is

$$\tan\beta = \frac{v_A \sin(180° - \theta)}{v_B + v_A \cos(180° - \theta)}$$

or $$\tan\beta = \frac{v_A \sin\theta}{v_B - v_A \cos\theta}$$

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8.11. APPLICATIONS OF RELATIVE VELOCITY

We shall discuss a few applications of relative velocity in two dimensions.

(*i*) Boat to cross the river along shortest path. Fig. 8.20 shows river flowing from right to left. Point A is the starting point of the boat and point B is directly opposite to point A on the other side of the river. It is desired that boat should follow the shortest path AB. The river flow will carry the boat downstream as the boat crosses the river. In order that the boat follows the path AB, it should be rowed upstream making an angle θ with AB as shown in Fig. 8.20.

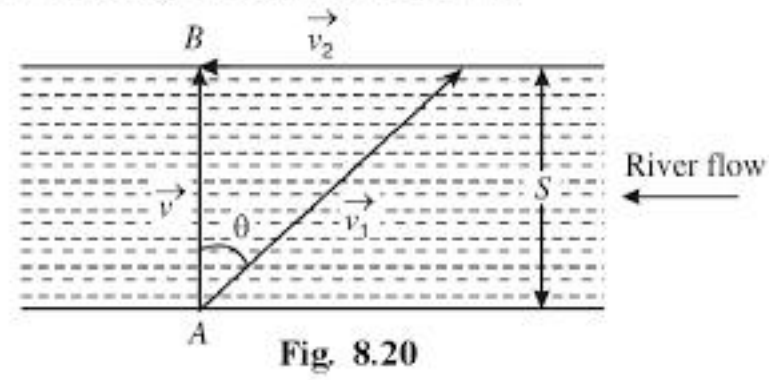


Fig. 8.20

Let $\overrightarrow{v_{BS}} = \vec{v}_1$ = Velocity of boat *w.r.t.* shore

$\overrightarrow{v_{WS}} = \vec{v}_2$ = Velocity of water *w.r.t.* shore

$\overrightarrow{v_{BW}} = \vec{v}$ = Velocity of boat *w.r.t.* water

$\therefore \quad \overrightarrow{v_{BW}} = \overrightarrow{v_{BS}} + \overrightarrow{v_{SW}} = \overrightarrow{v_{BS}} - \overrightarrow{v_{WS}}$

or $\vec{v} = \vec{v}_1 - \vec{v}_2$

The magnitude of $\vec{v}\ (=\overrightarrow{v_{BW}})$ is given by ;

$$v = \sqrt{v_1^2 - v_2^2}$$

Now the boat moves across AB with a velocity v.

$\therefore$ Time of crossing of boat, $t = \dfrac{AB}{v} = \dfrac{S}{v}$

Angle θ is given by ; $\sin\theta = v_2/v_1$

Note that angle θ is determined by the magnitudes of $\vec{v}_2$ and $\vec{v}_1$.

(*ii*) Boat to cross the river in shortest time. In order that the boat crosses the river in the shortest time, the river velocity $\vec{v}_2$ should help the boat velocity $\vec{v}_1$. Therefore, the boat should be rowed along AB. Now the resultant velocity $\vec{v}\ (=\overrightarrow{v_{BW}})$ is given by (See Fig. 8.21) ;

$$\vec{v} = \vec{v}_1 + \vec{v}_2$$

Magnitude of $\vec{v}$, $v = \sqrt{v_1^2 + v_2^2}$; $\tan\theta = \dfrac{v_2}{v_1}$

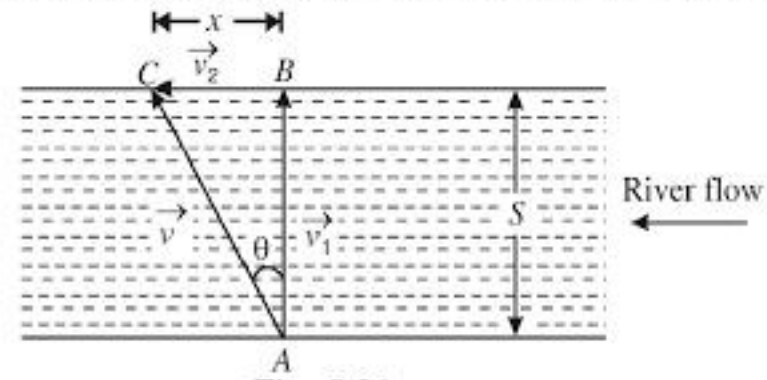


Fig. 8.21

Now the boat moves with velocity v along AC and will reach point C instead of point B. If $BC = x$, then,

$$AC = \sqrt{x^2 + S^2} \quad \text{where } S = \text{width of the river}$$

$\therefore$ Time of crossing of boat is

$$t = \frac{AC}{v} = \sqrt{\frac{x^2 + S^2}{v_1^2 + v_2^2}}$$

Now $\quad \tan\theta = \dfrac{v_2}{v_1} = \dfrac{x}{S} \quad \therefore \quad x = \dfrac{S v_2}{v_1}$

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(*iii*) Relative velocity of rain *w.r.t.* man. Consider a man walking east with velocity $\vec{v}_m$. Let the rain be falling vertically downward with velocity $\vec{v}_r$. In order to find the relative velocity of rain *w.r.t.* man $(\vec{v}_{rm})$, impress a velocity $-\vec{v}_m$ on rain as well as man. Now $\vec{v}_{rm}$ is the resultant of $-\vec{v}_m$ and $\vec{v}_r$ as shown in Fig. 8.22.

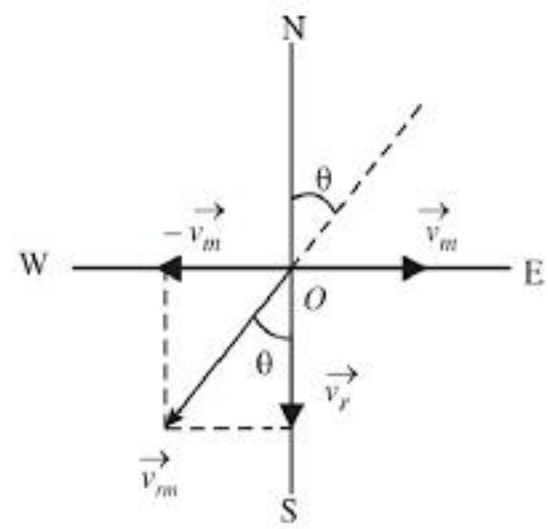


Fig. 8.22

Magnitude of $\vec{v}_{rm}$ is $v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90°}$

or $$v_{rm} = \sqrt{v_r^2 + v_m^2}$$

If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction, then,

$$\tan\theta = \frac{v_m}{v_r} \qquad \ldots (i)$$

In order to save himself from the rain, the man should hold his umbrella in the direction of relative velocity of rain *w.r.t.* man $(\vec{v}_{rm})$. In other words, the umbrella should be held at an angle θ with the vertical direction given by eq. (*i*) above.

Example 8.8. A river 1 km wide is flowing at 3 km/h. A swimmer whose velocity in still water is 4 km/h can swim only for 15 minutes. In what direction should he strike out in order to reach the other bank? What is total distance covered?

Solution. Fig. 8.23 shows the conditions of the problem. The point O represents the starting position of the swimmer. At this point, man's velocity $\overrightarrow{OB} = 4$ km/h and that of water current $\overrightarrow{OA} = 3$ km/h. The resultant velocity of man will be $\overrightarrow{OB'}$ and he will reach point P on the other side of the bank.

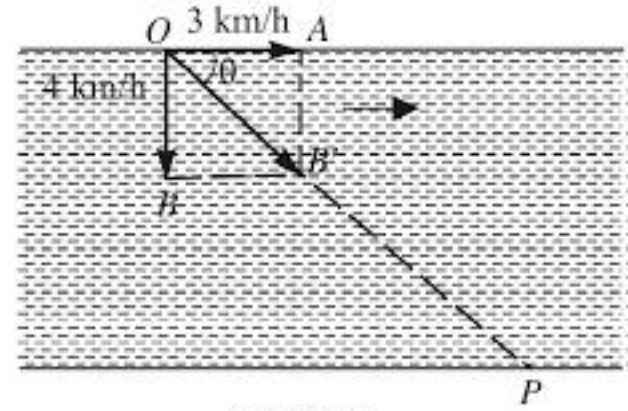


Fig. 8.23

$$\therefore \qquad \tan\theta = \frac{4}{3} \text{ or } \theta = \tan^{-1}\frac{4}{3} = 53.1°$$

Magnitude of resultant velocity is

$$OB' = \sqrt{(OA)^2 + (OB)^2} = \sqrt{(3)^2 + (4)^2} = 5 \text{ km/h}$$

$\therefore$ Distance covered by the swimmer $= OB' \times \text{time} = 5 \times \frac{15}{60} = 1.25$ km along OB'

Example 8.9. A boatman can row with a speed of 10 km/h in still water. If the river flows steadily at 5 km/hr, in which direction should the boatman row in order to reach a point on the other bank directly opposite to the point from where he started? The width of the river is 2 km.

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Solution. Suppose the river is flowing from left to right and O is the starting point. The boatman wants to reach point P directly opposite to the point O (See Fig. 8.24). If he starts along OP, he will be swayed away due to flow of water and may reach point P''. Therefore, in order to reach point P, he should start along OP' (upstream), making an angle θ with OP. The resultant velocity is now $\overrightarrow{OB'}$. $\overrightarrow{OB'} = \overrightarrow{OA} + \overrightarrow{OB}$

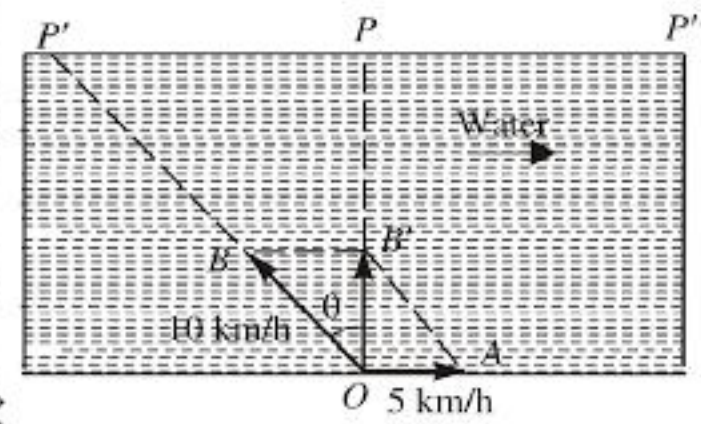


Fig. 8.24

In right angled triangle $OB'B$, $\sin\theta = \dfrac{BB'}{OB} = \dfrac{5}{10} = 0.5 \quad \therefore \quad \theta = 30°$

$\therefore$ Angle with the bank of river $= 90° + \theta = 90° + 30° =$ **120°**

Therefore, the boatman should row making an angle of 120° with the bank of the river.

Example 8.10. A man is going due east with a velocity of 3 km/h. Rain falls vertically downward with a speed of 10 km/h. Find the angle at which he should hold his umbrella so as to save himself from rain.

Solution. Fig. 8.25 shows the conditions of the problem.

$\vec{v}_m$ = velocity of man = 3 km/h due east

$\vec{v}_r$ = velocity of rain = 10 km/h vertically downward

$\vec{v}_{rm}$ = velocity of rain *w.r.t.* man

In order to save himself from the rain, the man should hold his umbrella in the direction of relative velocity of rain w.r.t. man (*i.e.* $\vec{v}_{rm}$).

Now $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m = \vec{v}_r + (-\vec{v}_m)$

In order to find $\vec{v}_{rm}$, reverse the direction of $\vec{v}_m$, thus producing $-\vec{v}_m$. Now find the sum of $\vec{v}_r$ and $-\vec{v}_m$ by parallelogram law of vector addition as shown in Fig. 8.25. The umbrella should be held at angle θ with the vertical where,

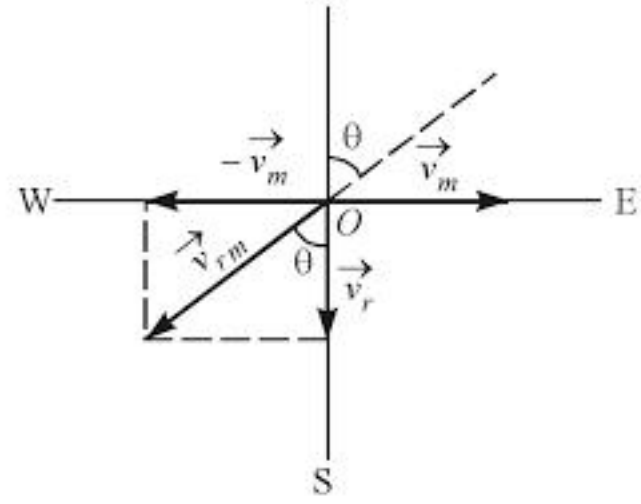


Fig. 8.25

$$\tan\theta = \frac{v_m}{v_r} = \frac{3}{10} = 0.3 \quad \therefore \quad \theta = \mathbf{16.7°}$$

Example 8.11. A boy is moving with a velocity of 3 km/h along east and the rain is falling vertically with a velocity of 4 km/h. Calculate the velocity of rain *w.r.t.* the boy.

Solution. Suppose $\vec{v}$ is the velocity of the boy and $\vec{u}$ that of rain. Then velocity of rain *w.r.t.* the boy is

$$\vec{V} = \vec{u} - \vec{v} = \vec{u} + (-\vec{v})$$

Taking x-axis along east and z-axis vertically upward (See Fig. 8.26), we have,

$$\vec{u} = -4\hat{k} \; ; \quad \vec{v} = 3\hat{i}$$

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$\therefore \quad \vec{V} = (-4\hat{k}) + (-3\hat{i}) = -4\hat{k} - 3\hat{i}$

Magnitude of $\vec{V}$ is given by ;

$$|\vec{V}| = \sqrt{(-4)^2 + (-3)^2} = \mathbf{5\ km/h}$$

If θ is the angle made by $\vec{V}$ with the vertical, then,

$$\tan\theta = +\frac{3}{4} = 0.75 \quad \therefore \quad \theta = \mathbf{37°}$$

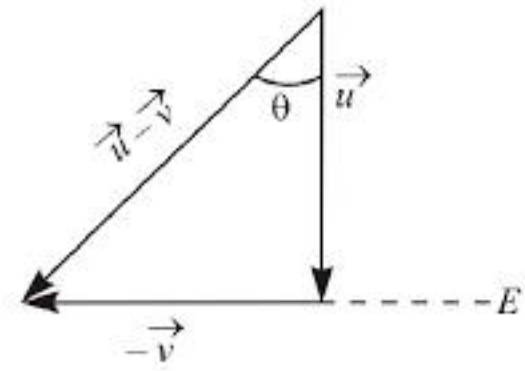


Fig. 8.26

Example 8.12. ***ABCD*** **is a parallelogram and** $\overrightarrow{AC}$ **and** $\overrightarrow{BD}$ **are its diagonals. Prove that :**

(*i*) $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{BC}$ **(*ii*)** $\overrightarrow{AC} - \overrightarrow{BD} = 2\overrightarrow{AB}$

Solution. Fig. 8.27 shows the conditions of the problem.

(*i*) Applying triangle law of vectors to ΔABC, we have,

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} \qquad \ldots (i)$$

Applying triangle law of vectors to ΔBCD, we have,

$$\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BC} - \overrightarrow{AB} \qquad \ldots (ii) \quad (\because \overrightarrow{CD} = -\overrightarrow{AB})$$

Adding eqs. (*i*) and (*ii*), we have,

$$\overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{BC} - \overrightarrow{AB}$$

$$\therefore \quad \overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{BC}$$

(*ii*) Subtracting eq. (*ii*) from eq. (*i*), we have,

$$\overrightarrow{AC} - \overrightarrow{BD} = (\overrightarrow{AB} + \overrightarrow{BC}) - (\overrightarrow{BC} - \overrightarrow{AB})$$

$$= \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{BC} + \overrightarrow{AB}$$

$$\therefore \quad \overrightarrow{AC} - \overrightarrow{BD} = 2\overrightarrow{AB}$$

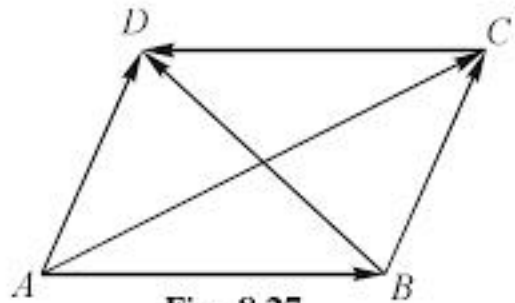


Fig. 8.27

PROBLEMS FOR PRACTICE

1. A boy goes 100 m north and then 70 m east. Find the displacement of the boy from the starting point. **[122 m]**
2. Two forces of 5 N and 10 N acting at a point have an angle of 120° between them. Find the magnitude and direction of the resultant force. **[8.66 N ; 30° with 10 N force]**
3. Two equal forces F have a resultant of 1.5 F. Find the angle between the forces. **[82.82°]**
4. A boat's speed in still water is 20 km/h. If the boat is to travel directly across a river whose current has a speed of 12 km/h, at what upstream angle must the boat head? **[126.9° with river bank]**
5. A swimmer is capable of swimming at 1.65 ms^{-1} in still water. (*i*) If she swims directly across a 180 m wide river whose current is 0.85 m/s, how far downstream (from a point opposite her starting point) will she land? (*ii*) How long will it take her to reach the other side? **[(*i*) 93 m (*ii*) 110 s]**

8.12. RESOLUTION OF A VECTOR IN A PLANE

When we add two vectors such as $\vec{A}$ and $\vec{B}$, we can replace the separate vectors by a single equivalent resultant vector, $\vec{R} = \vec{A} + \vec{B}$. We can do this in reverse *i.e.*, we can replace a single vector by any two (or more) vectors whose sum gives us back the original vector. This is called resolution of a vector.

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The process of splitting a single vector into two or more vectors in different directions in a plane such that their sum gives back the original vector is called ***resolution of a vector.***

The vectors into which the given vector $\vec{R}$ is resolved (or splitted) are called the **vector components** of $\vec{R}$. Fig. 8.28 shows the vector $\vec{R}$ resolved into two non-parallel vectors $\vec{A}$ and $\vec{B}$ such that :

$$\vec{R} = \vec{A} + \vec{B}$$

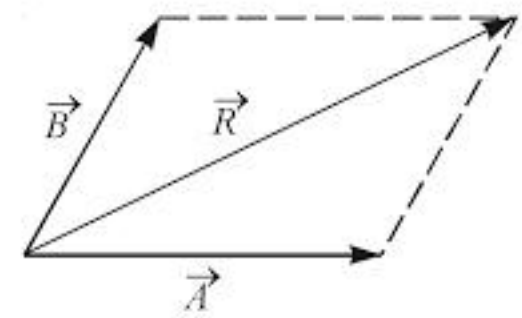


Fig. 8.28

Therefore, $\vec{A}$ and $\vec{B}$ are the vector components of $\vec{R}$. As any number of vectors can be combined to give a single equivalent vector, therefore, a vector can be resolved into any number of vector components. The resolution of a vector into its components often simplifies many problems related to vectors.

8.13. RECTANGULAR COMPONENTS OF A VECTOR IN A PLANE

When a vector in a plane is resolved (i.e., splitted) into two components at right angles to each other, the component vectors are called ***rectangular components*** *of the vector.*

Fig. 8.29(*i*) shows a vector $\vec{A}$ in the X-Y plane. It is resolved into two rectangular components $\vec{A}_x$ and $\vec{A}_y$ (along X-axis and Y-axis).

$$\therefore \qquad \vec{A} = \vec{A}_x + \vec{A}_y$$

The vector components $\vec{A}_x$ and $\vec{A}_y$ are the rectangular components of vector $\vec{A}$. Therefore, we can replace vector $\vec{A}$ by $\vec{A}_x$ and $\vec{A}_y$. From the geometry of Fig. 8.29(*i*), we see that magnitudes of components $\vec{A}_x$ and $\vec{A}_y$ are related to the magnitude of $\vec{A}$ by ;

$$A_x = A \cos\theta \; ; \; A_y = A \sin\theta$$

Note that A_x and A_y are not vectors ; these are x-component and y-component respectively of vector $\vec{A}$. Therefore, we can treat them as *algebraic* quantities. Thus, we can specify vector $\vec{A}$ in a plane by scalar components A_x (= $A \cos\theta$) and A_y (= $A \sin\theta$).

Magnitude of $\vec{A}$, $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(A\cos\theta)^2 + (A\sin\theta)^2}$

Direction of $\vec{A}$ with X-axis, $\tan\theta = \dfrac{A_y}{A_x}$

In short, we require two quantities to specify a vector such as $\vec{A}$ *viz.* either A and θ or A_x and A_y.

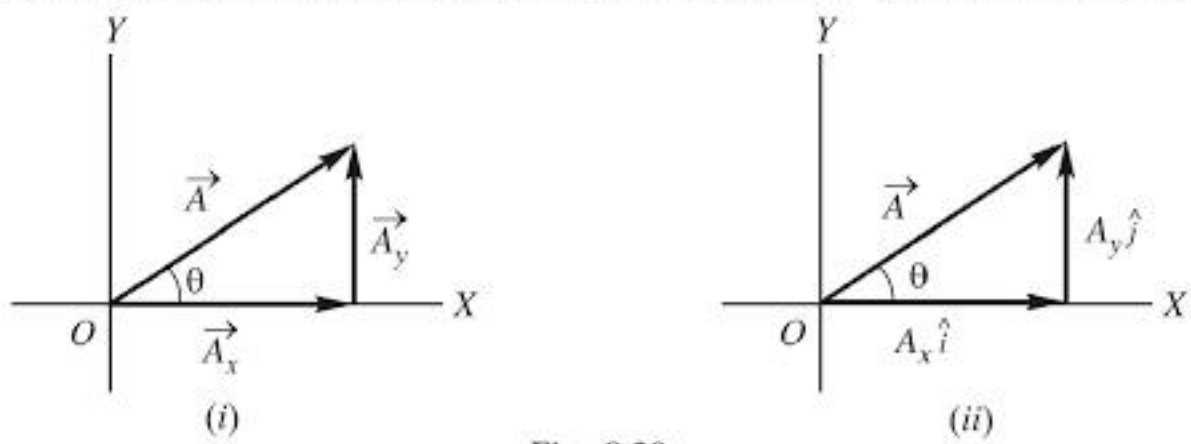


Fig. 8.29

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We can express the vectors $\vec{A}_x$ and $\vec{A}_y$ in terms of unit vectors $\hat{i}$ and $\hat{j}$ [See Fig. 8.29 (*ii*)].

$$\vec{A}_x = A_x \hat{i} \; ; \; \vec{A}_y = A_y \hat{j}$$

$$\therefore \qquad \vec{A} = A_x \hat{i} + A_y \hat{j}$$

Note that A_x (= $A \cos \theta$) is not a vector ; $A_x \hat{i}$ is a vector. Similarly, A_y (= $A \sin \theta$) is not a vector ; $A_y \hat{j}$ is a vector.

8.14. VECTOR ADDITION BY RECTANGULAR COMPONENTS

Adding vectors graphically is not very accurate and is not useful for vectors in three dimensions. A far more powerful and accurate method for adding vectors is to resolve the vectors into their rectangular components.

Fig. 8.30 shows two vectors $\vec{A}$ and $\vec{B}$ in the XY plane. By triangle law of vectors, their vector sum is $\vec{R}$. Note that vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$ are resolved into rectangular components. It is easy to see the component R_x of $\vec{R}$ along X-axis is equal to the algebraic sum of the components of the vectors $\vec{A}$ and $\vec{B}$ along X-axis *i.e.*,

$$R_x = A_x + B_x$$

Similarly, the component R_y of $\vec{R}$ along Y-axis is equal to the sum (algebraic) of the components of vector $\vec{A}$ and $\vec{B}$ along Y-axis *i.e.*,

$$R_y = A_y + B_y$$

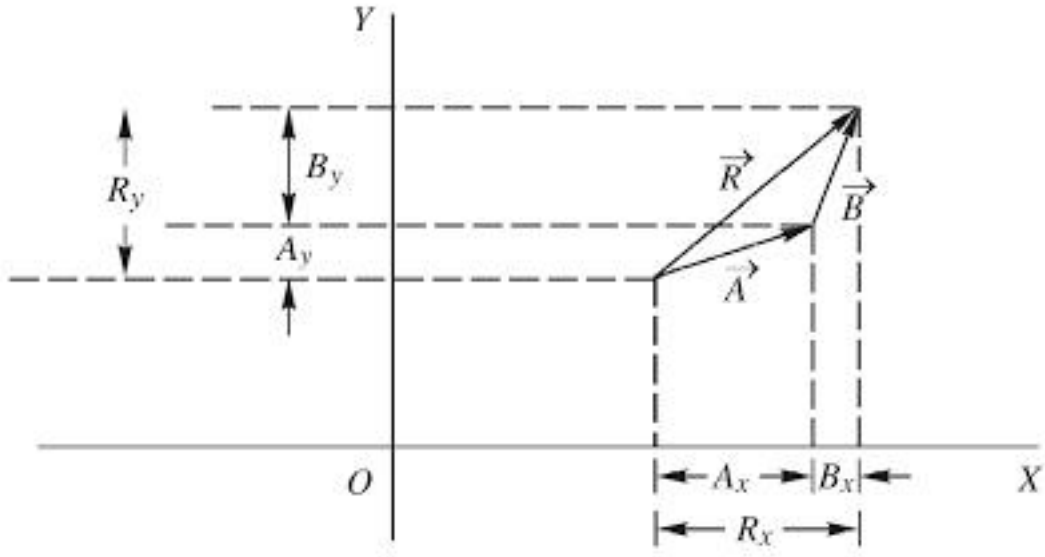


Fig. 8.30

The magnitude and direction of the resultant is given by ;

$$R = \sqrt{R_x^2 + R_y^2} \; ; \; \tan \theta = \frac{R_y}{R_x}$$

Note that angle θ (not shown) is the angle that $\vec{R}$ makes with the positive X-direction.

Example 8.13. Find the resultant of the following forces acting simultaneously at a point.

(*i*) a force of 50 N acting along *OX* axis

(*ii*) a force of 40 N acting at an angle of 60° to *OX* axis

(*iii*) a force of 60 N acting 330° with *OX* axis.

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Solution. Fig. 8.31 (*i*) shows the conditions of the problem. Let R be the magnitude of the resultant of the forces. Resolving the forces into rectangular components, we have,

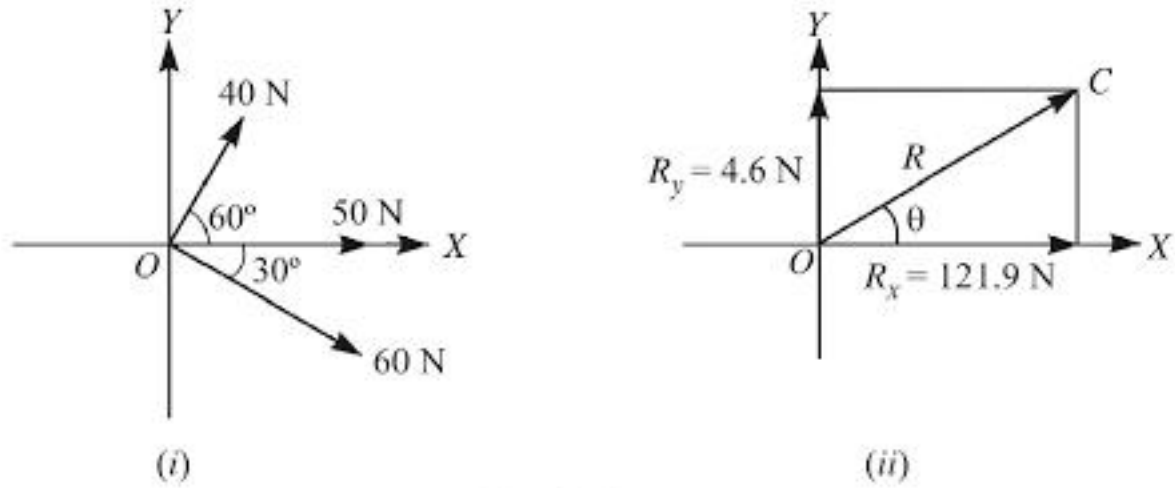


Fig. 8.31

The x-component of R, R_x = *Algebraic sum of x-components of forces

$= 50 \cos 0° + 40 \cos 60° + 60 \cos 30°$

$= 50 + 20 + 51.9 = 121.9$ N

The y-component of R, R_y = Algebraic sum of y-components of forces

$= 50 \sin 0° + 40 \sin 60° - 60 \sin 30°$

$= 0 + 34.6 - 30 = 4.6$ N

As shown in Fig. 8.31 (*ii*), R $(= OC)$ represents the magnitude of the resultant.

$$\therefore \quad R = \sqrt{R_x^2 + R_y^2} = \sqrt{(121.9)^2 + (4.6)^2} = \mathbf{122\ N}$$

$$\tan\theta = \frac{R_y}{R_x} = 0.0377 \quad \therefore \quad \theta = \tan^{-1}\ 0.0377 = 2.2°$$

Note. Following the usual convention, the angle θ is measured counterclockwise from the positive X-axis.

8.15. RECTANGULAR COMPONENTS IN THREE DIMENSIONS

So far we have considered the rectangular components of a vector in a plane. The treatment can be extended to three-dimensional space in which case a vector will have three rectangular components. Consider a vector $\vec{A}$ in space. It has three rectangular components $\vec{A}_x$, $\vec{A}_y$ and $\vec{A}_z$ as shown in Fig. 8.32.

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

Magnitude of $\vec{A}$, $^{**}A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Note that $\vec{A}_x$ is the x-component of $\vec{A}$; $\vec{A}_y$ is the y-component of $\vec{A}$ and $\vec{A}_z$ is the z-component of $\vec{A}$.

---

* An algebraic sum is one in which the sign of quantity is to be taken into account. The component along OX axis is positive while in opposite direction, it is negative. Similarly, component along OY is positive and that in the opposite direction is negative.

** Note that A_x and A_y are perpendicular to each other and their resultant is $R' = \sqrt{A_x^2 + A_y^2}$. Now A_z is $\perp$ R'. Therefore, magnitude of $A = \sqrt{R'^2 + A_z^2} = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

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We can also express vector $\vec{A}$ in terms of unit vectors as :

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

Note that A_x is not a vector ; $A_x\hat{i}$ is a vector. Same is true for other two components.

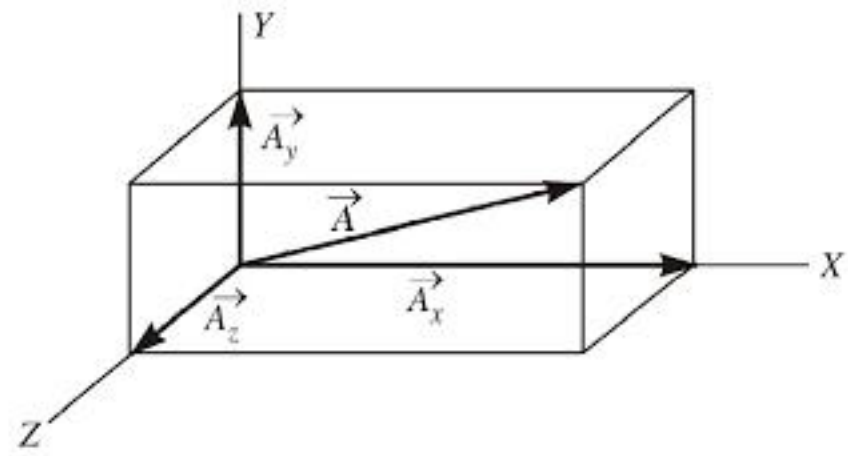


Fig. 8.32

Addition of vectors in three dimensions. To add vectors in three dimensions, each vector is resolved into three rectangular components and the treatment is carried out in a similar way as in the two dimensional case. Suppose that there are three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ in space and their sum is $\vec{R}$.

$$\therefore \qquad \vec{R} = \vec{A} + \vec{B} + \vec{C}$$

The three corresponding algebraic equations are :

$$R_x = A_x + B_x + C_x\ ;\ R_y = A_y + B_y + C_y\ ;\ R_z = A_z + B_z + C_z$$

Note the economy, generality and elegance of vector algebra. A single vector equation represents three component equations.

Direction cosines of vector. If α, β and γ are the angles which vector $\vec{A}$ makes with X, Y and Z axes respectively, then,

$$\cos\alpha = \frac{A_x}{A} \quad \text{or} \quad A_x = A\cos\alpha$$

$$\cos\beta = \frac{A_y}{A} \quad \text{or} \quad A_y = A\cos\beta$$

$$\cos\gamma = \frac{A_z}{A} \quad \text{or} \quad A_z = A\cos\gamma$$

Here $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called **direction cosines** of the vector $\vec{A}$.

Now $$A^2 = A_x^2 + A_y^2 + A_z^2$$

or $$A^2 = A^2\cos^2\alpha + A^2\cos^2\beta + A^2\cos^2\gamma$$

or $$1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$$

Therefore, the sum of the squares of the direction cosines of a vector is always unity.

Example 8.14. A force is represented by ;

$$\vec{F} = (2\hat{i} + 3\hat{j} + 6\hat{k}) \text{ newton.}$$

What is the magnitude of force?

Solution. Magnitude of force, $F = \sqrt{(2)^2 + (3)^2 + (6)^2} = \sqrt{49} = 7$ N

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Example 8.15. **Two vectors are given as $\vec{A} = 3\hat{i} + 9\hat{j} - 6\hat{k}$ and $\vec{B} = 8\hat{i} - 4\hat{j} + 8\hat{k}$. Find $|\vec{A} + \vec{B}|$.**

Solution. $\vec{A} + \vec{B} = (3\hat{i} + 9\hat{j} - 6\hat{k}) + (8\hat{i} - 4\hat{j} + 8\hat{k}) = 11\hat{i} + 5\hat{j} + 2\hat{k}$

$\therefore \quad |\vec{A} + \vec{B}| = \sqrt{11^2 + 5^2 + 2^2} = \sqrt{150}$

Example 8.16. **The components of a vector $\vec{A}$ are $A_x = 0$; $A_y = 22$. The components of vector $\vec{B}$ are $B_x = 33.2$ and $B_y = -33.2$. Write the vectors (*i*) in unit vector notation and (*ii*) perform the addition.**

Solution. (*i*) In unit vector notation, the vectors will be expressed as :

$$\vec{A} = 0\hat{i} + 22\hat{j} \; ; \; \vec{B} = 33.2\,\hat{i} - 33.2\,\hat{j}$$

(*ii*) $\vec{A} + \vec{B} = (0\,\hat{i} + 22\,\hat{j}) + (33.2\hat{i} - 33.2\,\hat{j}) = (0 + 33.2)\hat{i} + (22 - 33.2)\hat{j} = 33.2\,\hat{i} - 11.2\,\hat{j}$

Example 8.17. **Determine the vector which when added to the resultant of $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ gives unit vector along *y*-direction.**

Solution. Resultant vector, $\vec{R} = \vec{A} + \vec{B}$

$$= (3\hat{i} - 5\hat{j} + 7\hat{k}) + (2\hat{i} + 4\hat{j} - 3\hat{k}) = 5\hat{i} - \hat{j} + 4\hat{k}$$

Unit vector along *y*-axis $= \hat{j}$

$\therefore$ Required vector $= \hat{j} - (5\hat{i} - \hat{j} + 4\hat{k}) = -5\hat{i} + 2\hat{j} - 4\hat{k}$

Example. 8.18. **The *x*- and *y*-components of $\vec{A}$ are 4 and 6 and those of $\vec{A} + \vec{B}$ are 10 and 9. Find the components, magnitude and direction of $\vec{B}$.**

Solution. $\vec{A} = 4\hat{i} + 6\hat{j}$; $\vec{A} + \vec{B} = 10\hat{i} + 9\hat{j}$

$\therefore \quad \vec{B} = (10\hat{i} + 9\hat{j}) - \vec{A} = (10\hat{i} + 9\hat{j}) - (4\hat{i} + 6\hat{j}) = 6\hat{i} + 3\hat{j}$

Therefore, *x*- and *y*-components of $\vec{B}$ are 6 and 3.

$\therefore \quad |\vec{B}| = \sqrt{(6)^2 + (3)^2} = \sqrt{45}$

Direction of $\vec{B}$, $\tan\theta = \frac{3}{6} = \frac{1}{2}$ $\therefore$ $\theta = 26.56°$ with *x*-axis

Example 8.19. **Given $\vec{A} = (2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{B} = (3\hat{i} - 5\hat{j} + \hat{k})$. Find the angle between $\vec{A}$ and $\vec{B}$.**

Solution. $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k} \quad \therefore \quad A = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$

$\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k} \quad \therefore \quad B = \sqrt{3^2 + (-5)^2 + 1^2} = \sqrt{35}$

Resultant $\vec{R} = \vec{A} + \vec{B} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + (3\hat{i} - 5\hat{j} + \hat{k})$

$= 5\hat{i} - 2\hat{j} + 5\hat{k}$

$\therefore \quad R = \sqrt{5^2 + 2^2 + 5^2} = \sqrt{54}$

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Now $R^2 = A^2 + B^2 + 2AB\cos\theta$

$\therefore \quad \cos\theta = \dfrac{R^2 - A^2 - B^2}{2AB} = \dfrac{54 - 29 - 35}{2 \times \sqrt{29} \times \sqrt{35}} = -0.1567 \quad \therefore \quad \theta = 99°$

8.16. MULTIPLICATION OF VECTORS

Vectors have direction as well as magnitude. Therefore, vector multiplication cannot follow exactly the same rules as the algebraic rules of scalar multiplication. Instead, we must *define* the operation of vector multiplication in such a way that we find it useful in physics. It is very useful to define the following two kinds of multiplication operations for vectors :

(*i*) Multiplication of one vector by a second vector so as to produce a scalar. It is called *scalar product* or *dot product* of two vectors.

(*ii*) Multiplication of one vector by a second vector so as to produce another vector. It is called *vector product* or *cross product* of two vectors.

8.17. SCALAR PRODUCT OF TWO VECTORS

Consider two vectors $\vec{A}$ and $\vec{B}$ with angle θ between them as shown in Fig. 8.33. The scalar product of vectors $\vec{A}$ and $\vec{B}$ is defined as :

$$\vec{A}.\vec{B} = AB\cos\theta$$

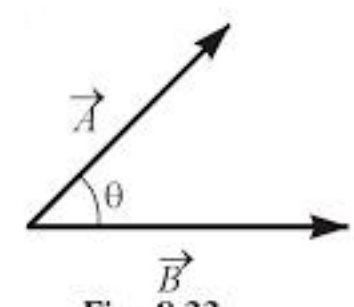


Fig. 8.33

where A and B are the magnitudes of the vectors and θ is the angle between them when their tails touch. Since A, B and $\cos\theta$ are scalars, the product $\vec{A}.\vec{B}$ (read "A dot B") is *also scalar. The equivalent definition of scalar product is as under :

The **scalar product** *of two vectors A and B is defined as the product of magnitude of one vector (say A) and the scalar component of the other vector ($B\cos\theta$) along the direction of the first vector $(\vec{A})$.*

Thus in Fig. 8.34 (*i*), $\vec{B}$ has a scalar component $B\cos\theta$ along the direction of $\vec{A}$.



Fig. 8.34

$\therefore \quad \vec{A}.\vec{B} = (A) \times$ scalar component of $\vec{B}$ along the direction of $\vec{A}$

$= (A)\ (B\cos\theta)$

or $\quad \vec{A}.\vec{B} = AB\cos\theta$

---

* This definition fits perfectly with our definition of work done by a constant force *i.e.*, we can write the work done by a constant force as the scalar product of force $(\vec{F})$ and displacement $(\vec{S})$.

Work done, $W = \vec{F}.\vec{S} = FS\cos\theta$

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Similarly, in Fig. 8.34 (*ii*), $\vec{B}.\vec{A} = (B)\ (A\cos\theta) = AB\cos\theta$

Note that the dot or scalar product of two vectors is a scalar quantity. Each of the vector $\vec{A}$ and $\vec{B}$ has a direction but the scalar product itself does not have a direction.

8.18. PROPERTIES OF SCALAR (OR DOT) PRODUCT

The following properties of scalar product (or dot product) are worth noting :

(*i*) *For given vectors* $\vec{A}$ *and* $\vec{B}$, *the value of the scalar product depends upon the angle* θ *between them* (See Fig. 8.35).

For $\theta = 0°$; $\quad \vec{A}.\vec{B} = AB\cos 0° = AB$

For $\theta = 180°$; $\quad \vec{A}.\vec{B} = AB\cos 180° = -AB$

For $\theta = 90°$; $\quad \vec{A}.\vec{B} = AB\cos 90° = 0$

Thus the dot product of two mutually perpendicular vectors is zero.

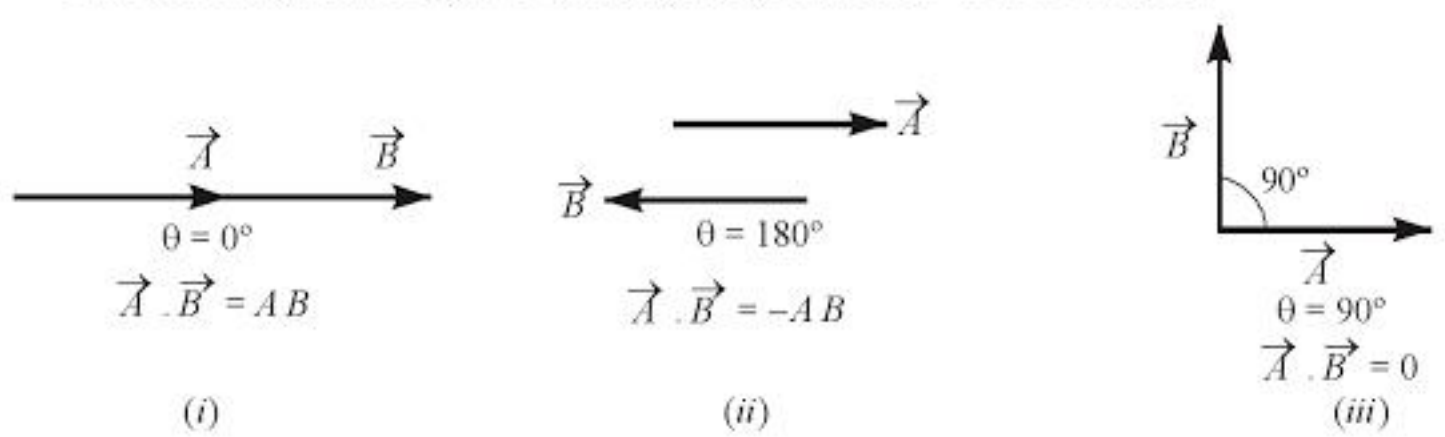


Fig. 8.35

(*ii*) *The dot product of two vectors obeys commutative law.* This directly follows from the definition of dot product.

$$\vec{A}.\vec{B} = A(B\cos\theta) = AB\cos\theta$$

$$\vec{B}.\vec{A} = B(A\cos\theta) = AB\cos\theta$$

$$\therefore \quad \vec{A}.\vec{B} = \vec{B}.\vec{A}$$

This simply means that the order of vectors in the dot product does not matter.

(*iii*) *The dot product obeys the distributive law.* If $\vec{C} = \vec{A} + \vec{B}$, th

$$\vec{D}.\vec{C} = \vec{D}.(\vec{A} + \vec{B})$$

$$= \vec{D}.\vec{A} + \vec{D}.\vec{B}$$

or $\quad \vec{D}.\vec{C} = \vec{D}.\vec{A} + \vec{D}.\vec{B}$

We can prove it by referring to Fig. 8.36. The component of vector $\vec{C}$ along the direction of $\vec{D}$ is the sum of the components of vector $\vec{A}$ and $\vec{B}$ along $\vec{D}$.

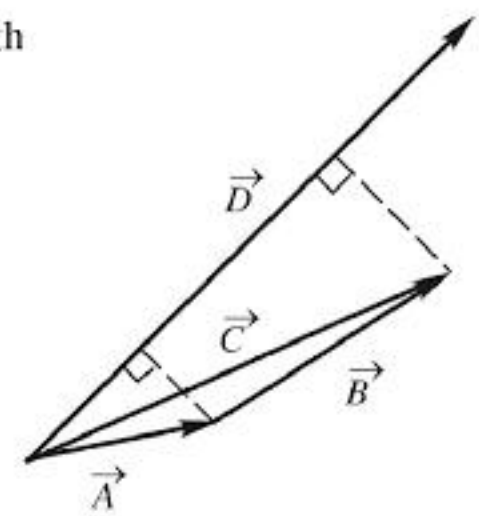


Fig. 8.36

(*iv*) *The dot product of a vector with itself gives square of its magnitude.*

$$\vec{A}.\vec{A} = (A)\ (A)\cos 0° \quad (\theta = 0°) \quad \therefore \quad \vec{A}.\vec{A} = A^2$$

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Note. If $\vec{A}$ is perpendicular to $\vec{B}$, then $\vec{A}.\vec{B} = 0$. But if the converse is given *i.e.*, $\vec{A}.\vec{B} = 0$, then there are three possibilities *viz.*, $\vec{A} = 0$, $\vec{B} = 0$ or $\vec{A} \perp \vec{B}$.

Examples of dot product of two vectors. In physics, there are many physical quantities (*e.g.* work, power, magnetic flux *etc.*) that can be described as the scalar or dot product of two vectors.

(*i*) Work done, $W = FS\cos\theta = \vec{F}.\vec{S}$

Thus work done by a constant force is the dot product of force ($\vec{F}$) and displacement ($\vec{S}$).

(*ii*) Instantaneous power, $P = Fv\cos\theta = \vec{F}.\vec{v}$

Thus instantaneous power is the dot product of force ($\vec{F}$) and velocity ($\vec{v}$).

8.19. UNIT VECTORS AND THE DOT PRODUCT

We have seen that if two vectors are aligned (*i.e.*, angle θ between them is zero), their dot product is equal to the product of their magnitudes. However, if the two vectors are perpendicular, their dot product is zero. Using these definitions of dot product, we can derive the following relationships for the unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$.

(*i*) Since $\hat{i}$ is parallel to $\hat{i}$ (*i.e.*, $\theta = 0°$) and each has a unit magnitude,

$\therefore\ \hat{i}.\hat{i} = (1)\ (1)\cos 0° = 1$. Similarly, $\hat{j}.\hat{j} = 1$ and $\hat{k}.\hat{k} = 1$.

$\therefore\ \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(*ii*) Since $\hat{i}$ and $\hat{j}$ are perpendicular and each has a unit magnitude,

$\therefore\ \hat{i}.\hat{j} = (1)\ (1)\cos 90° = 0$. Similarly, $\hat{i}.\hat{k} = 0$ and $\hat{j}.\hat{k} = 0$.

$\therefore\ \hat{i}.\hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{k} = 0$

Consider two three-dimensional vectors $\vec{A}$ and $\vec{B}$. These can be expressed in the rectangular form as :

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\ ;\quad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\therefore\quad \vec{A}.\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}).(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

With the distributive law, it will yield,

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

This is a very useful relation.

Also $\vec{A}.\vec{B} = AB\cos\theta \quad \therefore\quad \cos\theta = \dfrac{\vec{A}.\vec{B}}{AB} = \dfrac{A_xB_x + A_yB_y + A_zB_z}{AB}$

Thus we can find the angle θ between the vectors $\vec{A}$ and $\vec{B}$. Note that A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$ respectively *i.e.*,

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}\ ;\qquad B = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

Example 8.20. If $\vec{R} = \vec{A} - \vec{B}$, show that $R^2 = A^2 + B^2 - 2AB\cos\theta$ where θ is the angle between $\vec{A}$ and $\vec{B}$.

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Solution. $\vec{R} = \vec{A} - \vec{B}$

Taking dot product of $\vec{R}$ with itself, we have,

$$\vec{R}.\vec{R} = (\vec{A} - \vec{B}).(\vec{A} - \vec{B})$$

$$= \vec{A}.\vec{A} - \vec{A}.\vec{B} - \vec{B}.\vec{A} + \vec{B}.\vec{B}$$

or $$R^2 = A^2 - AB\cos\theta - BA\cos\theta + B^2$$

$\therefore$ $$R^2 = A^2 + B^2 - 2AB\cos\theta$$

Example 8.21. Find the angle between the vectors $\vec{A} = 3\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{B} = 5\hat{i} - 2\hat{j} - 3\hat{k}$.

Solution. $$\vec{A}.\vec{B} = AB\cos\theta \quad \therefore \quad \cos\theta = \frac{\vec{A}.\vec{B}}{AB}$$

Now $$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z = 3 \times 5 + 2\,(-2) + (1)\,(-3) = 8$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$$

$$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(5)^2 + (-2)^2 + (-3)^2} = \sqrt{38}$$

$\therefore$ $$\cos\theta = \frac{8}{\sqrt{14 \times 38}} = 0.35 \text{ or } \theta = \cos^{-1} 0.35 = \mathbf{69.5°}$$

Example 8.22. The sum and difference of two vectors are perpendicular to each other. Prove that the vectors are equal in magnitude.

Solution. Suppose the two vectors are $\vec{A}$ and $\vec{B}$. The sum and difference are $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$. Since these two are perpendicular to each other, their dot product is zero *i.e.*,

$$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0 \text{ or } \vec{A}.\vec{A} - \vec{A}.\vec{B} + \vec{B}.\vec{A} - \vec{B}.\vec{B} = 0$$

or $$A^2 - AB\cos\theta + BA\cos\theta - B^2 = 0 \quad \text{or} \quad A^2 - B^2 = 0 \quad \therefore \quad A = \pm B$$

Example 8.23. The sum and difference of two vectors are equal in magnitude *i.e.* $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$. Prove that vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Solution. $$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

or $$|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2 \qquad \ldots (i)$$

We know that dot product of a vector with itself is equal to the square of the magnitude of the vector. Therefore, expression given in eq. (*i*) can be written as :

$$(\vec{A} + \vec{B}).(\vec{A} + \vec{B}) = (\vec{A} - \vec{B}).(\vec{A} - \vec{B})$$

or $$\vec{A}.\vec{A} + \vec{A}.\vec{B} + \vec{B}.\vec{A} + \vec{B}.\vec{B} = \vec{A}.\vec{A} - \vec{A}.\vec{B} - \vec{B}.\vec{A} + \vec{B}.\vec{B}$$

or $$A^2 + AB\cos\theta + AB\cos\theta + B^2 = A^2 - AB\cos\theta - AB\cos\theta + B^2$$

or $$2AB\cos\theta = -2AB\cos\theta$$

or $$4AB\cos\theta = 0 \text{ or } \cos\theta = 0 \quad \therefore \quad \theta = \mathbf{90°}$$

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Example 8.24. If the magnitudes of two vectors are 3 and 4 and the magnitude of their scalar product is 6, find the angle between the vectors.

Solution. Let θ be the angle between the two vectors $\vec{A}$ and $\vec{B}$. Their scalar product is

$$\vec{A}.\vec{B} = AB\cos\theta$$

Here $\vec{A}.\vec{B} = 6\ ;\ A = 3\ ;\ B = 4$

$$\therefore \qquad 6 = 3\times 4\times\cos\theta \quad \text{or} \quad \cos\theta = \frac{6}{3\times 4} = 0.5 \quad \therefore \quad \theta = 60°$$

Example 8.25. Prove that vectors $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j}$ are perpendicular to each other.

Solution. $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}\ ;\quad \vec{B} = 2\hat{i} - \hat{j}$

The two vectors are perpendicular if $\vec{A}.\vec{B} = 0$.

Now $\vec{A}.\vec{B} = (\hat{i} + 2\hat{j} + 3\hat{k}).(2\hat{i} - \hat{j})$

$$= 1 \times 2 + 2 \times (-1) = 2 - 2 = 0$$

Since the dot product of $\vec{A}$ and $\vec{B}$ is zero, the two vectors are mutually perpendicular.

Example 8.26. The resultant vector of $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. On reversing the direction of $\vec{Q}$, the resultant vector becomes $\vec{S}$. Show that :

$$R^2 + S^2 = 2(P^2 + Q^2)$$

Solution. Let θ be the angle between P and Q.

Given : $\vec{R} = \vec{P} + \vec{Q}$ and $\vec{S} = \vec{P} - \vec{Q}$

Now $\vec{R}.\vec{R} = (\vec{P} + \vec{Q}).(\vec{P} + \vec{Q})$

or $R^2 = \vec{P}.\vec{P} + \vec{P}.\vec{Q} + \vec{Q}.\vec{P} + \vec{Q}.\vec{Q}$

or $R^2 = P^2 + PQ\cos\theta + PQ\cos\theta + Q^2$

or $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ ... (*i*)

Also $\vec{S}.\vec{S} = (\vec{P} - \vec{Q}).(\vec{P} - \vec{Q})$

or $S^2 = \vec{P}.\vec{P} - \vec{P}.\vec{Q} - \vec{Q}.\vec{P} + \vec{Q}.\vec{Q}$

or $S^2 = P^2 - PQ\cos\theta - PQ\cos\theta + Q^2$

or $S^2 = P^2 + Q^2 - 2PQ\cos\theta$ ... (*ii*)

Adding eqs. (*i*) and (*ii*), we have,

$$R^2 + S^2 = 2(P^2 + Q^2)$$

Example 8.27. A particle moves from position $(3\hat{i} + 2\hat{j} - 6\hat{k})$ to a position $(14\hat{i} + 13\hat{j} + 9\hat{k})$ in metre units and a constant force $(4\hat{i} + 2\hat{j} + 3\hat{k})$ newton acts on it. Calculate the work done by the force.

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Solution. $\vec{r_1} = 3\hat{i} + 2\hat{j} - 6\hat{k}$; $\vec{r_2} = 14\hat{i} + 13\hat{j} + 9\hat{k}$

Displacement of the particle, $\vec{S} = \vec{r_2} - \vec{r_1} = (14\hat{i} + 13\hat{j} + 9\hat{k}) - (3\hat{i} + 2\hat{j} - 6\hat{k})$

$= 11\hat{i} + 11\hat{j} + 15\hat{k}$

The work done by a constant force is equal to the dot product of force and displacement.

$\therefore$ Work done, $W = \vec{F}.\vec{S} = (4\hat{i} + 2\hat{j} + 3\hat{k}).(11\hat{i} + 11\hat{j} + 15\hat{k})$

$= 4\times11 + 2\times11 + 3\times15 = 44 + 22 + 45 = \mathbf{111\ J}$

Example 8.28. If unit vectors $\hat{A}$ and $\hat{B}$ are inclined at an angle θ, then prove that :

$$|\hat{A} - \hat{B}| = \mathbf{2\sin\frac{\theta}{2}}$$

Solution. We know that $|\hat{A} - \hat{B}|^2 = (\hat{A} - \hat{B}).(\hat{A} - \hat{B})$

or $|\hat{A} - \hat{B}|^2 = \hat{A}\cdot\hat{A} + \hat{B}\cdot\hat{B} - 2\hat{A}.\hat{B}$

$= (1)^2 + (1)^2 - 2(1)(1)\cos\theta$

$= 2 - 2\cos\theta = 2(1 - \cos\theta)$

$= 2\left[1 - \left(1 - 2\sin^2\frac{\theta}{2}\right)\right] = 4\sin^2\frac{\theta}{2}$

$\therefore$ $|\hat{A} - \hat{B}| = 2\sin\frac{\theta}{2}$

Example 8.29. Find the magnitude and direction of vector $\hat{i} + \hat{j}$.

Solution. Magnitude of vector $\hat{i} + \hat{j}$ is

$$|\hat{i} + \hat{j}| = [(1)^2 + (1)^2]^{1/2} = \sqrt{\mathbf{2}}$$

Suppose the vector $\hat{i} + \hat{j}$ makes an angle θ with x-axis (*i.e.* with unit vector $\hat{i}$). Then,

$(\hat{i} + \hat{j}).\hat{i} = |\hat{i} + \hat{j}|\,|\hat{i}|\cos\theta$ $\quad (\because\ \vec{A}.\vec{B} = AB\cos\theta)$

or $\hat{i}\cdot\hat{i} + \hat{j}\cdot\hat{i} = (\sqrt{2})(1)\cos\theta$

or $1 + 0 = \sqrt{2}\cos\theta \quad \therefore \quad \cos\theta = \frac{1}{\sqrt{2}}$ or $\theta = \mathbf{45°}$

Example 8.30. Find the unit vector parallel to the resultant of vectors $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$.

Solution. Let $\vec{R}$ be the resultant of $\vec{A}$ and $\vec{B}$.

$$\vec{R} = \vec{A} + \vec{B} = (2\hat{i} - 6\hat{j} - 3\hat{k}) + (4\hat{i} + 3\hat{j} - \hat{k}) = 6\hat{i} - 3\hat{j} - 4\hat{k}$$

$\therefore$ $R = |\vec{R}| = \sqrt{6^2 + (-3)^2 + (-4)^2} = \sqrt{61}$

The unit vector parallel to $\vec{R}$ is given by ;

$$\hat{R} = \frac{\vec{R}}{R} = \frac{6\hat{i} - 3\hat{j} - 4\hat{k}}{\sqrt{61}} = \frac{\mathbf{6}}{\sqrt{\mathbf{61}}}\hat{\mathbf{i}} - \frac{\mathbf{3}}{\sqrt{\mathbf{61}}}\hat{\mathbf{j}} - \frac{\mathbf{4}}{\sqrt{\mathbf{61}}}\hat{\mathbf{k}}$$

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PROBLEMS FOR PRACTICE

1. Find the angle between the X-axis and the vector $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$. $\left[\cos^{-1}\frac{2}{\sqrt{29}}\right]$
2. Find the angle between the vectors $\vec{A} = 2\hat{i} + 4\hat{j} + 4\hat{k}$ and $\vec{B} = 4\hat{i} + 2\hat{j} - 4\hat{k}$. **[90°]**
3. The magnitudes of two vectors $\vec{A}$ and $\vec{B}$ are 5 and 4 units respectively and the angle between them is 30°. Find the value of $\vec{A}.\vec{B}$. **[17.32 units]**
4. If $\vec{A} + \vec{B} = \vec{C}$ and $A^2 = B^2 + C^2$, then prove that $\vec{A}$ and $\vec{B}$ are perpendicular to each other.
5. If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are non zero vectors and $\vec{A}.\vec{B} = 0$ and $\vec{B}.\vec{B} = 0$, find the value of $\vec{A}.\vec{C}$. **[*AC*]**
6. Determine the value of a so that vectors $\vec{A} = 5\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{B} = 2\hat{i} + 2\hat{j} - a\hat{k}$ are perpendicular. **[– 8]**
7. Prove that vectors $\vec{A} = -2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} - 4\hat{k}$ are perpendicular to each other.
8. A point of application of force $\vec{F} = 5\hat{i} - 3\hat{j} + 2\hat{k}$ is moved from $\vec{r}_1 = 2\hat{i} + 7\hat{j} + 2\hat{k}$ to $\vec{r}_2 = -5\hat{i} + 2\hat{j} + 3\hat{k}$. Find the work done. **[– 22 units]**
9. Find the angle θ between vectors $\vec{A}$ and $\vec{B}$ where $\vec{A} = -2\hat{i} + 2\hat{j}$ and $\vec{B} = 2\sqrt{3}\,\hat{i} + 2\hat{j}$. **[105°]**
10. Show that $\vec{A}$ is perpendicular to $\vec{B}$ if $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$.

8.20. VECTOR PRODUCT

Consider two vectors $\vec{A}$ and $\vec{B}$ as shown in Fig. 8.37 (*i*) where the angle θ is the smaller of the angles between the two vectors. The vector or cross product of vectors $\vec{A}$ and $\vec{B}$ is another vector $\vec{C}$ given by :

$$\vec{C} = \vec{A} \times \vec{B}$$

It is called vector product of vectors $\vec{A}$ and $\vec{B}$ because $\vec{C}$ is itself a vector. The magnitude and direction of $\vec{C}(= \vec{A} \times \vec{B})$ are defined as under :

(*i*) The magnitude of $\vec{C}$ is given by : $C = AB\sin\theta$

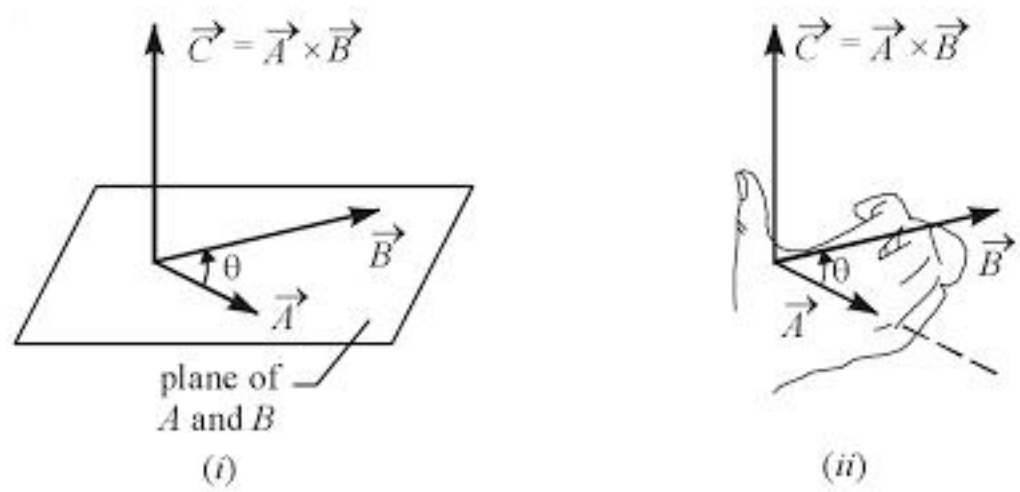


Fig. 8.37

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Here A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$ while θ is the smaller of the angles between vectors $\vec{A}$ and $\vec{B}$ [See Fig. 8.37 (*i*)].

(*ii*) The vector $\vec{C}$ is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ and its direction is given by **right-hand rule** [See Fig. 8.37 (*ii*)]. **Curl the fingers of the right hand in the sense that would rotate the first vector $\vec{A}$ into the second vector $\vec{B}$ through the smaller angle between them. The extended thumb gives the direction of $\vec{C}\,(=\vec{A}\times\vec{B})$.** Applying this rule to Fig. 8.37(*ii*), the direction of $\vec{C}$ is vertically upward.

Note. The direction of $\vec{C}\,(=\vec{A}\times\vec{B})$ can also be found by **Right hand screw rule** stated below :

If a right handed screw placed with its axis perpendicular to the plane containing the two vectors is rotated from $\vec{A}$ to $\vec{B}$ through smaller angle, then the direction of advance of the tip of the screw gives the direction of $\vec{C}\,(=\vec{A}\times\vec{B})$.

Applying this rule to Fig. 8.37(*ii*), the direction of $\vec{C}$ is vertically upward.

8.21. PROPERTIES OF VECTOR PRODUCT

The following are the important properties of vector product or cross product :

(*i*) *For given vectors $\vec{A}$ and $\vec{B}$, the value of the cross product for angle 90° between them is equal to the product of magnitudes of the two vectors i.e.,*

For $\theta = 90°$, $\vec{A}\times\vec{B} = AB\sin 90° = AB$

(*ii*) *The cross product of two vectors does not obey commutative law i.e.,* $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$.

Thus referring to Fig. 8.38, the direction of vector $\vec{A}\times\vec{B}$ is opposite to that of vector $\vec{B}\times\vec{A}$. Since the magnitude in each case is $AB\sin\theta$, it follows, therefore, that

$$\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A})$$

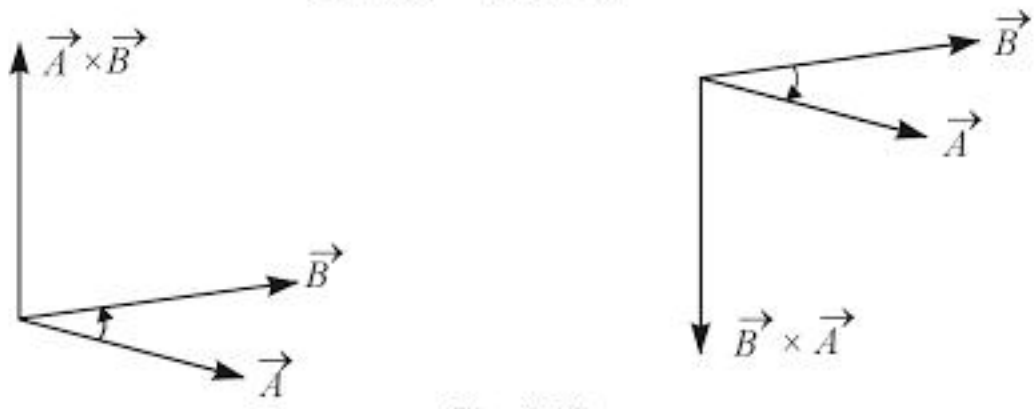


Fig. 8.38

Thus reversing the order of the vectors reverses the direction of the cross product. Therefore, commutative law does not hold for the cross product although it does for the dot product of two vectors.

(*iii*) *The cross product of a vector with itself is zero i.e.,*

$$\vec{A}\times\vec{A} = 0 \qquad (\because\ \theta = 0°)$$

(*iv*) Suppose two vectors $\vec{A}$ and $\vec{B}$ are parallel or antiparallel. The angle θ between them is either 0° or 180°. Then $\vec{A}\times\vec{B} = 0$. *The cross product of parallel (or antiparallel vectors) is zero.*

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(*v*) *The cross product obeys distributive law i.e.,*

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

(*vi*) *The magnitude of the cross product of two vectors is equal to the area of parallelogram formed by them.*

Suppose two vectors $\vec{A}$ and $\vec{B}$ are represented in magnitude and direction by the two adjacent sides $\overrightarrow{OM}$ and $\overrightarrow{OL}$ of the parallelogram *OLKM* (See Fig. 8.39). The area of the parallelogram is given by :

$$= OL \times MN = B(A \sin\theta) = AB \sin\theta$$

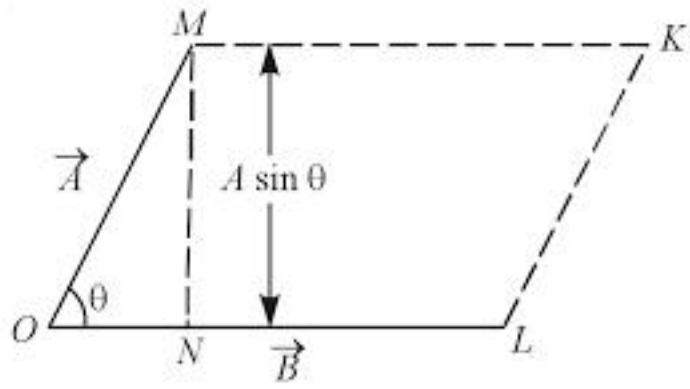


Fig. 8.39

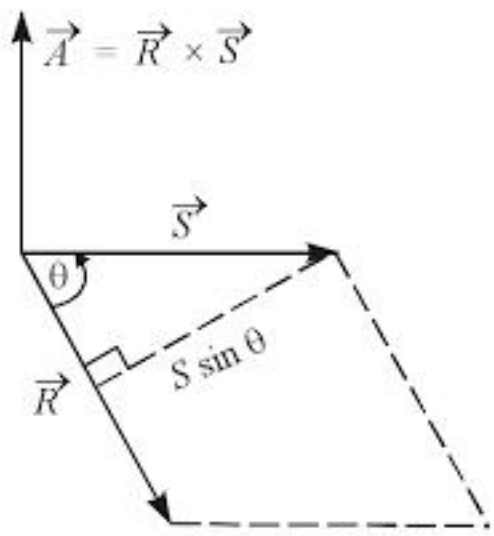


Fig. 8.40

But $AB \sin\theta$ is the magnitude of the cross product $\vec{A} \times \vec{B}$. Therefore, the magnitude of the cross product of two vectors is equal to the area of the parallelogram formed by them.

Note. The area of parallelogram formed by the two vectors is

Area *OLKM* = 2 × Area of triangle formed by two vectors

Hence the magnitude of the cross product of two vectors is equal to twice the area of triangle formed by them.

(*vii*) There is an interesting and useful geometrical interpretation of the cross product of two displacements. The displacements $\vec{R}$ and $\vec{S}$ in Fig. 8.40 form a parallelogram whose area $A = RS \sin\theta$.

This area is the magnitude of the cross product of $\vec{R}$ and $\vec{S}$ *i.e.*, $A = |\vec{R} \times \vec{S}| = RS \sin\theta$.

We define an oriented area element as a vector $\vec{A} = \vec{R} \times \vec{S}$ *whose magnitude is the area of the parallelogram formed by* $\vec{R}$ *and* $\vec{S}$ *and whose direction is perpendicular to plane of* $\vec{R}$ *and* $\vec{S}$. In this way, we specify the orientation of an element of surface area.

Examples of cross product of two vectors. In physics, there are many physical quantities (*i.e.* torque, linear velocity, centripetal acceleration *etc.*) that can be described as the vector or cross product of two vectors.

(*i*) Linear velocity, $\vec{v} = \vec{\omega} \times \vec{r}$

Thus linear velocity of a particle in rotational motion is equal to the cross product of its angular velocity $(\vec{\omega})$ and displacement vector $(\vec{r})$.

(*ii*) Centripetal acceleration, $\vec{a}_c = \vec{\omega} \times \vec{v}$

Thus the centripetal acceleration of a particle in rotational motion is equal to the cross product of its angular velocity $(\vec{\omega})$ and its linear velocity $(\vec{v})$.

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8.22. UNIT VECTORS AND THE CROSS PRODUCT

The cross product of two vectors can be evaluated in terms of the rectangular components. To do so, we must consider the cross product of the unit vectors.

(*i*) Let us evaluate $\hat{i}\times\hat{i}$. The result is zero. It is because the two vectors are parallel ($\sin\theta = 0$) and each has a unit magnitude.

$\therefore \qquad \hat{i}\times\hat{i} = (1)\ (1)\ \sin 0° = 0$

Similarly, $\hat{j}\times\hat{j} = 0$ and $\hat{k}\times\hat{k} = 0$

$\therefore \qquad \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

(*ii*) To evaluate $\hat{i}\times\hat{j}$, refer to Fig. 8.41 which shows the unit vectors on XYZ – *coordinate system. The magnitude of each unit vector is 1 and so the magnitude of $\hat{i}\times\hat{j}$ is 1 *i.e.*, $|\hat{i}\times\hat{j}| = (1)\ (1)\ \sin 90° = 1$. The direction of $\hat{i}\times\hat{j}$ is given by the right-hand rule and from Fig. 8.41, it is along the positive Z-axis. But this is just the unit vector $\hat{k}$. Therefore, we have $\hat{i}\times\hat{j} = \hat{k}$. Simply reversing the order of the unit vectors gives $\hat{j}\times\hat{i} = -\hat{k}$.

$\therefore \qquad \hat{i}\times\hat{j} = -(\hat{j}\times\hat{i}) = \hat{k}$

Similarly, $\hat{j}\times\hat{k} = -(\hat{k}\times\hat{j}) = \hat{i}$

and $\hat{k}\times\hat{i} = -(\hat{i}\times\hat{k}) = \hat{j}$

Consider three dimensional vectors $\vec{A}$ and $\vec{B}$. These vectors can be expressed in terms of rectangular vectors as:

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\ ;\ \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

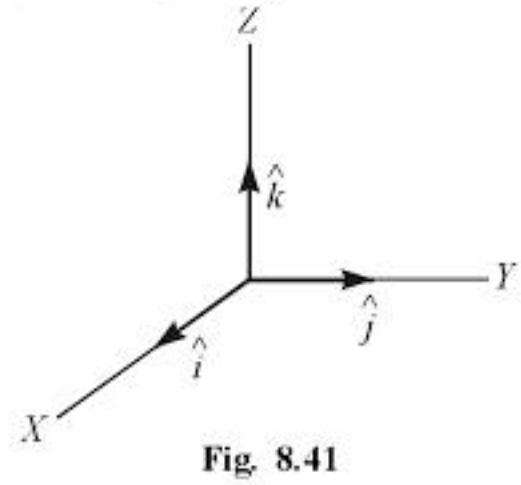


Fig. 8.41

We form cross product $\vec{C} = \vec{A}\times\vec{B}$, expressing $\vec{A}$ and $\vec{B}$ in terms of their components.

$\therefore \qquad \vec{A}\times\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

The distributive property allows us to carry out the multiplication and we get,

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k} \qquad \ldots (i)$$

In determinant form, this can be written as :

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Another way to express the result is

$$\vec{A}\times\vec{B} = \vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k} \qquad \ldots (ii)$$

---

* The coordinate system shown in Fig. 8.41 is called *right-handed coordinate system*. The unit vectors $\hat{i}, \hat{j}, \hat{k}$ associated with the X, Y and Z axes in that order are connected by the right-hand rule $\hat{i}\times\hat{j} = \hat{k}$.

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By comparison with eq. (*i*), the components of $\vec{C}$ are :

$$C_x = A_yB_z - A_zB_y \; ; C_y = A_zB_x - A_xB_z \; ; \; C_z = A_xB_y - A_yB_x$$

Example 8.31. Find the cross product of vectors $\vec{A} = 2\hat{i}$ and $\vec{B} = -2\hat{i} + 4\hat{j}$.

Solution. $\vec{A} \times \vec{B} = (2\hat{i}) \times (-2\hat{i} + 4\hat{j})$

$$= -4(\hat{i} \times \hat{i}) + 8(\hat{i} \times \hat{j})$$

$$= 0 + 8\hat{k} = \mathbf{8\hat{k}}$$

Note that in this example we see again that the plane of $\vec{A}$ and $\vec{B}$ (the $\hat{i} - \hat{j}$ plane) is perpendicular to the direction of $\vec{A} \times \vec{B}$ (the $\hat{k}$ direction).

Example 8.32. Determine the area of the parallelogram whose adjacent sides are $2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} - \hat{j}$.

Solution. Let $\vec{A}$ and $\vec{B}$ be the vectors representing the adjacent sides of the parallelogram.

Here $\vec{A} = 2\hat{i} + \hat{j} + 3\hat{k} \; ; \quad \vec{B} = \hat{i} - \hat{j}$

The area of parallelogram is equal to the magnitude of the cross product of $\vec{A}$ and $\vec{B}$.

Now $\vec{A} \times \vec{B} = (2\hat{i} + \hat{j} + 3\hat{k}) \times (\hat{i} - \hat{j})$

$$= 2(\hat{i} \times \hat{i}) - 2(\hat{i} \times \hat{j}) + (\hat{j} \times \hat{i}) - (\hat{j} \times \hat{j}) + 3(\hat{k} \times \hat{i}) - 3(\hat{k} \times \hat{j})$$

$$= 0 - 2k - k - 0 + 3j + 3i$$

$\therefore \quad \vec{A} \times \vec{B} = 3\hat{i} + 3\hat{j} - 3\hat{k}$

$\therefore$ Magnitude of the area of the parallelogram

$$= \sqrt{(3)^2 + (3)^2 + (-3)^2} = \sqrt{27} = \mathbf{3\sqrt{3}} \text{ sq units}$$

Example 8.33. Show that vectors $\vec{A} = \hat{i} - 5\hat{j}$ and $\vec{B} = 2\hat{i} - 10\hat{j}$ are parallel to each other.

Solution. If the two vectors are parallel, then their cross product must be zero *i.e.*, $\vec{A} \times \vec{B} = 0$.

Now $\vec{A} \times \vec{B} = (\hat{i} - 5\hat{j}) \times (2\hat{i} - 10\hat{j})$

$$= 2(\hat{i} \times \hat{i}) - 10(\hat{i} \times \hat{j}) - 10(\hat{j} \times \hat{i}) + 50(\hat{j} \times \hat{j})$$

$$= 2(0) - 10(\hat{k}) - 10(-\hat{k}) + 50(0) = 0$$

Hence vectors $\vec{A}$ and $\vec{B}$ are parallel to each other.

Example 8.34. Find a unit vector perpendicular to both the vectors $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$.

Solution. By definition, the cross product of two vectors $\vec{A}$ and $\vec{B}$ is a vector that is perpendicular to both $\vec{A}$ and $\vec{B}$.

Now $\vec{A} \times \vec{B} = (3\hat{i} + \hat{j} + 2\hat{k}) \times (2\hat{i} - 2\hat{j} + 4\hat{k}) = 8\hat{i} - 8\hat{j} - 8\hat{k}$

$\therefore \quad |\vec{A} \times \vec{B}| = \sqrt{(8)^2 + (-8)^2 + (-8)^2} = 8\sqrt{3}$

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$\therefore$ Unit vector perpendicular to $\vec{A}$ and $\vec{B}$ is

$$\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{8\hat{i} - 8\hat{j} - 8\hat{k}}{8\sqrt{3}} = \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$$

PROBLEMS FOR PRACTICE

1. Find the vector product of $\hat{i} + \hat{j} + \hat{k}$ and $-3\hat{i} + \hat{j} - 2\hat{k}$. **[$3\hat{i} + \hat{j} - 4\hat{k}$]**
2. Prove that vectors $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = 6\hat{i} + 9\hat{j} - 3\hat{k}$ are parallel.
3. If $\vec{A} = 18\hat{i} + 17\hat{j} - 7\hat{k}$ and $\vec{B} = 5\hat{i} - 3\hat{j} + \hat{k}$, find $|\vec{A} \times \vec{B}|$. **[32 units]**
4. Show that $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B}) = 2(\vec{B} \times \vec{A})$.
5. The adjacent sides of a parallelogram are represented by $4\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} - 3\hat{j} + 2\hat{k}$. Find the area of the parallelogram. **[$\sqrt{171}$ sq units]**

CONCEPTUAL QUESTIONS

Q.1. $\vec{A}$ and $\vec{B}$ are two vectors, can $\vec{A} + \vec{B}$ be zero?

Ans. $\vec{A} + \vec{B}$ will be zero if $\vec{A}$ and $\vec{B}$ have same magnitude but are opposite in direction.

Q.2. When are two three-dimensional vectors equal?

Ans. Suppose A and B are three dimensional vectors. These vectors will be equal if their corresponding rectangular components are equal *i.e.*

$\vec{A} = \vec{B}$ if $A_x = B_x$, $A_y = B_y$ and $A_z = B_z$

Q.3. What is the difference between A_x and $\vec{A}_x$?

Ans. A_x means the scalar component of vector $\vec{A}$ along X-axis. $\vec{A}_x$ is the vector component of $\vec{A}$ along X-axis.

Q.4. What is the importance of scalar product and vector product?

Ans. The scalar product and vector product are very useful in physics. It is because many quantities can be described as the scalar product or vector product of two vectors.

Q.5. Distinguish between cross product and dot product of two vectors $\vec{A}$ and $\vec{B}$.

Ans. The following are the distinguishing features between the cross product and dot product:

(*i*) The cross product of two vectors is a vector ; the dot product is a scalar.

(*ii*) The magnitude of the cross product is $AB \sin\theta$; the magnitude of dot product is $AB \cos\theta$.

(*iii*) The cross product of two vectors is zero when the vectors are parallel ; the dot product of two vectors is zero when the vectors are perpendicular.

(*iv*) The cross product does not obey commutative law ; the dot product does.

Q.6. A scalar may be a pure number or a physical quantity. What is the effect of multiplication of a vector by a scalar?

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Ans. (*i*) If we multiply vector $\vec{A}$ by a pure number n, the new vector is $n\vec{A}$. It means that the magnitude of new vector is n times the magnitude of vector $\vec{A}$. The new vector has the same direction as $\vec{A}$ if n is positive and the opposite direction if n is negative. Note that physical nature of the original vector has not changed in this case.

(*ii*) If we multiply a vector by a scalar (physical quantity), the physical nature of product differs from the original vector. For example,

mass × acceleration = Force

or $$m\vec{a} = \vec{F}$$

Here $\vec{a}$ is an acceleration (vector) and m is mass (positive scalar). Note that physical nature of $\vec{F}$ is different from the original vector $\vec{a}$.

Q.7. What is the necessary condition for adding vectors?

Ans. The necessary condition for adding (or subtracting) the vectors is that the vectors must be of same kind. We cannot add displacement vector to velocity vector.

Q.8. Can three vectors (*i*) lying in a plane (*ii*) not lying in a plane give zero resultant?

Ans. Consider three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$. If resultant of these vectors is zero, then,

$$\vec{A}+\vec{B}+\vec{C}=0 \qquad \text{or} \qquad \vec{C}=-(\vec{A}+\vec{B})$$

Therefore, they will produce zero resultant if $\vec{C}=-(\vec{A}+\vec{B})$ or vector sum of any two vectors is equal and opposite to the third vector. The vectors can give this result only if they lie in the same plane.

The three vectors cannot give zero resultant if they do not lie in the same plane.

Q.9. What are coplanar vectors?

Ans. Those vectors which act in the same plane are called coplanar vectors.

Q.10. What is the importance of vectors?

Ans. The laws of physics are generally expressed in vector form due to the following reasons:

(*i*) By the use of vectors, many physics laws can be expressed in compact form.

(*ii*) The analysis of many physcis laws is greatly simplified by the use of vectors.

Q.11. What are the characteristics of vector addition?

Ans. (*i*) Only vectors of the same nature can be added.

(*ii*) Vector addition obeys commutative law. It is because

$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

(*iii*) Vector addition obeys associative law. It is because

$$(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$$

Q.12. What is the condition for zero resultant vector for more than three vectors acting simultaneously on a particle?

Ans. If a number of vectors acting simultaneously on a particle can be represented in magnitude and direction by the sides of a polygon taken in the same order, their resultant vector is zero.

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Q.13. Walking of a man is an example of resolution of a vector. Discuss.

Ans. When a man walks, he presses the ground with his feet backward at an angle θ. The reaction R of the ground on the feet of the man is in the opposite direction. The reaction R is resolved into two rectangular components *viz* vertical component $R \sin\theta$ and horizontal component $R \cos\theta$. The vertical component $R \sin\theta$ balances the weight of the man while the horizontal component $R \cos\theta$ helps the man to walk forward [See Fig. 8.42].

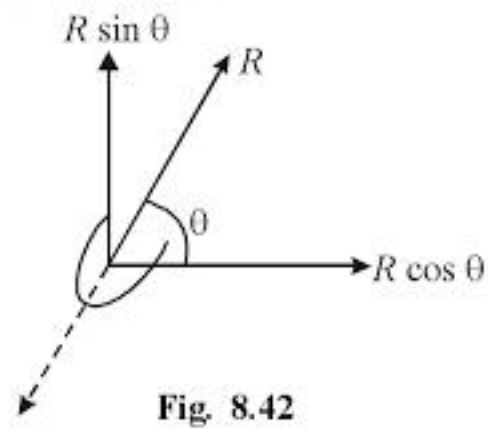


Fig. 8.42

Q.14. If $\vec{A} \times \vec{B} = \vec{C} \times \vec{B}$, show that $\vec{C}$ need not be equal to $\vec{A}$. When will $\vec{A}$ be equal to $\vec{C}$?

Ans. $\vec{A} \times \vec{B} = \vec{C} \times \vec{B}$ or $(\vec{A} - \vec{C}) \times \vec{B} = 0$

In other words, the cross product of $(\vec{A} - \vec{C})$ and $\vec{B}$ is zero. This is possible if any of the following three possibilities occur :

(*i*) $\vec{A} - \vec{C} = 0$ or $\vec{A} = \vec{C}$

(*ii*) $\vec{B} = 0$

(*iii*) $(\vec{A} - \vec{C})$ and $\vec{B}$ are parallel (*i.e.*, $\theta = 0°$)

From point (*iii*) above, we have, $\vec{A} = \vec{C} + m\vec{B}$ where m is a non-zero real number. The first statement is true if $\vec{B}$ is a null vector [point (*i*)] or $\vec{A}$ is equal to $(\vec{C} + m\vec{B})$ [point (*iii*)]. On the other hand, the second statement is true if $\vec{B}$ is not a null vector and $\vec{A}$ is not equal to $(C + mB)$.

VERY SHORT ANSWER QUESTIONS

Q.1. Which of the following is not a vector quantity? Mass, acceleration, momentum, force.

Ans. Mass.

Q.2. Is a quantity which has a magnitude and direction always a vector? Give examples.

Ans. No. For example, electric current possesses both magnitude and direction but is a scalar quantity. For a quantity to be a vector, it must obey the laws of vector algebra.

Q.3. Pick up the only vector quantity from the following: temperature, power, area, gravitational potential, coefficient of friction.

Ans. Only area is the vector quantity.

Q.4. Give an example of a physical quantity (*i*) which has neither unit nor direction (*ii*) has a direction but not a vector (*iii*) can be either a vector or scalar.

Ans. (*i*) Specific gravity (*ii*) current (*iii*) area.

Q.5. How is a vector represented?

Ans. A vector is represented by a straight line with an arrowhead. The length of the line represents the magnitude of the vector (on suitable scale) and the arrowhead indicates its direction.

Q.6. What condition should be satisfied for two vectors to be equal?

Ans. The two vectors will be equal if they have (*i*) the same magnitude and (*ii*) the same direction.

Q.7. Can you add velocity vector to the displacement vector?

Ans. No. It is because only vectors of the same kind can be added or subtracted.

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Q.8. The magnitude of the sum of two vectors is equal to the magnitude of difference between them. What is the angle between the two vectors?

Ans. $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

$\therefore \quad A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$ or $4AB\cos\theta = 0$ or $\theta = 90°$.

Q.9. What is the property of two vectors if $\vec{A} + \vec{B} = \vec{A} - \vec{B}$?

Ans. It means $\vec{B}$ is a null vector.

$$\vec{A} + \vec{B} = \vec{A} - \vec{B} \text{ or } 2\vec{B} = 0 \quad \therefore \quad \vec{B} = 0$$

Q.10. When can three vectors be added to give null vector?

Ans. When the three vectors are lying in the same plane and are represented in magnitude and direction by the three sides of a triangle taken in the same order.

Q.11. How can a vector be tripled?

Ans. By multiplying the vector by 3 (scalar) or by adding two similar vectors (vectors of same magnitude and direction) to the given vector.

Q.12. Does vector addition hold for any two vectors?

Ans. No. The vector addition (or subtraction) holds for two vectors of the same kind.

Q.13. What is the result of multiplying a vector by a real number n?

Ans. When a vector $\vec{A}$ is multiplied by a real positive number n, the result is $n\vec{A}$ *i.e.*, resultant vector has magnitude nA and has the same direction as that of $\vec{A}$. For negative real number, magnitude is nA but direction opposite to that of $\vec{A}$.

Q.14. Why vectors cannot be added algebraically?

Ans. Vectors possess magnitude as well as direction. In scalar addition, we have to add magnitudes of scalars. But vector addition requires the use of vector algebra.

Q.15. Why is $\vec{A} \times \vec{A} = 0$?

Ans. The angle between similar vectors is zero. Therefore, $\vec{A} \times \vec{A} = AA\sin\theta = A^2\sin 0° = 0$.

Q.16. What is the number of minimum forces that can give zero resultant?

Ans. Two, provided they are equal in magnitude but act in opposite directions.

Q.17. What is the minimum number of unequal forces that can give zero resultant?

Ans. Three, provided they act in the same plane and can be represented in magnitude and direction by the sides of a triangle taken in the same order.

Q.18. Will $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ be in the same or different planes?

Ans. They will be in the same plane.

Q.19. Is commutative law applicable for vector subtraction?

Ans. No, because $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$.

Q.20. Is associative law applicable for vector subtraction?

Ans. Yes, because $(\vec{A} + \vec{B}) - \vec{C} = \vec{A} + (\vec{B} - \vec{C})$.

Q.21. Can the magnitude of the resultant of two vectors be less than the magnitude of any of the vector?

Ans. Yes, if the angle between them is more than 90° but less than 270°. It is because in that case $\cos\theta$ is negative.

Q.22. What is the unit of a unit vector?

Ans. It has no units.

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Q.23. Can the scalar product of two vector quantities be negative?

Ans. The scalar product of two vectors can be negative if angle between them is between 90° to 270°.

Q.24. What is the maximum and minimum value of the resultant of two forces?

Ans. The resultant is maximum when both forces act in the same direction and is minimum when they act in opposite directions.

Q.25. Is the magnitude and direction of $(\vec{A}-\vec{B})$ is the same as that of $(\vec{B}-\vec{A})$?

Ans. The magnitude is the same but direction is opposite.

Q.26. Is it possible to add vectors representing force of 5 N and 200 dynes?

Ans. Yes, it is because both vectors represent force. However, they must be represented on the same scale.

Q.27. If a vector is added or subtracted from a vector, the resultant is a vector. Is this also true in case of multiplication of two vectors?

Ans. This may or may not be true. In case of cross product of two vectors, the result is a vector quantity. But in case of dot product of two vectors, the result is a scalar quantity.

Q.28. Consider a vector $\vec{F}=3\hat{i}-4\hat{j}$. What is a unit vector perpendicular to $\vec{F}$?

Ans. It will be $\hat{k}$ (*i.e.*, along Z-axis) because $\vec{F}$ lies in the X-Y plane.

SHORT ANSWER QUESTIONS

Q.1. Can two vectors of the same magnitude have resultant equal to either of them?

Ans. Yes when the two vectors are inclined at 120° angle.

$P^2 = P^2 + P^2 + 2P.P\cos\theta$ or $\cos\theta = -1/2 \quad \therefore \quad \theta = 120°$

Q.2. In the above question, what will be the magnitude of the difference of the two vectors?

Ans. In this case θ will be 60°.

$$\therefore \qquad R = \sqrt{P^2+P^2+2\,PP.\cos 60°} = \sqrt{2P^2+2P^2\times(0.5)} = \sqrt{3}P$$

Q.3. Can any of the two rectangular components of a given vector have a magnitude greater than the vector itself?

Ans. No. The rectangular components of any vector $\vec{A}$ have values $A\cos\theta$ and $A\sin\theta$. Since the value of $\cos\theta$ or $\sin\theta$ cannot be greater than 1, the values of $A\cos\theta$ and $A\sin\theta$ cannot be greater than A.

Q.4. Show that $\vec{P}+\vec{Q}=\vec{Q}+\vec{P}$.

Ans. $R^2 = P^2 + Q^2 + 2P.Q\cos\theta$. If we interchange $\vec{P}$ and $\vec{Q}$, the magnitude and direction of resultant remains the same. Therefore, $\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$.

Q.5. Prove that the magnitude of the resultant of two vectors $\vec{A}$ and $\vec{B}$ cannot be greater than $(A + B)$ and smaller than $(A - B)$ or $(B - A)$.

Ans. $R = \sqrt{A^2+B^2+2AB\cos\theta}$

$R_{max} = \sqrt{A^2+B^2+2AB\cos 0°} = (A+B)$; $R_{min} = \sqrt{A^2+B^2+2AB\cos 180°} = (A-B)$ or $(B-A)$

Q.6. A boat is moving with a velocity of $(3\hat{i}+4\hat{j})$ *w.r.t.* ground. The water in the river is moving with a velocity $(-3\hat{i}-4\hat{j})$ *w.r.t* ground. What is the relative velocity of boat *w.r.t.* water?

Ans. $v_{BG} = (3\hat{i}+4\hat{j})$; $v_{WG} = (-3\hat{i}-4\hat{j})$

$\therefore$ Velocity of boat *w.r.t.* water, $v_{BW} = v_{BG} - v_{WG} = (3\hat{i}+4\hat{j}) - (-3\hat{i}-4\hat{j}) = 6\hat{i}+8\hat{j}$

Q.7. Give two examples of (*i*) dot product and (*ii*) vector product of two vectors.

Ans. Refer to Art. 8.18 and Art. 8.21.

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Q.8. There are two displacements, one of magnitude 3 m and another of magnitude 4 m. How the displacement vectors should be added to get a resultant displacement of 7 m?

Ans. $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$. Here $A = 3$ m ; $B = 4$ m and $R = 7$ m.

$\therefore \quad 7 = \sqrt{3^2 + 4^2 + 2\times3\times4\cos\theta}$ or $\cos\theta = 1 \quad \therefore \quad \theta = 0°$

Q.9. A force is represented by : $\vec{F} = (2\hat{i} + 3\hat{j} + 6\hat{k})$N. What is the magnitude of force?

Ans. Magnitude of force $= \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7$ N

Q.10. At what angle the two forces $A + B$ and $A - B$ act so that their resultant is $\sqrt{3A^2 + B^2}$?

Ans. Here $P = A + B$; $Q = A - B$ and $R = \sqrt{3A^2 + B^2}$

$$R^2 = (A + B)^2 + (A - B)^2 + 2(A + B)(A - B)\cos\theta$$

or $3A^2 + B^2 = 2(A^2 + B^2) + 2(A^2 - B^2)\cos\theta$ or $\cos\theta = 1/2 \quad \therefore \quad \theta = 60°$

Q.11. What is the magnitude of the resultant of two forces, one 6N due east and the other 8N due north?

Ans. Resultant $= \sqrt{6^2 + 8^2} = 10$ N $(\because \theta = 90°)$

Q.12. What conclusions you draw from the following equations? (*i*) $\vec{A} + \vec{B} = \vec{C}$ and $A + B = C$ (*ii*) $\vec{A} + \vec{B} = \vec{A} - \vec{B}$ (*iii*) $\vec{A} + \vec{B} = \vec{C}$ and $A^2 + B^2 = C^2$.

Ans. (*i*) In this case, the two vectors are in the same direction because $A + B = C$.

(*ii*) In this case, $\vec{B}$ must be zero because $\vec{A} + \vec{B} = \vec{A} - \vec{B}$ or $\vec{B} = 0$.

(*iii*) In this case, $\vec{A}$ is perpendicular to $\vec{B}$ because resultant $C = \sqrt{A^2 + B^2}$.

Q.13. A force on a body is 100 N, 60° west of south. Find the north and east components of the force.

Ans. Fig. 8.43 shows the conditions of the problem. It is clear that force F (= 100 N) makes an angle of 120° with north direction and 150° with east direction.

$\therefore F_{north} = 100\cos 120° = -50$ N; $F_{east} = 100\cos 150° = -86.6$ N

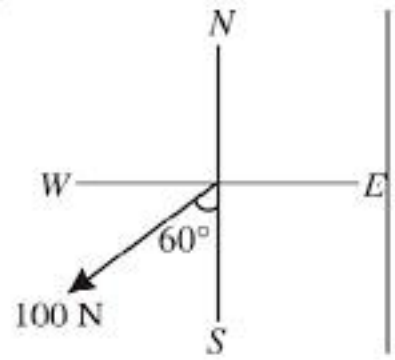


Fig. 8.43

Q.14. Find the magnitude of the resultant of two forces impressed on a particle: 3 N due north-east and 4N south-east.

Ans. Resultant $= \sqrt{3^2 + 4^2} = 5$ N $(\because \theta = 90°)$

Q.15. What is the maximum number of components into which a vector can be resolved?

Ans. A vector can be resolved into any number of components (infinite). However, a vector can be resolved into only two rectangular components.

Q.16. What is the magnitude and direction of $(\hat{i} + \hat{j})$?

Ans. $|\hat{i} + \hat{j}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$. Suppose the vector $(\hat{i} + \hat{j})$ makes an angle θ with the direction of $\hat{i}$. Then, $\tan\theta = 1/1 = 1$ or $\theta = 45°$.

Q.17. If $\vec{A} = 2\hat{i} + 4\hat{j} - 5\hat{k}$, find (*i*) magnitude of $\vec{A}$ (*ii*) the direction cosines of $\vec{A}$.

Ans. (*i*) $|\vec{A}| = \sqrt{(2)^2 + (4)^2 + (-5)^2} = \sqrt{45}$

(*ii*) $\cos\alpha = 2/\sqrt{45}$; $\cos\beta = 4/\sqrt{45}$; $\cos\gamma = -5/\sqrt{45}$

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Q.18. Two forces of 5 N and 10 N are inclined to each other at an angle of 120°. What is the angle which the resultant makes with 10N?

Ans. Here $P = 10$ N ; $Q = 5$ N ; $\theta = 120°$; $\alpha = ?$

$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta} = \frac{5 \sin 120°}{10 + 5 \cos 120°} = \frac{1}{\sqrt{3}} \quad \therefore \alpha = 30°$$

Q.19. If the magnitudes of vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are 3, 4 and 5 units respectively and if $\vec{A} + \vec{B} = \vec{C}$, find (*i*) angle between $\vec{A}$ and $\vec{B}$ (*ii*) angle between $\vec{A}$ and $\vec{C}$.

Ans. (*i*) Since $3^2 + 4^2 = 5^2$ (*i.e.*, $A^2 + B^2 = C^2$), the angle between $\vec{A}$ and $\vec{B}$ is 90°.

(*ii*) If θ is the angle between $\vec{A}$ and $\vec{C}$, then $\cos \theta = A/C = 3/5$. Therefore, $\theta = \cos^{-1}(3/5)$.

Q.20. Determine the vector which when added to the resultant of $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ gives unit vector along *y*-direction.

Ans. $\vec{A} + \vec{B} = (3\hat{i} - 5\hat{j} + 7\hat{k}) + (2\hat{i} + 4\hat{j} - 3\hat{k}) = 5\hat{i} - \hat{j} + 4\hat{k}$

The unit vector along *y*-direction $= \hat{j}$. Therefore, required vector $= \hat{j} - (5\hat{i} - \hat{j} + 4\hat{k}) = -5\hat{i} + 2\hat{j} - 4\hat{k}$.

Q.21. What is the angle between $\vec{A}$ and $\vec{B}$, if $\vec{A}$ and $\vec{B}$ denote the adjacent sides of a parallelogram drawn from a point and the area of the parallelogram is $\frac{1}{2}AB$?

Ans. Area of parallelogram $= |\vec{A} \times \vec{B}| = AB \sin \theta$

But it is given that area of parallelogram $= AB/2$.

$\therefore$ $AB \sin \theta = AB/2$ or $\sin \theta = 1/2$ $\therefore$ $\theta = 30°$.

Q.22. Can a vector be multiplied with both dimensional and non-dimensional scalars?

Ans. Yes. When a vector is multiplied with a dimensional scalar, the resultant vector will have different dimensions. For example, if velocity vector is multiplied with mass (a dimensional scalar), the resultant vector has the dimensions of momentum. When a vector is multiplied by a non-dimensional scalar, it will be a vector having dimensions as that of the given vector.

LONG ANSWER QUESTIONS

1. Explain (*i*) negative vectors (*ii*) equal vectors. **[Refer to Art. 8.2]**
2. Explain (*i*) unit vector and (*ii*) null vector. **[Refer to Art. 8.2]**
3. What do you mean by resultant of two vectors ? How will you add two vectors graphically? **[Refer to Art. 8.4]**
4. State parallelogram law of vector addition. Find analytically the magnitude and direction of the resultant of two vectors when they are inclined at an angle θ. **[Refer to Arts. 8.4 and 8.6]**
5. State and explain triangle law and polygon law of addition of vectors. **[Refer to Arts. 8.4, 8.5 and 8.6]**
6. Explain dot product and vector product of two vectors. **[Refer to Arts. 8.17 and 8.20]**
7. Explain rectangular components of a vector in a plane. **[Refer to Art. 8.13]**

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COMPETITION SUCCESS MATERIAL

Useful Concepts/Information

1. Vectors are quantities that have both magnitude and direction. Scalars are quantities that have only magnitude.
2. Vectors are used to describe motion in two or three dimensions.
3. Vectors can be added using either the *triangle method* or *parallelogram method*. In the triangle method [See Fig. 8.44 (*i*)], the vector $\vec{R} = \vec{P} + \vec{Q}$ runs from the tail of $\vec{P}$ to the tip of $\vec{Q}$. In parallelogram method [See Fig. 8.44 (*ii*)], $\vec{R}$ is the diagonal of the parallelogram having $\vec{P}$ and $\vec{Q}$ as its adjacent sides. The order in which we add vectors does not change the resultant vector *i.e.*,

$$\vec{R} = \vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

We can find the magnitude and direction of the resultant vector analytically.

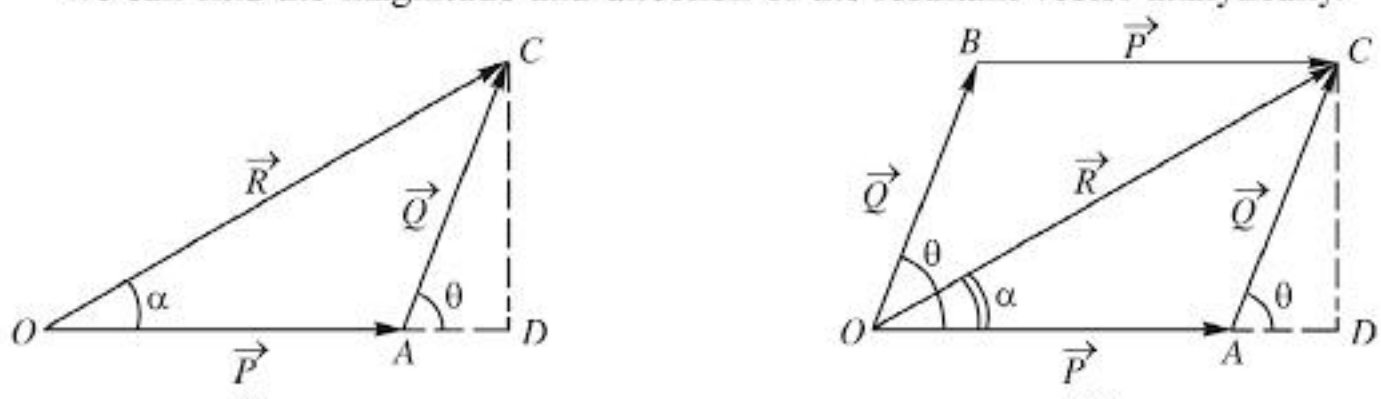


Fig. 8.44

Magnitude of resultant in Fig. 8.44 (*i*) or Fig 8.44 (*ii*) is

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

The direction which the resultant vector $\vec{R}$ makes with $\vec{P}$ is

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

4. A vector in a plane can also be described by giving its components on a chosen set of coordinates. The x component of vector $\vec{A}$ is equivalent to its projection along x-axis *i.e.*, $A_x = A\cos\theta$ as shown in Fig. 8.45. Here θ is the angle (measured CCW) that vector $\vec{A}$ makes with positive direction of x-axis. Similarly, the y component of $\vec{A}$ is its projection along the y-axis *i.e.*, $A_y = A\sin\theta$.

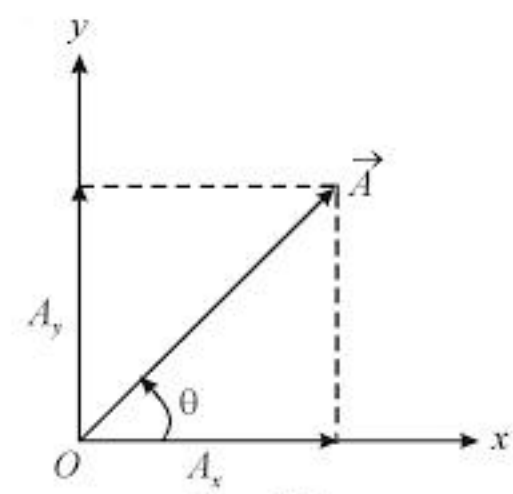


Fig. 8.45

5. The vectors in a plane can be added by using the method of components. Thus to find the resultant vector $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \ldots$, we add algebraically the x components of the vectors to find R_x and then add y components algebraically to find R_y.

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$$R_x = A_x + B_x + C_x + \ldots ; \; R_y = A_y + B_y + C_y + \ldots$$

Magnitude of resultant, $R = \sqrt{R_x^2 + R_y^2}$

The angle θ (measured CCW) that resultant vector makes with positive x-axis is

$$\tan\theta = \frac{R_y}{R_x}$$

6. If a vector $\vec{A}$ in a plane has an x component equal to A_x and y component equal to A_y, the vector can be represented in unit vector form as :

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

In this notation, $\hat{i}$ is unit vector in the positive x direction and $\hat{j}$ is a unit vector in the positive y direction. Since $\hat{i}$ and $\hat{j}$ are unit vectors, $|\hat{i}| = |\hat{j}| = 1$.

7. A vector $\vec{A}$ in three dimensions can be represented as $\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$.

8. **Dot or scalar product of two vectors.** The scalar or dot product of two vectors $\vec{A}$ and $\vec{B}$ is defined as a scalar quantity equal to the product of the magnitudes of two vectors and cosine of smaller angle between them *i.e.*

$$\vec{A}.\vec{B} = AB\cos\theta$$

where θ is the smaller angle between $\vec{A}$ and $\vec{B}$

as shown in Fig. 8.46. Note that $\vec{A}$ and $\vec{B}$ need not have the same units. Note in Fig. 8.46, $B\cos\theta$ is the projection of $\vec{B}$ onto $\vec{A}$ *i.e.*, $\vec{A}.\vec{B}$ =(Magnitude of $\vec{A}$) (Projection of $\vec{B}$ onto $\vec{A}$).

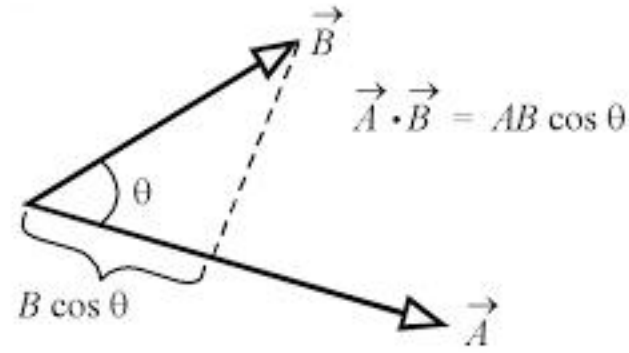


Fig. 8.46

The following properties of dot product are worth noting :

(*i*) $$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

i.e. order of the dot product can be reversed.

(*ii*) The dot product obeys the *distributive law of multiplication* so that :

$$\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$$

(*iii*) The scalar or dot product of unit vectors are:

$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1; \; \hat{i}.\hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{k} = 0$$

(*iv*) The two vectors $\vec{A}$ and $\vec{B}$ can be expressed in component form as :

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}; \; \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

Using the property of dot product of unit vectors, we have,

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

In special case when $\vec{A} = \vec{B}$, we have,

$$\vec{A}.\vec{A} = A_x^2 + A_y^2 + A_z^2 = A^2$$

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9. Cross or vector product. Consider two vectors $\vec{A}$ and $\vec{B}$ as shown in Fig. 8.47 where angle θ is the smaller angle between the two vectors. The vector or cross product of vectors $\vec{A}$ and $\vec{B}$ is another vector $\vec{C}$ given by ; $\vec{C} = \vec{A} \times \vec{B}$

It is called vector product of $\vec{A}$ and $\vec{B}$ because $\vec{C}$ is itself a vector.

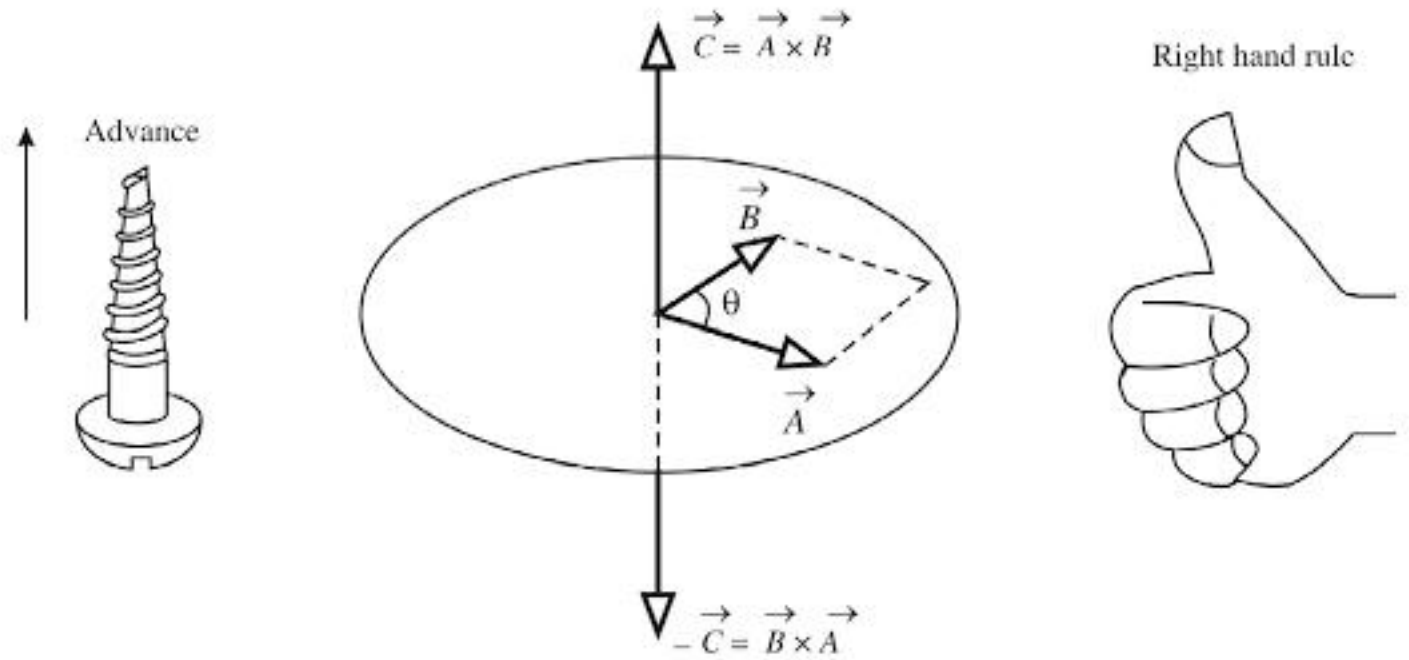


Fig. 8.47

Magnitude of $\vec{C}$. The magnitude of $\vec{C}$ is given by ;

$$C = |\vec{C}| = AB \sin\theta$$

Note that the quantity $AB\sin\theta$ is equal to the area of the parallelogram formed by $\vec{A}$ and $\vec{B}$ as shown in Fig. 8.47.

Direction and sense of $\vec{C}$. The direction of $\vec{C}$ $(= \vec{A} \times \vec{B})$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$ as shown in Fig. 8.47 and its sense is determined by the advance of a right-handed screw when turned from $\vec{A}$ to $\vec{B}$ through the *smaller* angle θ. A more convenient rule to use for the direction of $\vec{A} \times \vec{B}$ is the right-hand rule illustrated in Fig. 8.47. The four fingers of the right hand are pointed along $\vec{A}$ and then "wrapped" into $\vec{B}$ through the angle θ. The direction of the erect right thumb is the direction of $\vec{A} \times \vec{B}$. Because of the notation, $\vec{A} \times \vec{B}$ is often read "A cross B", hence the term *cross product.* Some properties of the vector product which follow from its definition are :

(*i*) Unlike the scalar product, the order in which the two vectors are multiplied in a cross product is important *i.e.*

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

Therefore, if you change the order of cross product, you must change the sign. You can easily verify this relation with right-hand rule (See Fig. 8.47).

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(*ii*) If $\vec{A}$ is parallel to $\vec{B}$ (*i.e.* $\theta = 0°$ or 180°), then $\vec{A} \times \vec{B} = 0$. It follows, therefore, $\vec{A} \times \vec{A} = 0$.

(*iii*) If $\vec{A}$ is perpendicular to $\vec{B}$, then $|\vec{A} \times \vec{B}| = AB$.

(*iv*) It is also important to note that vector product obeys the *distributive law i.e.*

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

(*v*) The derivative of cross product w.r.t. some variable such as t is given by ;

$$\frac{d}{dt}(\vec{A} \times \vec{B}) = \vec{A} \times \frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt} \times \vec{B}$$

Here it is important to preserve the multiplication order of $\vec{A}$ and $\vec{B}$.

(*vi*) The cross products of unit vectors are given below :

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0; \quad \hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$$

$$\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}; \quad \hat{k} \times \hat{i} = -\hat{i} \times \hat{k} = \hat{j}$$

Note that signs are interchangeable. For example,

$$\hat{i} \times (-\hat{j}) = -\hat{i} \times \hat{j} = -\hat{k}$$

(*vii*) The cross product of any two vectors $\vec{A}$ and $\vec{B}$ can be expressed in the following determinant form :

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Expanding this determinant gives the result :

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. If $\vec{P}.\vec{Q} = |\vec{P} \times \vec{Q}|$, then the angle between $\vec{P}$ and $\vec{Q}$ is **[AFMC – Pune 2001]**

(*a*) 0 (*b*) $\frac{\pi}{2}$

(*c*) $\frac{\pi}{4}$ (*d*) π

2. If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ will be **[CBSE PMT 2001]**

(*a*) 90° (*b*) 120°

(*c*) 0° (*d*) 60°

3. Two equal vectors have a resultant equal to either of them. The angle between them is **[AIIMS 2001]**

(*a*) 60° (*b*) 90°

(*c*) 100° (*d*) 120°

4. A force $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$ acting on a particle causes displacement $\vec{S} = -4\hat{i} + 2\hat{j} + 3\hat{k}$ in its own direction. If work done is 6 J, then value of 'c' is **[Pb PMT 2001]**

(*a*) 0 (*b*) 1

(*c*) 6 (*d*) 12

5. $\vec{P}.\vec{Q} = PQ$, then angle between $\vec{P}$ and $\vec{Q}$ is **[AIIMS 1999]**

(*a*) 0° (*b*) 30°

(*c*) 45° (*d*) 60°

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6. The vectors $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular to each other. The positive value of 'a' is **[AFMC-Pune 2000]**

(a) 3 (b) 4
(c) 9 (d) 13

7. For a body, angular velocity $\vec{\omega} = \hat{i} - 2\hat{j} + 3\hat{k}$ and radius vector $\vec{r} = \hat{i} + \hat{j} + \hat{k}$. Then linear velocity is **[CBSE PMT 1999]**

(a) $-5\hat{i} + 2\hat{j} + 3\hat{k}$ (b) $-5\hat{i} + 2\hat{j} - 3\hat{k}$
(c) $-5\hat{i} - 2\hat{j} + 3\hat{k}$ (d) $-5\hat{i} - 2\hat{j} - 3\hat{k}$

8. The force acting on a body of 10 kg is $(2\hat{i} + \hat{j} - \hat{k})$ N. If the body is initially at rest, then velocity at the end of 20 sec will be (in m/s) **[Pb. PMT 1997]**

(a) $2\sqrt{6}$ (b) $6\sqrt{2}$
(c) $3\sqrt{2}$ (d) $2\sqrt{3}$

9. The angle between two vectors of magnitude 12 and 18 units when their resultant is 24 units is **[AIIMS 1997]**

(a) 63.50° (b) 75.52°
(c) 82.30° (d) 89.16°

10. The angle between two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ will be **[AIIMS 1996]**

(a) zero (b) 45°
(c) 90° (d) 180°

11. A bus is going due north at a speed of 50 km/h. It makes 90° left turn without changing its speed. The change in the velocity of the bus is **[CBSE PMT 1990]**

(a) 50 km/h towards west
(b) 50 km/h towards south-west
(c) 70.7 km/h towards south-west
(d) 70.7 km/h towards north-west

12. Two vectors $\vec{A}$ and $\vec{B}$ are such that $\left|\vec{A} + \vec{B}\right| = \left|\vec{A} - \vec{B}\right|$. Then the angle between the two vectors will be **[CBSE PMT 1991]**

(a) 45° (b) 60°
(c) 75° (d) 90°

13. A force $\vec{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})$ N acting on a mass produces an acceleration of 1 m/s^2. What will be the mass of the body? **[CBSE PMT 1996]**

(a) $10\sqrt{2}$ kg (b) $2\sqrt{10}$ kg
(c) 10 kg (d) 20 kg

14. Two quantities A and B have different dimensions. Which mathematical operation given below is physically meaningful? **[CPMT 1997]**

(a) A/B (b) $A + B$
(c) $A - B$ (d) none

15. The sum of two forces at a point is 16 N. If their resultant is normal to the smaller force and has a magnitude of 8N, then the forces are **[Pb. PMT 1995]**

(a) 6N and 10N (b) 8N and 8N
(c) 4N and 12N (d) 2N and 14N

16. A boat is sent across a river with a velocity of 8 km/h. If the resultant velocity of boat is 10 km/h, then velocity of river is **[CBSE PMT 1994]**

(a) 12.8 km/h (b) 6 km/h
(c) 8 km/h (d) 10 km/h

17. Which of the following cannot be the resultant of the vectors of magnitudes 5 and 10? **[AFMC-Pune 1996]**

(a) 7 (b) 8
(c) 5 (d) 2

18. The vector sum of two forces is perpendicular to their vector difference. In that case, the forces **[CBSE PMT 2003]**

(a) are not equal to each other in magnitude
(b) cannot be predicted
(c) are equal to each other
(d) are equal to each other in magnitude

19. An aeroplane flies 400 m due north and then 300 m due south and then flies 1200 m upwards, the net displacement is: **[AFMC 2004]**

(a) greater than 1200 m
(b) less than 1200 m
(c) 1400 m
(d) 1500 m

20. Minimum number of unequal vectors which can give zero resultant are: **[AFMC 2005]**

(a) two (b) three
(c) four (d) more than four

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ANSWERS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. (*c*)	**2.** (*b*)	**3.** (*d*)	**4.** (*c*)	**5.** (*a*)
6. (*a*)	**7.** (*a*)	**8.** (*a*)	**9.** (*b*)	**10.** (*c*)
11. (*d*)	**12.** (*d*)	**13.** (*a*)	**14.** (*a*)	**15.** (*a*)
16. (*b*)	**17.** (*d*)	**18.** (*d*)	**19.** (*a*)	**20.** (*b*)

HINTS TO MCQ FROM MEDICAL ENTRANCE EXAMINATIONS

1. $\vec{P}.\vec{Q} = PQ\cos\theta$ and $|\vec{P}\times\vec{Q}| = PQ\sin\theta$

As per problem, $PQ\cos\theta = PQ\sin\theta$ or $\tan\theta = 1$ $\therefore$ $\theta = \pi/4$

2. Let angle between $\vec{A}$ and $\vec{B}$ be θ.

$\therefore$ $A^2 + B^2 + 2AB\cos\theta = A^2$ or $A^2 + A^2 + 2A^2\cos\theta = A^2$

or $2A^2(1+\cos\theta) = A^2$ or $\cos\theta = -\frac{1}{2}$ $\therefore$ $\theta = 120°$

3. Solution is the same as MCQ No. 2.

4. Work done $= \vec{F}.\vec{S}$ or $6 = (3\hat{i} + c\hat{j} + 2\hat{k}).(-4\hat{i} + 2\hat{j} + 3\hat{k})$

or $6 = -12 + 2c + 6$ $\therefore$ $c = 6$

5. $\vec{P}.\vec{Q} = PQ$ ... given or $PQ\cos\theta = PQ$ or $\cos\theta = \frac{PQ}{PQ} = 1$ $\therefore$ $\theta = 0°$

6. We know that if two vectors are perpendicular to each other, then their dot product is zero.

$\therefore$ $\vec{P}.\vec{Q} = 0$ or $(a\hat{i} + a\hat{j} + 3\hat{k}).(a\hat{i} - 2\hat{j} - \hat{k}) = 0$ or $a^2 - 2a - 3 = 0$

On solving this equation, $a = 3$ or -1. Therefore, positive value of 'a' is **3.**

7. We know that linear velocity $\vec{v}$ is the cross product of angular velocity $(\vec{\omega})$ and radius vector $(\vec{r})$.

$$\therefore \quad \vec{v} = \vec{\omega}\times\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k})\times(\hat{i} + \hat{j} + \hat{k}) \quad \text{or} \quad \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$

or $\vec{v} = (-2-3)\hat{i} + (3-1)\hat{j} + (1+2)\hat{k}$ or $\vec{v} = \mathbf{-5\hat{i} + 2\hat{j} + 3\hat{k}}$

8. Acceleration, $\vec{a} = \frac{\vec{F}}{m} = \frac{2\hat{i} + \hat{j} - \hat{k}}{10}$ m/s^2. Here $\vec{u} = 0$; $t = 20$ sec ; $\vec{v} = ?$

$$\vec{v} = \vec{u} + \vec{a}t = 0 + \left(\frac{2\hat{i} + \hat{j} - \hat{k}}{10}\right)\times 20 = 4\hat{i} + 2\hat{j} - 2\hat{k}$$

$\therefore$ Magnitude of velocity, $v = \sqrt{4^2 + 2^2 + (-2)^2} = \mathbf{2\sqrt{6}}$ **m/s**

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9. $$R^2 = A^2 + B^2 + 2AB\cos\theta$$

or $$(24)^2 = (12)^2 + (18)^2 + 2\times 12\times 18\cos\theta$$

or $$\cos\theta = 0.25 \text{ or } \theta = \mathbf{75.52°}$$

10. $$\vec{A}.\vec{B} = (3\hat{i} + 4\hat{j} + 5\hat{k}).(3\hat{i} + 4\hat{j} - 5\hat{k}) = 9 + 16 - 25 = 0$$

But $$\vec{A}.\vec{B} = AB\cos\theta \quad \therefore \quad \cos\theta = \frac{\vec{A}.\vec{B}}{AB} = \frac{0}{AB} = 0 \quad \text{or} \quad \theta = \mathbf{90°}$$

11. Change in velocity $= \sqrt{(50)^2 + (50)^2} = \mathbf{70.7\ km/h}$

The direction of this change in velocity is **north-west.**

12. Let θ be the angle between the two vectors.

$$|\vec{A} + \vec{B}|^2 = A^2 + B^2 + 2AB\cos\theta \quad \text{and} \quad |\vec{A} - \vec{B}|^2 = A^2 + B^2 - 2AB\cos\theta$$

As per the condition in the problem,

$$A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta \quad \therefore \quad 4AB\cos\theta = 0$$

or $$\cos\theta = \frac{0}{4AB} = 0 \quad \therefore \quad \theta = \mathbf{90°}$$

13. $$\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$$

Magnitude of force, $$F = \sqrt{(6)^2 + (-8)^2 + (10)^2} = 10\sqrt{2} \text{ N}$$

$$\therefore \quad \text{Mass} = \frac{F}{a} = \frac{10\sqrt{2}}{1} = \mathbf{10\sqrt{2}\ kg}$$

14. The quantities having different dimensions can only be multiplied or divided but cannot be added or subtracted.

15. Let P and Q be the forces and R their resultant. $P + Q = 16$ N and $R = 8$N.

If P is the smaller force, then,

$$R^2 = Q^2 - P^2 = (Q+P)(Q-P) \text{ or } (8)^2 = (16)(Q-P) \therefore Q - P = 4$$

On solving, $P = \mathbf{6N}$, $Q = \mathbf{10N}$

16. River velocity $= \sqrt{10^2 - 8^2} = \mathbf{6\ km/h}$

17. The resultant must lie between (10 + 5) and (10 – 5) *i.e.* between 15 and 5.

18. Let $\vec{A}$ and $\vec{B}$ be the two forces.

Vector sum : $\vec{F_1} = \vec{A} + \vec{B}$; Vector difference : $\vec{F_2} = \vec{A} - \vec{B}$

Since $\vec{F_1}$ is perpendicular to $\vec{F_2}$, $\vec{F_1}.\vec{F_2} = 0$.

$\therefore \quad (\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$

or $\quad A^2 - AB + AB - B^2 = 0 \quad \therefore \quad |\vec{A}| = |\vec{B}|$

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19. Fig. 8.48 shows the conditions of the problem.

OA = 400 m (due north)

AB = 300 m (due south)

$\therefore$ $OB = OA - AB$

$= 400 - 300 = 100$ m

Also BC = 1200 m (upward)

It is clear from Fig. 8.48 that net displacement is OC. In the right angled triangle OBC,

$$OC = \sqrt{OB^2 + BC^2} = \sqrt{(100)^2 + (1200)^2} = 1204 \text{ m}$$

Hence net displacement is **greater than 1200 m**.

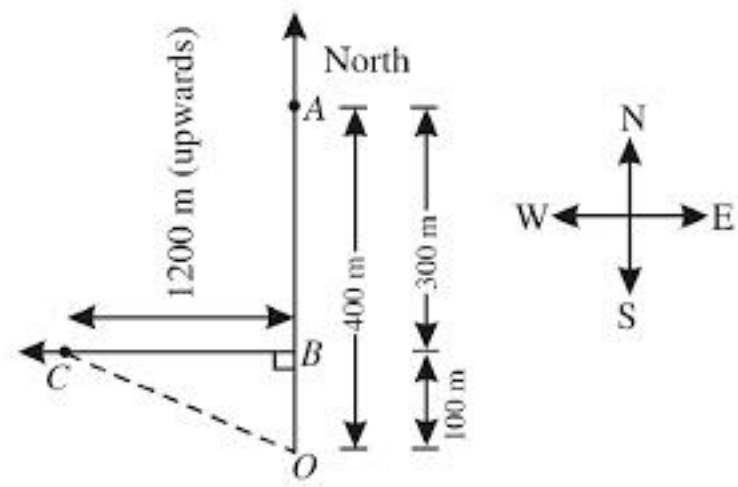


Fig. 8.48

20. According to triangle law of vectors, the minimum number of unequal vectors required to give zero resultant is **3**.

MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. The angle between two vectors $-2\hat{i}+3\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$ is **[EAMCET 1990]**

(a) 0° (b) 90°

(c) 180° (d) none

2. Find the torque of a force $\vec{F} = -3\hat{i}+\hat{j}+5\hat{k}$ acting at the point $\vec{r} = 7\hat{i}+3\hat{j}+\hat{k}$. **[Pb CET 1998]**

(a) $14\hat{i}-38\hat{j}+16\hat{k}$ (b) $4\hat{i}+4\hat{j}+16\hat{k}$

(c) $-14\hat{i}+38\hat{j}-16\hat{k}$ (d) $-21\hat{i}+3\hat{j}-5\hat{k}$

3. If vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ have magnitudes 5, 12 and 13 units and $\vec{P}+\vec{Q}=\vec{R}$, then angle between $\vec{Q}$ and $\vec{R}$ is **[Haryana CET 1998]**

(a) $\cos^{-1}\frac{5}{12}$ (b) $\cos^{-1}\frac{5}{13}$

(c) $\cos^{-1}\frac{12}{13}$ (d) $\cos^{-1}\frac{7}{13}$

4. The radius vector is $2\hat{i}+\hat{j}+\hat{k}$ while linear momentum is $2\hat{i}-3\hat{j}+\hat{k}$. The angular momentum is **[BHU 1997]**

(a) $2\hat{i}-4\hat{k}$ (b) $4\hat{i}-8\hat{k}$

(c) $2\hat{i}-4\hat{j}+2\hat{k}$ (d) $-4\hat{i}+8\hat{k}$

5. If $\vec{A} = 5\hat{i}+7\hat{j}-3\hat{k}$ and $\vec{B} = 2\hat{i}+2\hat{j}-c\hat{k}$ are perpendicular vectors, then value of c is **[CET 1991]**

(a) − 2 (b) 8

(c) − 7 (d) − 8

6. Find the unit vector perpendicular to $3\hat{i}+\hat{j}+2\hat{k}$ and $2\hat{i}-2\hat{j}+4\hat{k}$. **[Haryana CET 1996]**

(a) $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$ (b) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}+\hat{k})$

(c) $(\hat{i}-\hat{j}-\hat{k})$ (d) $\sqrt{3}\,\hat{i}-\hat{j}-\hat{k}$

7. Two forces $\vec{F_1} = 5\hat{i}+10\hat{j}-20\hat{k}$ and $\vec{F_2} = 10\hat{i}-5\hat{j}-15\hat{k}$ act on a single point. The angle between $\vec{F_1}$ and $\vec{F_2}$ is **[Haryana CET 1998]**

(a) 60° (b) 90°

(c) 45° (d) 30°

8. The relative velocity of a ship moving at V m/s East with reference to another ship moving at V m/s North is **[Haryana CET 1993]**

(a) $\sqrt{2}$ V SE (b) $\sqrt{2}$ V NE

(c) $\sqrt{2}$ V NW (d) $\sqrt{2}$ V SW

9. The angle between $\vec{A}$ and $\vec{B}$ is θ. The value of triple product $\vec{A}\cdot\vec{B}\times\vec{A}$ is **[Haryana CET 1996]**

(a) $A^2B\cos\theta$ (b) $A^2B\sin\theta$

(c) A^2B (d) zero

10. The sum of the magnitudes of two forces acting at a point is 18 and the magnitude of

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resultant is 12. If the resultant is at 90° with the force of smaller magnitude, then their magnitudes are **[Roorkee 1992]**

(*a*) 3, 15 (*b*) 4, 14

(*c*) 5, 13 (*d*) 6, 12

11. Three forces start acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle *ABC* (See Fig. 8.49). The particle will now move with velocity: **[AIEEE 2003]**

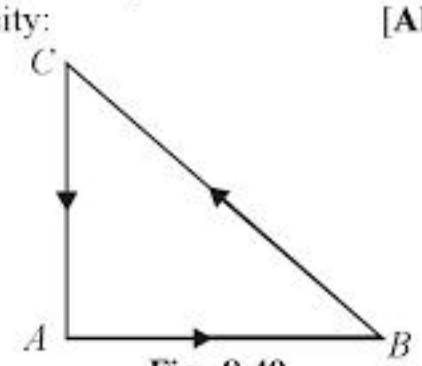


Fig. 8.49

(*a*) less than $\vec{v}$ (*b*) greater than $\vec{v}$

(*c*) $|\vec{v}|$ in the direction of largest force *BC*

(*d*) $\vec{v}$, remaining unchanged

12. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin. Then: **[AIEEE 2003]**

(*a*) $\vec{r}.\vec{F}=0$ and $\vec{F}.\vec{\tau}\neq 0$

(*b*) $\vec{r}.\vec{\tau}\neq 0$ and $\vec{F}.\vec{\tau}=0$

(*c*) $\vec{r}.\vec{\tau}\neq 0$ and $\vec{F}.\vec{\tau}\neq 0$

(*d*) $\vec{r}.\vec{\tau}=0$ and $\vec{F}.\vec{\tau}=0$

13. The coordinates of a moving particle at any time t are given by $x=\alpha t^3$ and $y=\beta t^3$. The speed of the particle at time t is given by : **[AIEEE 2003]**

(*a*) $3t\sqrt{\alpha^2+\beta^2}$ (*b*) $3t^2\sqrt{\alpha^2+\beta^2}$

(*c*) $t^2\sqrt{\alpha^2+\beta^2}$ (*d*) $\sqrt{\alpha^2+\beta^2}$

14. If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$, then the angle between $\vec{A}$ and $\vec{B}$ is : **[AIEEE 2004]**

(*a*) π (*b*) $\pi/3$

(*c*) $\pi/2$ (*d*) $\pi/4$

15. Two forces are acting at a point. The magnitude of each force is 2N and the magnitude of their resultant is also 2N. Then the angle between the two forces is **[DCE 2004]**

(*a*) 90° (*b*) 60°

(*c*) 120° (*d*) 0°

ANSWERS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. (*b*)	**2.** (*a*)	**3.** (*c*)	**4.** (*b*)	**5.** (*d*)
6. (*a*)	**7.** (*c*)	**8.** (*a*)	**9.** (*d*)	**10.** (*c*)
11. (*d*)	**12.** (*d*)	**13.** (*b*)	**14.** (*a*)	**15.** (*c*)

HINTS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. $\cos\theta=\dfrac{\vec{A}.\vec{B}}{AB}$. Now $\vec{A}.\vec{B}=(-2\hat{i}+3\hat{j}+\hat{k}).(\hat{i}+2\hat{j}-4\hat{k})=-2+6-4=0$

$\therefore \quad \cos\theta=\dfrac{0}{AB}=0$ or $\theta=90°$

2. Torque $=\vec{r}\times\vec{F}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\7&3&1\\-3&1&5\end{vmatrix}=\hat{i}(15-1)+\hat{j}(-3-35)+\hat{k}(7+9)=\mathbf{14\hat{i}-38\hat{j}+16\hat{k}}$

3. The conditions of the problem suggest $(13^2=5^2+12^2)$ that the angle between $\vec{P}$ and $\vec{Q}$ is 90°. If θ is the angle between $\vec{Q}$ and $\vec{R}$, then $\cos\theta=12/13$.

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4. Angular momentum is given by ;

$$\vec{L} = \vec{r} \times \vec{p} = (2\hat{i} + \hat{j} + \hat{k}) \times (2\hat{i} - 3\hat{j} + \hat{k}) = \mathbf{4\hat{i} - 8\hat{k}}$$

5. The scalar product of perpendicular vectors is zero.

$\therefore \quad \vec{A} . \vec{B} = 0$ or $A_x B_x + A_y B_y + A_z B_z = 0$ or $10 + 14 + 3c = 0 \quad \therefore \quad c = -8$

6. $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$; $\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

The unit vector perpendicular to $\vec{A}$ and $\vec{B}$ is $\vec{n} = \dfrac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

$$\vec{A} \times \vec{B} = (4 + 4)\hat{i} + (4 - 12)\hat{j} + (-6 - 2)\hat{k} = 8\hat{i} - 8\hat{j} - 8\hat{k}$$

$$|\vec{A} \times \vec{B}| = \sqrt{8^2 + 8^2 + 8^2} = 8\sqrt{3} \quad \therefore \quad \vec{n} = \frac{\mathbf{1}}{\sqrt{\mathbf{3}}}(\hat{\boldsymbol{i}} - \hat{\boldsymbol{j}} - \hat{\boldsymbol{k}})$$

7. $\cos\theta = \dfrac{\vec{F_1} . \vec{F_2}}{F_1 F_2} = \dfrac{(5\hat{i} + 10\hat{j} - 20\hat{k}).(10\hat{i} - 5\hat{j} - 15\hat{k})}{(\sqrt{5^2 + 10^2 + (-20)^2})\sqrt{10^2 + (-5)^2 + (-15)^2}}$

$$= \frac{5 \times 10 - 10 \times 5 + (-20) \times (-15)}{(\sqrt{525})(\sqrt{350})} \quad 0.7$$

$\therefore \quad \theta \quad 45°$

8. Fig. 8.50 shows the conditions of the problem.

Relative velocity, $V_R = \sqrt{V^2 + V^2} = \sqrt{2}$ **V SE**

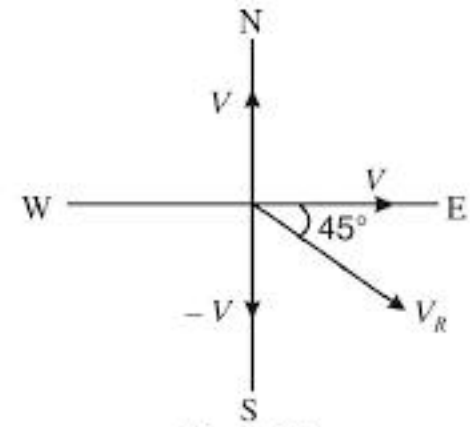


Fig. 8.50

9. $\vec{B} \times \vec{A}$ is a vector which is perpendicular to $\vec{A}$ and $\vec{B}$. Its dot product with $\vec{A}$ will be **zero** as $\vec{A}$ and $\vec{B} \times \vec{A}$ are perpendicular to each other.

10. Fig. 8.51 shows the conditions of the problem.

$R = 12$; $Q + P = 18$; $Q^2 = P^2 + R^2$ or $Q^2 - P^2 = R^2 = 144$

$\therefore \quad (Q - P)\ (Q + P) = 144$

or $$(Q - P) = \frac{144}{Q + P} = \frac{144}{18} = 8$$

$Q + P = 18$ and $Q - P = 8$. On solving, $Q = 13$ and $P = 5$.

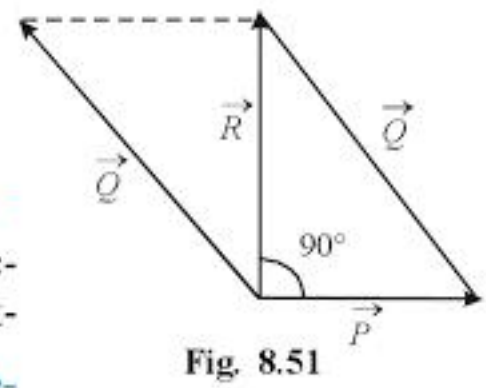


Fig. 8.51

11. Since the three forces are represented in magnitude and direction by the three sides of a triangle in the same order, the resultant force is zero. Therefore, the particle velocity $(\vec{v})$ **will remain unchanged.**

12. $\vec{\tau} = \vec{r} \times \vec{F}$ means that $\vec{r}, \vec{F}$ and $\vec{\tau}$ are perpendicular to each other.

$\therefore \quad \vec{\tau} . \vec{r} = \mathbf{0}$ **and** $\vec{F} . \vec{\tau} = \mathbf{0}$

13. $x = \alpha t^3$; $y = \beta t^3$

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$$\therefore\ v_x = \frac{dx}{dt} = \frac{d}{dt}(\alpha t^3) = 3\alpha t^2\ ;\ v_y = \frac{dy}{dt} = \frac{d}{dt}(\beta t^3) = 3\beta t^2$$

$$\therefore\ \text{Resultant velocity} = \sqrt{v_x^2 + v_y^2} = \sqrt{(3\alpha t^2)^2 + (3\beta t^2)^2} = \mathbf{3t^2\sqrt{\alpha^2 + \beta^2}}$$

14. $\vec{A} \times \vec{B} = AB \sin\theta\ ;\ \vec{B} \times \vec{A} = BA \sin\theta = -AB \sin\theta$

Now $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ or $AB \sin\theta = -AB \sin\theta$

$\therefore\ AB \sin\theta + AB \sin\theta = 0$ or $2AB \sin\theta = 0$

Since A and B are not zero, $\sin\theta = 0$ or $\theta = 0$ or π

15. $$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos\theta$$

or $$2^2 = 2^2 + 2^2 + 2 \times 2 \times 2 \cos\theta$$

$\therefore$ $\cos\theta = -4/8 = -1/2$ or $\theta = 120°$

NUMERICAL PROBLEMS FOR COMPETITIVE EXAMINATIONS

1. At what angle do the two forces $(P + Q)$ and $(P - Q)$ act so that the resultant is $\sqrt{3P^2 + Q^2}$?

Hint. Let $X = P + Q$ and $Y = P - Q$. Now $R = \sqrt{3P^2 + Q^2}$. Suppose the angle between X and Y is θ.

Now $$R^2 = X^2 + Y^2 + 2XY\cos\theta$$

or $$3P^2 + Q^2 = (P+Q)^2 + (P-Q)^2 + 2(P+Q)(P-Q)\cos\theta$$

or $$3P^2 + Q^2 = 2P^2 + 2Q^2 + 2(P^2 - Q^2)\cos\theta$$

or $$P^2 - Q^2 = 2(P^2 - Q^2)\cos\theta$$

$\therefore$ $$\cos\theta = \frac{P^2 - Q^2}{2(P^2 - Q^2)} = \frac{1}{2} \quad \text{or} \quad \theta = 60°$$

2. A ball is given simultaneously two velocities, one of 10 m/s due east and the other 20 m/s due north-west. Calculate the resultant velocity of the ball.

Hint. Figure 8.52 shows the conditions of the problem.

$$v_1 = 10\ \text{ms}^{-1};\ v_2 = 20\ \text{ms}^{-1}\ ;\ \theta = 135°$$

The resultant velocity v is given by ;

$$v^2 = v_1^2 + v_2^2 + 2v_1v_2\cos\theta$$

$$= (10)^2 + (20)^2 + 2 \times 10 \times 20 \cos 135° = 217.2$$

$\therefore$ $$v = \sqrt{217.2} = 14.73\ \text{ms}^{-1}$$

Suppose α is the angle which the resultant makes with velocity v_2 (= 20 ms^{-1}). Then,

$$\tan\alpha = \frac{v_1 \sin\theta}{v_2 + v_1\cos\theta} = \frac{10\sin 135°}{20 + 10\cos 135°} = 0.5518$$

$\therefore$ $$\alpha = \tan^{-1} 0.5518 = 28.9°$$

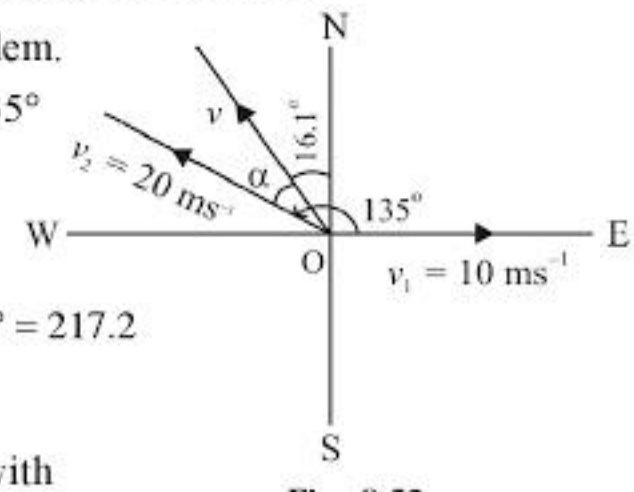


Fig. 8.52

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$\therefore$ Angle which v makes with north = 45° – 28.9° = 16.1°

Hence, the resultant velocity is **14.73 ms^{-1} and it acts 16.1° west of north.**

3. A river is flowing from west to east at a speed of 5 m/min. In what direction should a man on the south bank of the river capable of swimming at 10 m/min in still water should swim to cross the river (i) in shortest time (ii) along shortest path. [I.I.T.]

Hint. (*i*) Suppose the man is at point O on the southern bank of the river and B is a point, opposite to O, on the other side of the bank [See Fig. 8.53]. Suppose the man swims with velocity v in a direction OA making an angle θ with OB as shown in Fig. 8.53. The velocity v can be resolved into two rectangular components, *viz.*, $v\cos\theta$ along OB and $v\sin\theta$ acting opposite to river flow. Only the component $v\cos\theta$ is helping the swimmer to cross the river. If d is the width of the river, then time t taken by the swimmer to cross the river is given by;

$$t = \frac{d}{v\cos\theta}$$

The time t will be minimum when $\cos\theta$ is maximum *i.e.*, $\cos\theta = 1$ or $\theta = 0°$. Therefore, in order to cross the river in minimum time, the man should swim **perpendicular to the flow of river.**

$$t_{min} = \frac{d}{v}$$

Note that due to the flow of water, the man will reach point C on the opposite bank where

$$BC = v_w \times t_{min} = v_w \times \frac{d}{v}$$

Here v_w = velocity of river flow

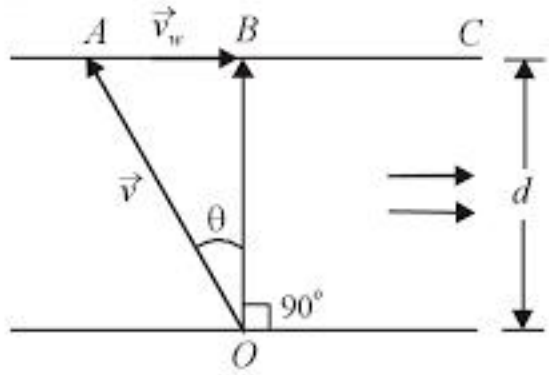


Fig. 8.53

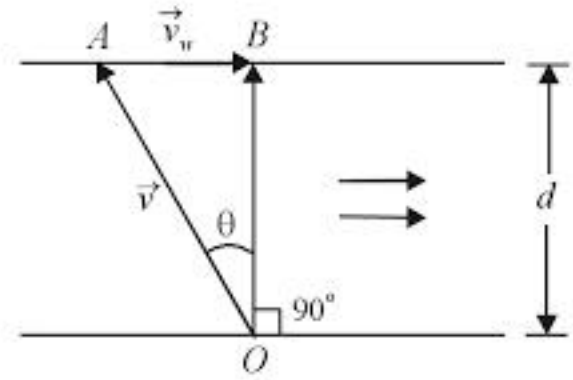


Fig. 8.54

(*ii*) The shortest path between the two banks of the river is OB [See Fig. 8.54]. If the river water had been still, the man could directly go from O to B. In order to cross the flowing river along OB, the man should swim along OA making an angle θ such that the horizontal component $v\sin\theta$ of v cancels the velocity v_w of river flow. Note that $v\sin\theta$ and v_w act in opposite directions.

$\therefore$ $$v_w = v\sin\theta$$

or $$\sin\theta = \frac{v_w}{v} = \frac{5}{10} = \frac{1}{2}$$

or $$\theta = \textbf{30° west of north}$$

The time t' taken by the man to cross the river is

$$t' = \frac{d}{v\cos\theta} = \frac{t_{min}}{\cos\theta}$$

Note that time taken in this case is not minimum.

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4. Find the angle between X-axis and the force represented by ;

$$\vec{F} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

Hint. Magnitude of force is given by ;

$$F = \sqrt{(2)^2 + (3)^2 + (4)^2} = \sqrt{29}$$

$$\therefore \quad \cos\theta = \frac{F_x}{F} = \frac{2}{\sqrt{29}} \quad \therefore \ \theta = \cos^{-1}\frac{2}{\sqrt{29}}$$

5. A force of $7\hat{i} + 6\hat{k}$ newton makes a body move on a rough plane with a velocity $3\hat{j} + 4\hat{k}$ ms^{-1}. Calculate the power in watt.

Hint. Power $= \vec{F}.\vec{v} = (7\hat{i} + 6\hat{k}).(3\hat{j} + 4\hat{k}) = 6 \times 4 = 24$ **W**

6. The resultant of two vectors $\vec{u}$ and $\vec{v}$ is perpendicular to $\vec{u}$ and its magnitude is equal to half of the magnitude of vector $\vec{v}$. Find out the angle between $\vec{u}$ and $\vec{v}$.

Hint. Figure 8.55 shows the conditions of the problem.

$$|\vec{v}| = v \ ; \quad |\vec{R}| = \frac{v}{2}$$

$$\therefore \quad \sin\theta = \frac{v/2}{v} = \frac{1}{2}$$

or $\quad \theta = 30°$

$\therefore$ Angle between u and $v = 180° - 30° = 150°$

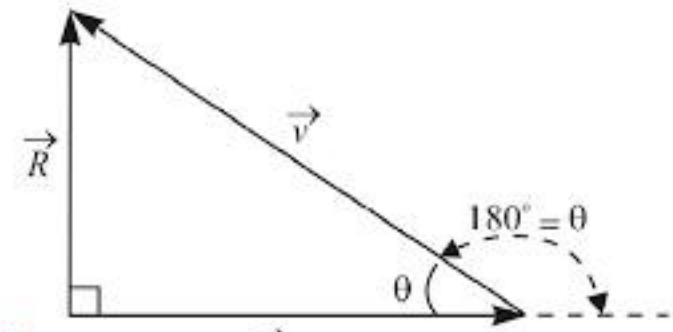


Fig. 8.55

7. (*i*) For two vectors $\vec{A}$ and $\vec{B}$ if $\vec{A} + \vec{B} = \vec{C}$ and $A + B = C$, then prove that vectors $\vec{A}$ and $\vec{B}$ are parallel to each other.

(*ii*) If $\vec{A} + \vec{B} = \vec{C}$ and $A^2 + B^2 = C^2$, prove that vectors $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

Hint. (*i*) $\vec{A} + \vec{B} = \vec{C}$

Taking the self-dot product of both sides, we have,

$$(\vec{A} + \vec{B}).(\vec{A} + \vec{B}) = \vec{C}.\vec{C} \quad \text{or} \quad \vec{A}.\vec{A} + \vec{A}.\vec{B} + \vec{B}.\vec{A} + \vec{B}.\vec{B} = \vec{C}.\vec{C}$$

Now $\quad \vec{A}.\vec{B} = \vec{B}.\vec{A} = AB\cos\theta$ where θ is the angle between vectors $\vec{A}$ and $\vec{B}$.

$$\therefore \quad A^2 + AB\cos\theta + AB\cos\theta + B^2 = C^2$$

or $\quad A^2 + B^2 + 2AB\cos\theta = C^2 \quad \ldots (i)$

It is given that $A + B = C$ or $A^2 + B^2 + 2AB = C^2 \quad \ldots (ii)$

Comparing eqs. (*i*) and (*ii*), $\cos\theta = 1$ or $\theta = 0°$. Hence, vectors $\vec{A}$ and $\vec{B}$ are parallel and act in the same direction.

(*ii*) $\quad \vec{A} + \vec{B} = \vec{C}$ or $(\vec{A} + \vec{B}).(\vec{A} + \vec{B}) = \vec{C}.\vec{C}$

As found above, $A^2 + B^2 + 2AB\cos\theta = C^2 \quad \ldots (i)$

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It is given that $A^2 + B^2 = C^2$. ... (*ii*)

Comparing eqs. (*i*) and (*ii*), $2AB\cos\theta = 0$. Since A and B are not zero, $\cos\theta = 0$ or $\theta = 90°$. Hence, $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

8. A particle is moving eastward with a velocity of 5 ms^{-1}. In 10 seconds, the velocity changes to 5 ms^{-1} northwards. What is the average acceleration in this time? [I.I.T.]

Hint. Figure 8.56 shows the conditions of the problem.

Initial velocity, $\vec{v_1} = 5\ ms^{-1}$ along east $= 5\,\hat{i}$

Final velocity, $\vec{v_2} = 5\ ms^{-1}$ along north $= 5\hat{j}$

Change in velocity, $\vec{\Delta v} = \vec{v_2} - \vec{v_1} = 5\hat{j} - 5\,\hat{i}$

$$\therefore \quad |\vec{\Delta v}| = \sqrt{5^2 + (-5)^2} = 5\sqrt{2}$$

The direction of $\vec{\Delta v}$ is north-west.

Average acceleration, $\vec{a} = \dfrac{\vec{\Delta v}}{\Delta t} = \dfrac{5\sqrt{2}}{10} = \dfrac{1}{\sqrt{2}}\ ms^{-1}$ north-west

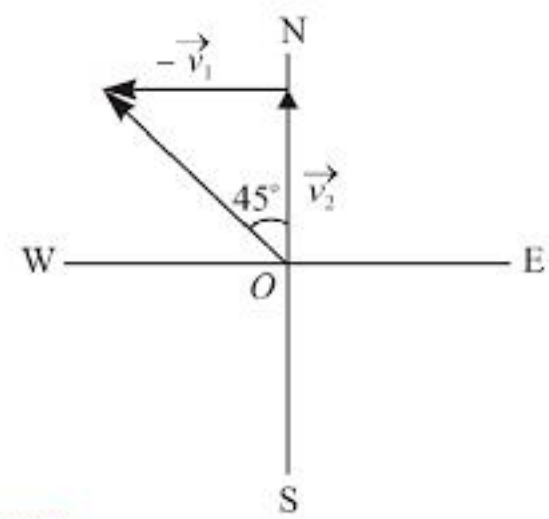


Fig. 8.56

9. Find the resultant of three vectors $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ shown in Fig. 8.57. Radius of the circle is R.

Hint. $|OA| = |OB| = |OC| = R$

The resultant of $\vec{OA} + \vec{OC}$ is along $\vec{OB}$ (bisector) and its magnitude

$$= 2R\cos 45° = R\sqrt{2}$$

The resultant of $(\vec{OA} + \vec{OC})$ and $\vec{OB}$ is along $\vec{OB}$ and its magnitude

$$= R\sqrt{2} + R = \boldsymbol{R(1+\sqrt{2})}$$

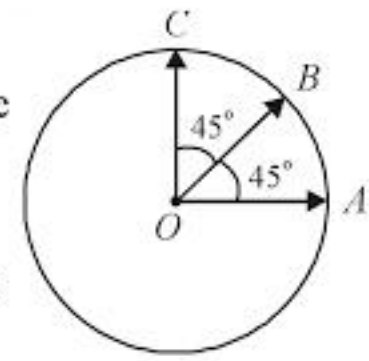


Fig. 8.57

10. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$, then find the vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$.

Hint. Let the required vector be

$$\vec{C} = C_x\hat{i} + C_y\hat{j}$$

It is given that $\vec{C}$ is parallel to $\vec{A}$. Therefore, $\dfrac{C_y}{C_x} = \dfrac{4}{3}$. ... (*i*)

It is also given that $|\vec{C}| = |\vec{B}|$. $\therefore \quad |\vec{C}| = \sqrt{(7)^2 + (24)^2} = \sqrt{625}$

or $\sqrt{C_x^2 + C_y^2} = \sqrt{625}$ or $C_x^2 + C_y^2 = 625$ ... (*ii*)

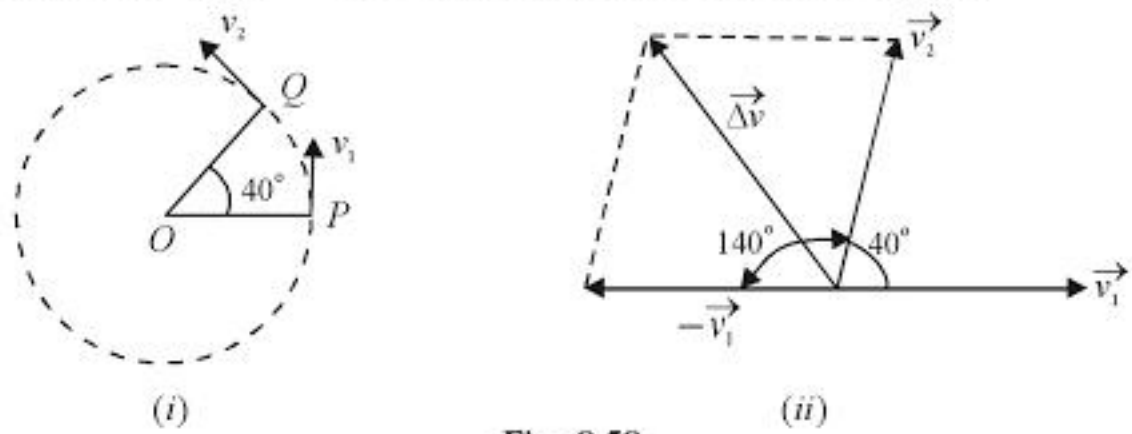
Solving eqs. (*i*) and (*ii*), we get, $C_x = 15$; $C_y = 20$

$$\therefore \quad \vec{C} = \mathbf{15\hat{i} + 20\hat{j}}$$

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11. A particle is moving in a circle of radius r centred at O with constant speed v. What is the change in velocity in moving from P to Q? Given that $\angle POQ = 40°$.

Hint. Fig. 8.58 (*i*) shows the conditions of the problem. Suppose the velocity of the particle is $\vec{v_1}$ at P and $\vec{v_2}$ at Q. Note that magnitudes of the velocities are the same ; only direction changes *i.e.*, $|\vec{v_1}| = |\vec{v_2}| = v$. Hence, the particle has acceleration.



(*i*) (*ii*)

Fig. 8.58

Referring to Fig. 8.58 (*ii*), change in velocity $\vec{\Delta v}$ is given by ;

$$\vec{\Delta v} = \vec{v_2} - \vec{v_1}$$

Magnitude of $\vec{\Delta v} = 2v\cos\dfrac{140°}{2} = 2v \cos 70° = 2v \sin 20°$

12. An airplane is flying with velocity of 70 km/h in north-east direction. The wind is blowing 30 km/h from north to south. What is the resultant displacement of the airplane in 4 hours?

Hint. Suppose $\vec{v_1}$ is the velocity of airplane and $\vec{v_2}$ that of the wind. Then the resultant velocity $\vec{v}$ of the airplane is given by (See Fig. 8.59) ;

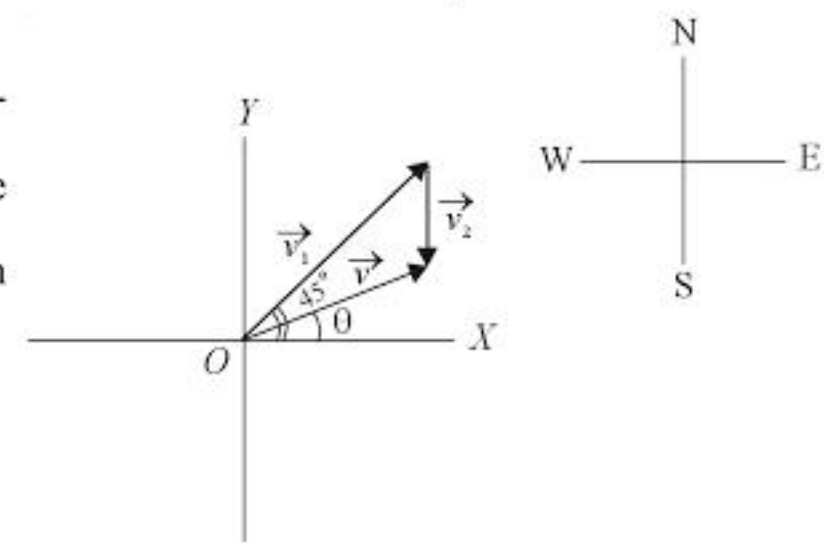


Fig. 8.59

$$\vec{v} = \vec{v_1} + \vec{v_2}$$

Now $\vec{v_1}$ = 70 km/h north-east

$\vec{v_2}$ = 30 km/h north to south

Taking X-axis along east and Y-axis along north, we have,

$$\vec{v_1} = 70\cos 45°\,\hat{i} + 70\cos 45°\hat{j} = \frac{70}{\sqrt{2}}(\hat{i} + \hat{j}) \text{ and } \vec{v_2} = -30\hat{j}$$

$$\therefore \quad \vec{v} = \vec{v_1} + \vec{v_2} = \frac{70}{\sqrt{2}}(\hat{i} + \hat{j}) - 30\hat{j} = (49.5\hat{i} + 19.5\hat{j}) \text{ km/h}$$

$\therefore$ Displacement in time t (= 4 hours) is

$$\vec{S} = \vec{v}\,t = (49.5\hat{i} + 19.5\hat{j}) \times 4 = (198\hat{i} + 78\hat{j}) \text{ km}$$

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Magnitude of displacement is given by ;

$$|\vec{S}| = \sqrt{(198)^2 + (78)^2} = \mathbf{212\ km}$$

If θ is the angle made by the resultant displacement with X-axis (*i.e.*, east), then,

$$\tan\theta = \frac{78}{198} = 0.394 \quad \therefore\ \theta = \mathbf{21.5°}$$

13. A vector $\vec{A}$ has magnitude of 6 units and is directed along positive X-axis. Another vector $\vec{B}$ has magnitude of 4 units and lies in the XY-plane, making an angle of 30° with positive X-axis and an angle of 60° with positive Y-axis. Find the vector product of $\vec{A}$ and $\vec{B}$.

Hint. Taking unit vector $\hat{i}$ along positive X-axis and unit vector $\hat{j}$ along positive Y-axis, the two vectors can be expressed as :

$$\vec{A} = 6\hat{i}\ ;\ \vec{B} = 4\cos 30°\,\hat{i} + 4\cos 60°\,\hat{j} = 2\sqrt{3}\,\hat{i} + 2\hat{j}$$

$$\therefore \quad \vec{A} \times \vec{B} = 6\hat{i} \times (2\sqrt{3}\hat{i} + 2\hat{j}) = 12\hat{k}$$

Thus, the vector $\vec{A} \times \vec{B}$ has a magnitude of **12 units and is directed along positive Z-axis.**

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9

Motion in a Plane

INTRODUCTION

In one dimensional motion, the object moves in a straight line (*e.g.*, along X-axis or Y-axis). Therefore, only one number (x or y) is required to specify the position of the object. But in two dimensional motion, the object moves in a plane —say XY plane. Consequently, we require two numbers (x, y) to specify the position of the object in the plane. However, in three dimensional motion, the object moves in space and we need three numbers (x, y, z) to locate the position of the object in space. It is because the space we live in is three dimensional. Two dimensional motion is very important due to two principal reasons. First, many interesting phenomena are associated with two dimensional motion. Secondly, this type of motion can be easily illustrated on a paper or blackboard. For this reason, most of our discussion in this chapter will be confined to two dimensional motion.

In one dimensional motion, we can handle vector quantities (*e.g.*, displacement, velocity, acceleration *etc.*) quite easily because their directional aspect can be taken care of by +ve and –ve signs. This is due to the fact that in one dimension, only two directions are possible. However, the situation is not that simple in two dimensional motion (*i.e.*, motion in a plane) or three dimensional motion (*i.e.*, motion in space). Here we have to use vectors to deal with vector quantities. We shall use the properties of vectors discussed in the previous chapter to describe the motion of an object in a plane.

9.1. MOTION IN A PLANE

When an object moves in a plane, say XY plane, we require two rectangular coordinates (x, y) to specify its position in the plane. Consider the simple case of an object moving along a straight line AB in the XY plane as shown in Fig. 9.1. When the object is at point A, its coordinates are (x_1, y_1) and when it is at point B, its coordinates are (x_2, y_2). Thus we can locate the position of an object in a plane by rectangular coordinates (x, y). However, we can also locate the position of an object in a plane by position vector of the object. Thus in Fig. 9.1, the position vector of the object at point A is $\overrightarrow{OA}$ $(= \vec{r}_1)$ and at point B, the position vector of the object is $\overrightarrow{OB}$ $(= \vec{r}_2)$. The two methods are equivalent and are frequently used to describe the motion of an object in a plane.

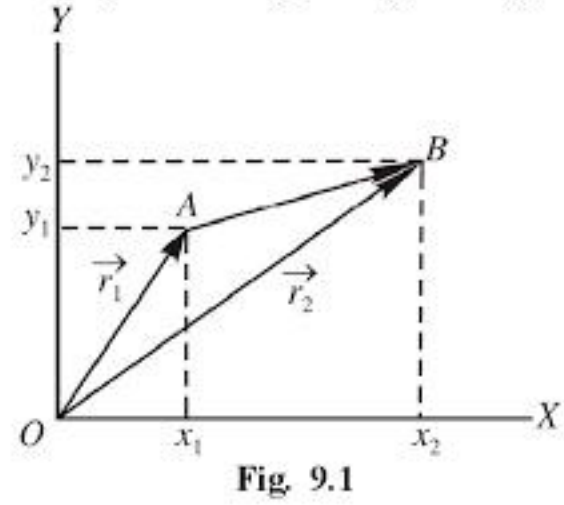


Fig. 9.1

Uniform velocity. *An object is said to be moving with a uniform velocity in a plane if it undergoes equal displacements in equal intervals of time, however small these time intervals may be.*

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Note that uniform velocity means that the magnitude as well as direction of the velocity remains the same. This is possible only if the object moves in a straight line with constant speed in the plane. *Therefore, we arrive at an important conclusion that the path of an object moving with uniform velocity in a plane is a straight line.*

9.2. MOTION WITH UNIFORM VELOCITY IN A PLANE

Consider an object moving with a uniform velocity $\vec{v}$ in the XY plane as shown in Fig. 9.2. Suppose at $t = 0$, the object is at point $A(x_0, y_0)$ and the position vector of the object is $\overrightarrow{OA}\ (= \vec{r}_0)$. Let at time t, the object be at point $B(x, y)$ and its position vector be $\overrightarrow{OB}\ (= \vec{r})$.

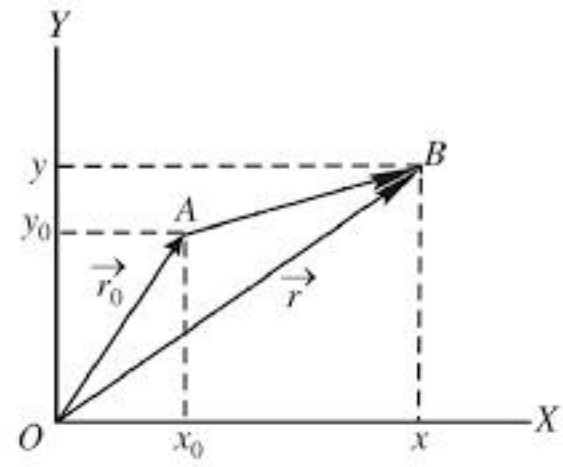


Fig. 9.2

(*i*) Displacement. The displacement of the object in the time interval $(t - 0)$ is $\overrightarrow{AB}$. According to triangle law of vectors,

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

or $$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$\therefore$ Displacement, $$\overrightarrow{AB} = \vec{r} - \vec{r}_0 \qquad ...(i)$$

Eq. (*i*) gives the displacement $\overrightarrow{AB}$ of the object in terms of its position vectors.

We can also express displacement $\overrightarrow{AB}$ in terms of rectangular components (or coordinates).

Now $$\vec{r}_0 = x_0\hat{i} + y_0\hat{j}\ ;\ \vec{r} = x\hat{i} + y\hat{j}$$

$\therefore$ Displacement, $$\overrightarrow{AB} = \vec{r} - \vec{r}_0 = (x\hat{i} + y\hat{j}) - (x_0\hat{i} + y_0\hat{j})$$

or $$\overrightarrow{AB} = (x - x_0)\hat{i} + (y - y_0)\hat{j} \qquad ...(ii)$$

Eq. (*ii*) gives the displacement $\overrightarrow{AB}$ of the object in terms of rectangular components.

(*ii*) Velocity. The velocity of the object is given by ;

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time interval}}$$

or $$\vec{v} = \frac{\vec{r} - \vec{r}_0}{t - 0} = \frac{\vec{r} - \vec{r}_0}{t}$$

$\therefore$ $$\vec{v} = \frac{\vec{r} - \vec{r}_0}{t} \qquad ...(iii)$$

Eq. (*iii*) gives the velocity $\vec{v}$ of the object in terms of its position vectors. Since the object is moving with uniform velocity ($\vec{v}$), the magnitude as well as direction of the velocity is the same at every instant.

(*iii*) Equation of motion of object. From eq. (*iii*), we have,

$$\vec{r} - \vec{r}_0 = \vec{v}\,t$$

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or $$\vec{r} = \vec{r_0} + \vec{v}\,t \qquad ...(iv)$$

Eq. (*iv*) gives the position $(\vec{r})$ of the object moving with uniform velocity $(\vec{v})$ at any time t. Note that $\vec{r_0}$ is the position vector of the object at $t = 0$.

In terms of rectangular components. Let us express eq. (*iv*) in terms of rectangular components (or coordinates). Suppose v_x and v_y are the magnitudes of the rectangular components of $\vec{v}$ along X-axis and Y-axis respectively. Then,

$$\vec{v} = v_x\hat{i} + v_y\hat{j} \quad \text{where } v = \sqrt{v_x^2 + v_y^2}$$

Since magnitude of $\vec{v}$ is constant, v_x = constant and v_y = constant.

Now $$\vec{r} = \vec{r_0} + \vec{v}\,t$$

Here $$\vec{r} = x\hat{i} + y\hat{j} \;;\; \vec{r_0} = x_0\hat{i} + y_0\hat{j} \;;\; \vec{v} = v_x\hat{i} + v_y\hat{j}$$

$\therefore$ $$x\hat{i} + y\hat{j} = (x_0\hat{i} + y_0\hat{j}) + (v_x\hat{i} + v_y\hat{j})t$$

or $$x\hat{i} + y\hat{j} = (x_0 + v_x t)\hat{i} + (y_0 + v_y t)\hat{j} \qquad ...(v)$$

The coefficients $\hat{i}$ and $\hat{j}$ on both sides of the eq. (*v*) must be equal.

$\therefore$ $$x = x_0 + v_x t \qquad ...(vi)$$

$$y = y_0 + v_y t \qquad ...(vii)$$

Thus eq. (*iv*) can be replaced by two scalar equations (*vi*) and (*vii*). Now eqs. (*vi*) and (*vii*) represent motion along two perpendicular straight lines—motion along X-axis with uniform velocity v_x and motion along Y-axis with uniform velocity v_y.

Therefore, two dimensional motion (i.e., motion in a plane) with uniform velocity is a combination of two one-dimensional uniform motions along perpendicular directions.

(*iv*) Path of the object. An object moving with a uniform velocity in a plane follows a straight line path. We have already given the physical explanation for it. Let us now turn to mathematical explanation.

Now $$x = x_0 + v_x t \;;\; y = y_0 + v_y t$$

It is clear that $t = \dfrac{x - x_0}{v_x}$. Putting this value of t in the equation for y, we have,

$$y = y_0 + \frac{v_y}{v_x}(x - x_0)$$

It is clear that this is the equation of a straight line having slope = v_y / v_x. *Therefore, the path of the object moving with uniform velocity in a plane is a straight line.*

9.3. MOTION WITH UNIFORM ACCELERATION IN A PLANE

We now discuss the motion of an object moving with uniform acceleration $\vec{a}$ in a plane. *An object is said to be moving with uniform acceleration in a plane if its velocity vector changes by equal amounts in equal intervals of time, however small these time intervals may be.* Note that uniform acceleration means that magnitude as well as direction of acceleration remains the same. As we shall see, the path of the object moving with uniform acceleration in a plane is parabolic.

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(*i*) Acceleration. Suppose at $t = 0$, the velocity of the object is $\vec{v}_0$ and at $t = t$, its velocity is $\vec{v}$. Then uniform acceleration $\vec{a}$ of the object is

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time interval}}$$

or $$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t - 0} = \frac{\vec{v} - \vec{v}_0}{t}$$

$\therefore$ $$\vec{a} = \frac{\vec{v} - \vec{v}_0}{t} \qquad ...(i)$$

Eq. (*i*) gives the expression for the uniform acceleration $\vec{a}$ of the object. Since the object is moving with uniform (constant) acceleration, the magnitude as well as direction of acceleration remains constant at every instant of motion.

(*ii*) Equation of motion. From eq. (*i*), we have,

$$\vec{v} - \vec{v}_0 = \vec{a}\,t$$

or $$\vec{v} = \vec{v}_0 + \vec{a}\,t \qquad ...(ii)$$

Eq. (*ii*) gives the velocity $(\vec{v})$ of the object moving with uniform acceleration $(\vec{a})$ at any time t. Note that $\vec{v}_0$ is the velocity of the object at $t = 0$.

In terms of rectangular components. Let us express eq. (*ii*) in terms of rectangular components (or coordinates). Suppose a_x and a_y are the magnitudes of the rectangular components of $\vec{a}$ along X-axis and Y-axis respectively. Then,

$$\vec{a} = a_x\hat{i} + a_y\hat{j} \quad \text{where} \quad a = \sqrt{a_x^2 + a_y^2}$$

Since magnitude of $\vec{a}$ is constant, a_x = constant and a_y = constant.

$$\text{Now} \quad \vec{v} = \vec{v}_0 + \vec{a}\,t$$

Here $$\vec{v} = v_x\hat{i} + v_y\hat{j}\,;\ \vec{v}_0 = v_{x0}\hat{i} + v_{y0}\hat{j}\,;\ \vec{a} = a_x\hat{i} + a_y\hat{j}$$

$\therefore$ $$v_x\hat{i} + v_y\hat{j} = (v_{x0}\hat{i} + v_{y0}\hat{j}) + (a_x\hat{i} + a_y\hat{j})t$$

or $$v_x\hat{i} + v_y\hat{j} = (v_{x0} + a_x t)\hat{i} + (v_{y0} + a_y t)\hat{j} \qquad ...(iii)$$

The coefficients of $\hat{i}$ and $\hat{j}$ on both sides of eq. (*iii*) must be the same.

$\therefore$ $$v_x = v_{x0} + a_x t \qquad ...(iv)$$

$$v_y = v_{y0} + a_y t \qquad ...(v)$$

Thus eq. (*ii*) can be replaced by two equations (*iv*) and (*v*). Now eqs. (*iv*) and (*v*) represent motion along two perpendicular straight lines—motion along X-axis with uniform acceleration a_x and motion along Y-axis with uniform acceleration a_y.

Therefore, two dimensional motion (i.e, motion in a plane) with uniform acceleration is a combination of two one dimensional uniformly accelerated motions along perpendicular directions.

(*iii*) Expression for displacement for uniform acceleration. Suppose at $t = 0$, the velocity of the object is $\vec{v}_0$ and at $t = t$, it is $\vec{v}$. The average velocity $\vec{v}_{av}$ during the time interval $0 - t$ is

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$$\vec{v}_{av} = \left(\frac{\vec{v}_0 + \vec{v}}{2}\right) = \left(\frac{\vec{v}_0 + \vec{v}_0 + \vec{a}\,t}{2}\right) \qquad \left(\because \vec{v} = \vec{v}_0 + \vec{a}\,t\right)$$

$$\therefore \quad \vec{v}_{av} = \vec{v}_0 + \frac{1}{2}\vec{a}\,t$$

If $\vec{r}_0$ and $\vec{r}$ are the position vectors of the object at $t = 0$ and $t = t$ respectively, then $\vec{v}_{av}$ is given by ;

$$\vec{v}_{av} = \frac{\vec{r} - \vec{r}_0}{t - 0} = \frac{\vec{r} - \vec{r}_0}{t}$$

or $$\vec{r} - \vec{r}_0 = \vec{v}_{av}\,t$$

or $$\vec{r} = \vec{r}_0 + \vec{v}_{av}\,t = \vec{r}_0 + (\vec{v}_0 + \frac{1}{2}\vec{a}\,t)t \qquad \left(\because \vec{v}_{av} = \vec{v}_0 + \frac{1}{2}\vec{a}\,t\right)$$

$$\therefore \quad \vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}\,t^2 \qquad \ldots (vi)$$

Eq. (*vi*) can be written in terms of rectangular components as :

$$x\hat{i} + y\,\hat{j} = \left(x_0\hat{i} + y_0\,\hat{j}\right) + \left(v_{x0}\hat{i} + v_{y0}\hat{j}\right)t + \frac{1}{2}\left(a_x\hat{i} + a_y\hat{j}\right)t^2$$

or $$x\hat{i} + y\,\hat{j} = \left(x_0 + v_{x0}\,t + \frac{1}{2}a_x t^2\right)\hat{i} + \left(y_0 + v_{y0}\,t + \frac{1}{2}a_y t^2\right)\hat{j}$$

$$\therefore \quad x = x_0 + v_{x0}\,t + \frac{1}{2}a_x t^2 \qquad \ldots (vii)$$

and $$y = y_0 + v_{y0}\,t + \frac{1}{2}a_y t^2 \qquad \ldots (viii)$$

Note that eqs. (*vii*) and (*viii*) are similar to that of the position-time relation for the motion of the object with uniform acceleration in one dimension.

(*iv*) Path of the object. If we eliminate t from eqs. (*vii*) and (*viii*), we get second degree equation in x and y. Therefore, path of the object will be parabolic. *Therefore, path of an object moving with uniform acceleration in a plane is parabolic.*

Example 9.1. The position vector of a particle at $t = 0$ is $\vec{r}_1 = -3\hat{i} + 2\,\hat{j}$ and later at $t = t$, it is $\vec{r}_2 = 9\hat{i} + 2\,\hat{j}$. What is the displacement from $\vec{r}_1$ to $\vec{r}_2$?

Solution. $$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \left(9\hat{i} + 2\hat{j}\right) - \left(-3\hat{i} + 2\hat{j}\right) = 12\,\hat{i}$$

Example 9.2. A particle moves in the *XY* plane in such a way that its coordinates x and y vary with time as:

$$x = t^3 - 32\,t\,; \qquad y = 5\,t^2 + 12$$

Here x and y are in metres and t in seconds. Find the position, velocity and acceleration of the particle at $t = 3$ s.

Solution. The position of the particle is given by ;

$$\vec{r} = x\hat{i} + y\hat{j} = \left(t^3 - 32t\right)\hat{i} + \left(5t^2 + 12\right)\hat{j}$$

$$= [(3)^3 - (32 \times 3)]\,\hat{i} + [5\,(3)^2 + 12]\,\hat{j} = -69\;\hat{i} + 57\,\hat{j}\ \text{m}$$

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or $$y = (v_0 \sin\theta)t - \frac{1}{2}gt^2 \qquad \left[\because v_{y0} = v_0 \sin\theta; y_0 = 0 \text{ and } a_y = -g\right]$$

$\therefore$ $$y = (v_0 \sin\theta)t - \frac{1}{2}gt^2 \qquad ...(ii)$$

At time t, the projectile is at $P(x, y)$ and the vertical component of its velocity at this time is given by ;

$$v_y = v_{y0} - gt$$

or $$v_y = v_0 \sin\theta - g\,t \qquad ...(iii)$$

Trajectory of projectile. The path or trajectory of the projectile (air friction neglected) is a parabola. To show this, we rewrite eq. (*ii*).

$$y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$$

From eq. (*i*), $t = x/v_0\cos\theta$. Putting this value of t in the above equation, we have,

$$y = (v_0 \sin\theta)\frac{x}{v_0\cos\theta} - \frac{1}{2}g\left(\frac{x}{v_0\cos\theta}\right)^2$$

or $$y = x\tan\theta - \left(\frac{g}{2v_0^2\cos^2\theta}\right)x^2$$

From this equation, we see that y as a function of x has the form :

$$y = ax - bx^2 \qquad \text{... Equation of parabola}$$

where $a\ (=\tan\theta)$ and $b\left(=\dfrac{g}{2v_0^2\cos^2\theta}\right)$ are constants for any given projectile. *Therefore, the path or the trajectory of the projectile is parabolic.*

9.6. PARAMETERS OF PROJECTILE MOTION

Consider the projectile motion shown in Fig. 9.4 (Refer back). The important parameters of the projectile motion are :

1. Time of flight (T)
2. Maximum height attained (H_{max})
3. Horizontal range (R)

The first two parameters (*i.e.*, T and H_{max}) are determined from the vertical motion of the projectile while the third (*i.e.*, R) is calculated from the horizontal motion of the projectile.

1. Time of flight (T). *It is the time elapsing from the launching to the time the projectile returns to the ground again.* Obviously, it is the time for which the projectile remains in air before coming to original height (ground). The position y of the projectile along Y-axis at any time t is

$$y = (v_0 \sin\theta)\,t - \frac{1}{2}g\,t^2$$

When projectile returns to ground, its vertical displacement is zero (*i.e.*, $y = 0$).

$\therefore$ $$0 = (v_0 \sin\theta)\,t - \frac{1}{2}g\,t^2$$

or $$0 = t\left(v_0 \sin\theta - \frac{1}{2}gt\right)$$

$\therefore$ $$t = 0 \;;\quad t = \frac{2v_0 \sin\theta}{g}$$

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The first solution for t (*i.e.*, $t = 0$) corresponds to the projecting point O. The second solution for t is the time of flight (T) of the projectile.

$$\therefore \qquad \text{Time of flight, } T = \frac{2v_0 \sin\theta}{g}$$

Note. The time taken by the projectile to go from the starting point O to the highest point B is called the *time of ascent*. On the other hand, the time taken by the projectile to go from the highest point B to the level of point O (*i.e.*, point D in Fig. 9.4) is the *time of descent*. Since the trajectory of the projectile is *symmetrical about the mid-point of its flight, the two times are equal *i.e.*,

$$\text{Time of ascent} = \text{Time of descent} = \frac{T}{2} = \frac{v_0 \sin\theta}{g}$$

2. Maximum height attained (H_{max}). *It is the maximum height to which the projectile rises above the launching point (origin).* The position of the projectile along Y-axis at any time t is given by:

$$y = (v_0 \sin\theta)\, t - \frac{1}{2} g t^2$$

$$\text{When } \quad t = \frac{T}{2} = \frac{v_0 \sin\theta}{g}, \text{ then, } y = H_{max}$$

$$\therefore \qquad H_{max} = (v_0 \sin\theta)\frac{(v_0 \sin\theta)}{g} - \frac{1}{2} g \left(\frac{v_0 \sin\theta}{g}\right)^2$$

$$\text{or} \qquad H_{max} = \frac{v_0^2 \sin^2\theta}{2g}$$

3. Horizontal Range (R). *It is the horizontal distance travelled by the projectile before returning to the ground (i.e., original height).* In other words, it is the horizontal distance travelled by the projectile during time T (= time of flight). The position of the projectile along X-axis at any time t is given by ;

$$x = (v_0 \cos\theta)\, t$$

$$\text{When } \quad t = T = \frac{2v_0 \sin\theta}{g} \; ; x = R$$

$$\therefore \qquad R = (v_0 \cos\theta)\left(\frac{2v_0 \sin\theta}{g}\right) = v_0^2\left(\frac{2\sin\theta\cos\theta}{g}\right)$$

$$\text{or} \qquad R = \frac{v_0^2 \sin 2\theta}{g} \qquad (\because \sin 2\theta = 2\sin\theta\cos\theta)$$

Note that the horizontal range (R) of a projectile depends upon the projection angle (θ) and initial speed (v_0). For fixed θ, $R \propto v_0^2$; doubling the projection speed will increase the range four times.

Condition for maximum horizontal range. For a given initial speed v_0, the horizontal range will be maximum when $\sin 2\theta = 1$ or $\theta = 45°$.

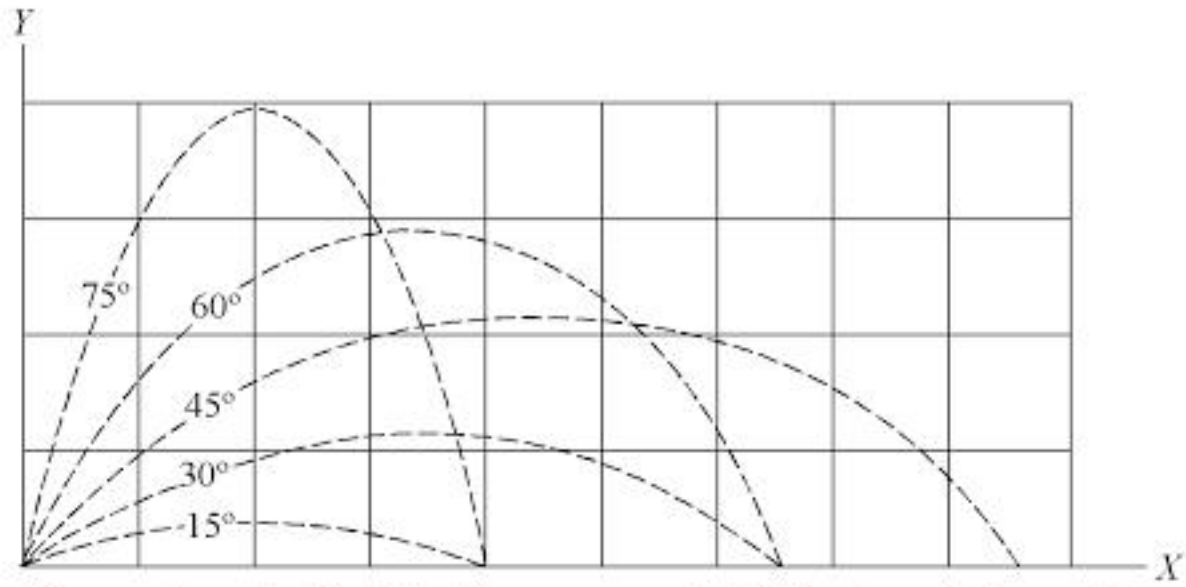
$$\therefore \qquad \text{Maximum range, } R_{max} = \frac{v_0^2}{g} \qquad \text{...when } \theta = 45°$$

---

* This symmetry is due to the fact that the acceleration due to gravity (g) is the same for upward and downward motion.

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Hence for a given initial speed, the horizontal range of the projectile will be maximum when the angle of projection is 45° (neglecting air friction).

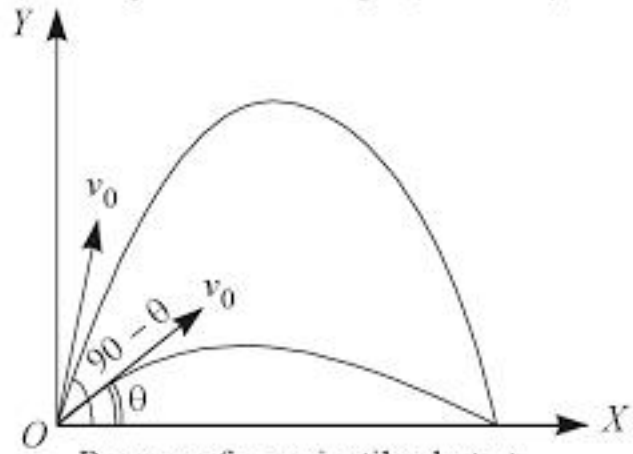


Ranges of a projectile shot at the same speed at different projection angles

Fig. 9.5

Fig. 9.5 shows the paths of several projectiles, all with the same initial speed (v_0) but with different projection angles. Note that horizontal range is maximum for the projectile which is launched at a projection angle of 45°. You may see that for small projection angles ($\theta < 45°$), the horizontal range is *small. It is also small when the projection angles are **large ($\theta > 45°$).

Two projection angles for same *R*. The reader may note another remarkable thing in Fig. 9.5 above. For the given initial speed v_0, there are two projection angles θ and $(90° - \theta)$ for which the horizontal range of the projectile is the same. Thus referring to Fig. 9.5, the horizontal range of the projectile is the same for $\theta = 30°$ and $(90° - \theta) = 90° - 30° = 60°$. In other words, if two projectiles are launched at projection angles of 30° and 60° with same initial speed, their horizontal range will be the same. Does this result surprise you? We can prove it mathematically by referring to Fig. 9.6. For projection angle θ, the horizontal range is



Ranges of a projectile shot at the same speed at different projection angles.

Fig. 9.6

$$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{v_0^2 2 \sin\theta \cos\theta}{g} \quad \ldots (i)$$

For projection angle $(90° - \theta)$, the horizontal range is

$$R = v_0^2 \left[\frac{2\sin(90° - \theta)\cos(90° - \theta)}{g} \right]$$

or

$$R = v_0^2 \left(\frac{2\cos\theta \sin\theta}{g} \right)$$

---

* If the projection angle is small, the horizontal component of velocity ($v_0 \cos\theta$) is relatively high but the time of flight ($T = 2v_0 \sin\theta/g$) will be very short. As a result, R [$= (v_0 \cos\theta)T$] will be small.

** In this case, the time of flight is long but horizontal component of velocity is small. Again horizontal range will be small.

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This is just the same equation as eq. (*i*). Although projection angles θ and $(90° - \theta)$ give the same horizontal range (of course, for same initial speed v_0), the two paths differ in time of flight (T) and maximum height (H_{max}) attained. This is not surprising because the values of T and H_{max} depend upon the angle of projection (θ).

9.7. VELOCITY OF PROJECTILE AT ANY TIME

During the flight of the projectile, it is under the action of two velocities *viz.*

(*i*) a constant horizontal velocity v_{x0}.

(*ii*) uniformly changing velocity along the vertical or y-axis.

Therefore, the velocity v of the projectile changes with time. At $t = 0$, the vertical component of velocity is $v_0 \sin\theta$ and goes on decreasing uniformly till it becomes zero at the highest point. From now onward, the vertical component of velocity goes on increasing till it becomes $v_0 \sin\theta$ when projectile hits the ground [See Fig. 9.7].

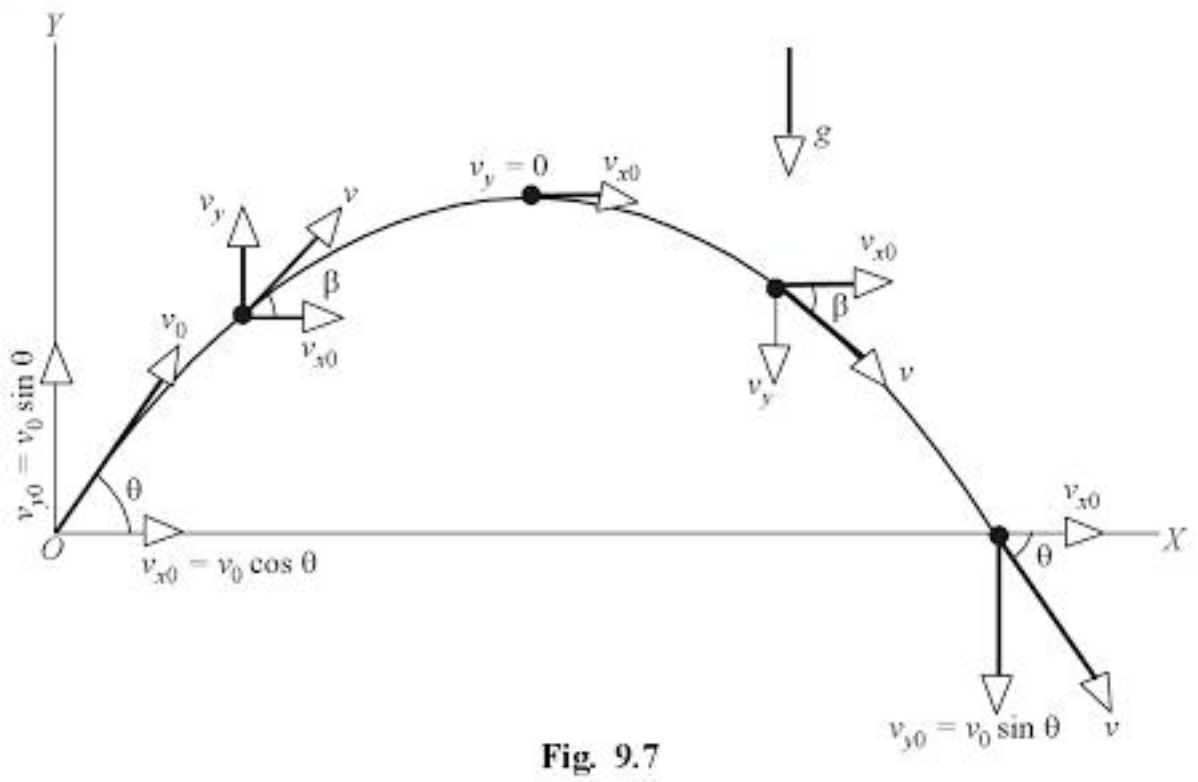


Fig. 9.7

At any time t, the vertical component of velocity is

$$v_y = v_0 \sin\theta - gt$$

Horizontal component of velocity, $v_x = v_{x0} = v_0 \cos\theta$

Since the velocities v_y and v_{x0} are always at right angles to each other, the projectile velocity is given by ;

$$v = \sqrt{(v_{x0})^2 + (v_y)^2} = \sqrt{(v_0\cos\theta)^2 + (v_0\sin\theta - gt)^2}$$

$$= \sqrt{v_0^2\cos^2\theta + v_0^2\sin^2\theta + g^2t^2 - 2v_0gt\sin\theta}$$

$$= \sqrt{v_0^2(\cos^2\theta + \sin^2\theta) + g^2t^2 - 2v_0gt\sin\theta}$$

$$\therefore \quad v = \sqrt{v_0^2 + g^2t^2 - 2v_0gt\sin\theta}$$

Since v is tangent to the path at any instant, the angle β which it makes with the horizontal at the considered instant is

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$$\tan\beta = \frac{v_y}{v_{x0}} = \frac{v_0\sin\theta - gt}{v_0\cos\theta} = \frac{v_0\sin\theta}{v_0\cos\theta} - \frac{gt}{v_0\cos\theta}$$

$$\therefore \quad \tan\beta = \tan\theta - \frac{gt}{v_0\cos\theta}$$

Note that angle β goes on changing with time t.

9.8. SPECIAL CASES OF PROJECTILE MOTION

So far we have considered the general case of projectile motion *i.e.*, projectile launched from the origin at an angle θ above the horizontal. However, the following two special cases of projectile motion will be discussed :

1. Projectile fired at an angle with the vertical.
2. Projectile fired horizontally.

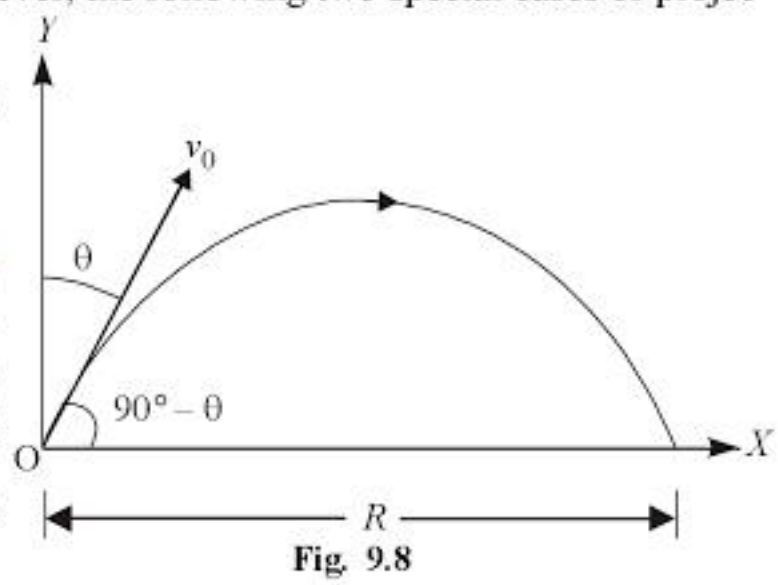


Fig. 9.8

1. Projectile fired at an angle with the vertical. Fig. 9.8 shows a projectile fired with initial velocity v_0 at an angle θ with the vertical. Therefore, firing angle with the horizontal is $90° - \theta$. In order to find the various parameters of projectile motion, change θ to $90° - \theta$ in the expressions derived in Arts. 9.5, 9.6 and 9.7.

(*i*) Trajectory of projectile. Changing θ to $90° - \theta$ in Art. 9.5, we have,

$$y = x\tan(90° - \theta) - \left(\frac{g}{2v_0^2\cos^2(90° - \theta)}\right)x^2$$

$$= x\cot\theta - \left(\frac{g}{2v_0^2\sin^2\theta}\right)x^2$$

$$\therefore \quad y = x\cot\theta - \frac{gx^2}{2v_0^2\sin^2\theta}$$

It is clear that the *trajectory of the projectile is parabolic.*

(*ii*) Time of flight. Changing θ to $90° - \theta$, time of flight is given by;

$$\text{Time of flight, } T = \frac{2v_0\sin(90° - \theta)}{g} = \frac{2v_0\cos\theta}{g}$$

(*iii*) Maximum height attained. Changing θ to $90° - \theta$, maximum height attained is given by ;

$$\text{Max. height attained, } H_{max} = \frac{v_0^2\sin^2(90° - \theta)}{2g} = \frac{v_0^2\cos^2\theta}{2g}$$

(*iv*) Horizontal range. Changing θ to $90° - \theta$, horizontal range is given by;

$$\text{Horizontal range, } R = \frac{v_0^2\sin 2(90° - \theta)}{g} = \frac{v_0^2\sin(180° - 2\theta)}{g} = \frac{v_0^2\sin 2\theta}{g}$$

(*v*) Velocity of projectile at any time *t*. Changing θ to $90° - \theta$, the velocity (v) of the projectile is given by ;

$$v = \sqrt{v_0^2 + g^2t^2 - 2v_0gt\sin(90° - \theta)}$$

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or $$v = \sqrt{v_0^2 + g^2t^2 - 2v_0gt\cos\theta}$$

The angle β which v makes with horizontal is

$$\tan\beta = \tan(90° - \theta) - \frac{gt}{v_0\cos(90° - \theta)}$$

or $$\tan\beta = \cot\theta - \frac{gt}{v_0\sin\theta}$$

2. Projectile fired horizontally. Suppose a projectile is fired horizontally with velocity v_0 ($= v_x = v_{x0}$) at a height H above the ground (See Fig. 9.9). The horizontal velocity v_0 ($= v_x = v_{x0}$) remains constant throughout the projectile motion. The vertical downward velocity is zero at the time of firing the projectile and goes on increasing uniformly with time till the projectile hits the ground. Let us choose the firing point as the origin. In order to find the various expressions for this projectile motion, we should take vertical displacement and velocity *negative, $\theta = 0°$ in all the expressions derived in Arts. 9.5, 9.6 and 9.7.

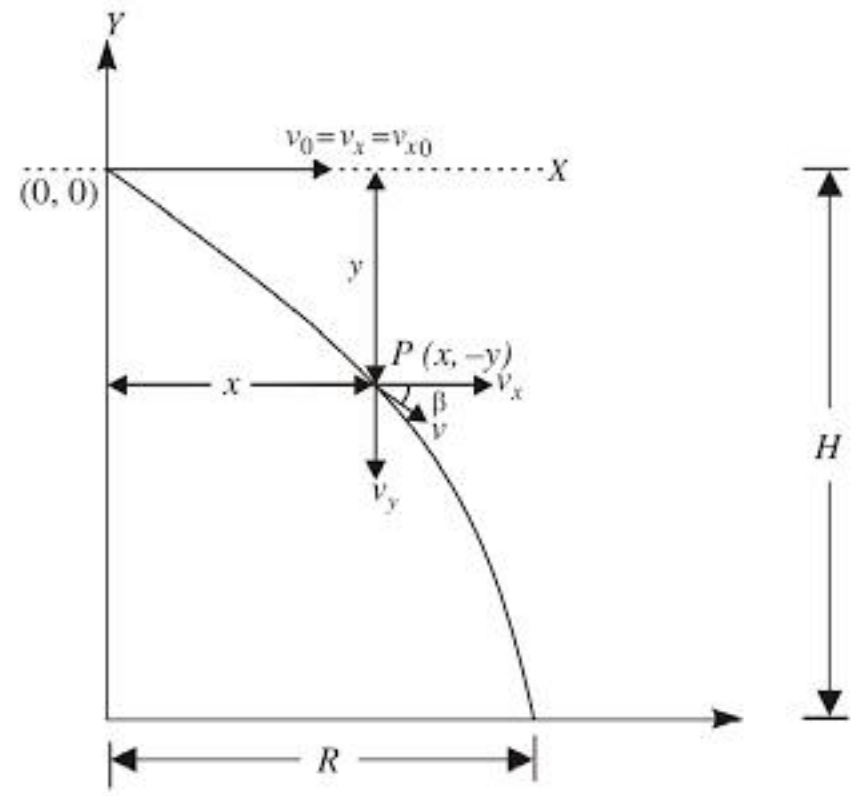


Fig. 9.9

(i) Motion along the horizontal. Suppose at any time t, the projectile is at point P $(x, -y)$. The horizontal displacement covered by the projectile is

$$x = (v_0\cos 0°)\ t = v_0\ t \qquad ...(i)$$

(ii) Motion along the vertical. At the considered point P, the vertical displacement is $-y$.

$$\therefore \quad -y = (v_0\sin 0°)\ t - \frac{1}{2}gt^2$$

or $$y = \frac{1}{2}gt^2 \qquad ...(ii)$$

Note that time of flight (T) and maximum height (H_{max}) attained by the projectile are determined by the vertical motion of the projectile.

(iii) Trajectory of projectile

$$y = \frac{1}{2}gt^2$$

From eq. (i), $t = x/v_0$

$$\therefore \quad y = \frac{1}{2}g\left(\frac{x}{v_0}\right)^2 = \left(\frac{1}{2}\frac{g}{v_0^2}\right)x^2$$

or $$y = kx^2 \qquad \left(\because \frac{1}{2}\frac{g}{v_0^2} = \text{constant} = k\right)$$

* Since motion is downward.

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This is the equation of parabola. *Therefore, the path or trajectory of the projectile is parabolic.*

(iv) Time of flight (T). At the considered point P, the vertical displacement is

$$-y = (v_0 \sin 0°)t - \frac{1}{2} gt^2$$

At $t = T$ = time of flight, $y = H$.

$$\therefore \qquad -H = (v_0 \sin 0°)\, T - \frac{1}{2} gT^2 = -\frac{1}{2} gT^2$$

$$\therefore \qquad \text{Time of flight, } T = \sqrt{\frac{2H}{g}}$$

(v) Horizontal range (R). At the considered point P, the horizontal displacement is

$$x = (v_0 \cos 0°)\, t$$

At $t = T$, $x = R$ = horizontal range.

$$\therefore \qquad \text{Horizontal range, } R = (v_0 \cos 0°)\, T = v_0\, T = v_0 \sqrt{\frac{2H}{g}}$$

$$\therefore \qquad R = v_0 \sqrt{\frac{2H}{g}}$$

(vi) Velocity of projectile at time t. At any time t, velocity v of the projectile is

$$v = \sqrt{v_0^2 + g^2 t^2 - 2v_0 gt \sin 0°}$$

or
$$v = \sqrt{v_0^2 + g^2 t^2}$$

The angle β which it makes with the horizontal at the considered instant is

$$\tan \beta = \frac{-v_y}{v_{x0}} = \frac{-(v_0 \sin 0° - gt)}{v_0 \cos 0°} = \frac{gt}{v_0}$$

Example 9.3. How far will a stone travel over level ground if it is thrown upward at an angle of 30° with the horizontal and with a speed of 12 ms^{-1}? What maximum range could be achieved with the same initial speed?

Solution. Horizontal range, $R = \frac{v_0^2}{g} \sin 2\theta$

Here $v_0 = 12\ ms^{-1}$; $\theta = 30°$; $g = 9.8\ ms^{-2}$

$$\therefore \qquad R = \frac{(12)^2}{9.8} \sin (2 \times 30°) = \mathbf{12.7\ m}$$

The horizontal range will be maximum when projection angle $\theta = 45°$ *i.e.*, $\sin 2\theta = 1$.

$$\therefore \qquad R_{max} = \frac{v_0^2}{g} = \frac{(12)^2}{9.8} = \mathbf{14.7\ m}$$

Note that the horizontal range is increased by about 16%.

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Example 9.4. A ball is thrown with a speed of 17 ms^{-1} at a projection angle of 58° above the horizontal. Determine (*i*) the time of maximum height (*ii*) the maximum height above the release point. Neglect air friction.

Solution. Here $v_0 = 17\ ms^{-1}$; $\theta = 58°$; $g = 9.8\ ms^{-2}$

(*i*) The time (t_m) taken by the ball to reach maximum height is

$$t_m = \frac{v_0 \sin\theta}{g} = \frac{17 \times \sin 58°}{9.8} = \mathbf{1.5\ s}$$

(*ii*) Maximum height (H_{max}) attained by the ball is

$$H_{max} = \frac{v_0^2 \sin^2\theta}{2g} = \frac{(17)^2 \times (\sin 58°)^2}{2 \times 9.8} = \mathbf{11\ m}$$

Example 9.5. An aeroplane travelling horizontally at 78 ms^{-1} at a height of 210 m drops a bomb to hit the target.

(*i*) At what horizontal distance from the target should the bomb be dropped ?

(*ii*) Calculate the velocity of the bomb as it reaches the ground.

Solution. Fig. 9.10 shows the conditions of the problem. We have placed the origin of the coordinate system at the ground directly under the position of the plane when it drops the bomb.

(*i*) The time t taken by the bomb to reach the ground will be determined from the vertical motion of the bomb.

$$y - y_0 = v_{y0}\, t + \frac{1}{2} a_y\, t^2$$

Here $y - y_0 = -210\ m$; $v_{y0} = 0$;

$$a_y = -g = -9.8\ ms^{-2}$$

$$\therefore \quad -210 = -\frac{1}{2} \times 9.8 \times t^2$$

or $$t = \sqrt{\frac{210 \times 2}{9.8}} = \mathbf{6.5\ s}$$

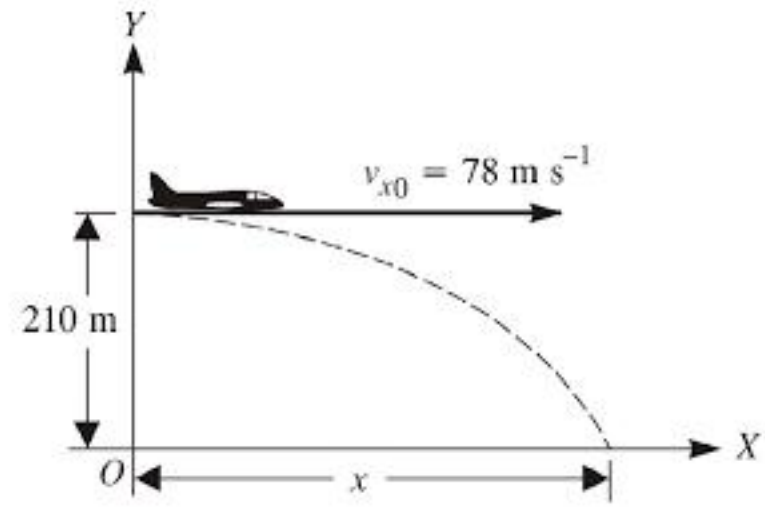


Fig. 9.10

Horizontal range, $R = x = {}^{*}v_x \times t = 78 \times 6.5 = \mathbf{507\ m}$

(*ii*) The initial horizontal component (v_{x0}) of bomb's velocity remains 78 ms^{-1}. The initial vertical component (v_{y0}) of bomb's velocity changes with time. At any time t, its value is

$$v_y = v_{y0} - gt = 0 - 9.8 \times 6.5 = -64\ ms^{-1}$$

$\therefore$ Magnitude of bomb's velocity (v) when it reaches the ground is

$$v = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(78)^2 + (-64)^2} = \mathbf{100\ ms^{-1}}$$

Example 9.6. An object is thrown horizontally with a velocity of 10 ms^{-1} from the top of a 20 m high building as shown in Fig. 9.11. Where does the object strike the ground?

Solution. In order to find time t taken by the object to strike the ground, we consider the vertical motion of the object. Taking the downward direction **positive,

$$v_{y0} = 0\ ;\ a_y = g = 9.8\ ms^{-2}\ ;\ y - y_0 = 20\ m$$

---

* $v_{x0} = v_x = 78\ ms^{-1}$. The horizontal speed of the bomb remains 78 ms^{-1} till it hits the ground. It is because there is no acceleration in the horizontal direction.

** It is arbitrary whether we choose y to be positive or negative in the upward or downward direction, but we must be consistent about it throughout the problem.

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(*ii*) The ball strikes the ground at point *B*. At point *B*, the ball has two velocities (*a*) horizontal velocity v_{x0} and (*b*) vertical velocity $v_y = v_{y0} + gt = 0 + gt = gt$. Referring to Fig. 9.13, $\alpha = 45°$.

Now $\tan\alpha = \dfrac{v_y}{v_{x0}} = \dfrac{gt}{v_{x0}}$ $\therefore$ $v_{x0} = \dfrac{gt}{\tan\alpha} = \dfrac{9.8\times 3}{\tan 45°} = \mathbf{29.4\ ms^{-1}}$

Example 9.12. A grasshopper can jump a maximum horizontal distance of 1 m. If it spends minimum time on the ground, what is the speed of travel along the road?

Solution. Horizontal range, $R = \dfrac{v_0^2 \sin 2\theta}{g}$; Maximum range, $R_{max} = \dfrac{v_0^2}{g}$ (For $\theta = 45°$)

$\therefore$ $v_0 = \sqrt{g R_{max}} = \sqrt{9.8\times 1} = 3.13\,\text{ms}^{-1}$

Velocity along road $= v_0 \cos\theta = 3.13 \times \cos 45° = \mathbf{2.21\ ms^{-1}}$

Example 9.13. From the top of a building 45 m high, a stone is thrown at an angle of 30° to the horizontal with an initial velocity of 20 m/s. Find (*i*) the time of flight (*ii*) the distance from the foot of the building where it strikes the ground.

Solution. Fig. 9.14 shows the conditions of the problem. The x and y components of initial velocity v_0 (= 20 m/s) are :

$v_{x0} = v_0 \cos\theta = 20 \cos 30° = 17.3$ m/s

$v_{y0} = v_0 \sin\theta = 20 \sin 30° = 10$ m/s

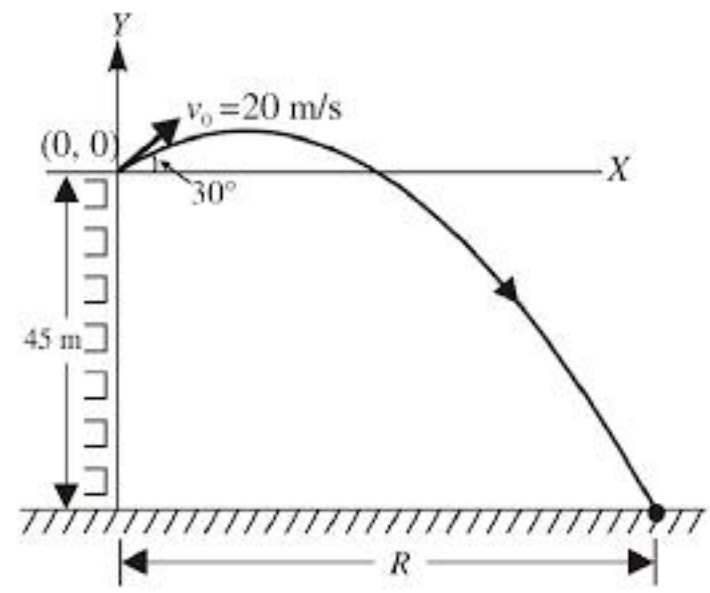


Fig. 9.14

(*i*) Let us choose the firing point as the origin. The time of flight is determined by vertical motion only. For vertical motion,

$$h = v_{y0}t + \frac{1}{2}gt^2$$

Here $h = -45$ m; $v_{y0} = 10$ m/s; $g = -9.8$ m/s^2; $t = T$

$\therefore$ $-45 = 10\,T - \dfrac{1}{2} \times 9.8\,T^2$

or $-45 = 10T - 4.9\,T^2$

Solving this quadratic equation, $T = \mathbf{4.22\ s}$ (The negative value of T has no physical meaning since time intervals are always positive).

(*ii*) Horizontal range, $R = v_{x0} \times T = 17.3 \times 4.22 = \mathbf{73\ m}$

Example 9.14. A boy wants to throw a letter wrapped over a stone to his friend across the street 40 m wide. The boy's window is 10m below friend's window. How should he throw the ball ?

Solution. Fig. 9.15 shows the conditions of the problem. The boy's window is at O and friend's window is at A. Let the boy throw the stone with a velocity v_0 making an angle θ with the horizontal so as to enter the window at A. The stone will follow the parabolic path with A as the highest point on the trajectory of stone.

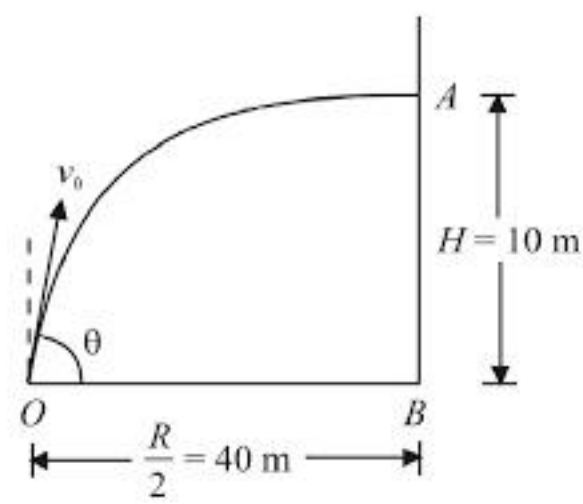


Fig. 9.15

$\therefore$ $\dfrac{R}{2} = 40$ or Horizontal range, $R = 2 \times 40 = 80$ m

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Maximum height attained, $H = 10$ m

Now $$R = \frac{v_0^2 \sin 2\theta}{g} = \frac{v_0^2\, 2 \sin\theta \cos\theta}{g} \quad ...(i)$$

and $$H = \frac{v_0^2 \sin^2\theta}{2g} \quad ... (ii)$$

$\therefore$ $$\frac{H}{R} = \frac{\tan\theta}{4} \text{ or } \tan\theta = 4 \times \frac{H}{R} = 4 \times \frac{10}{80} = \frac{1}{2} \quad \therefore \theta = \mathbf{26.56°}$$

The projection velocity v_0 can be found by putting the value of θ in eq. (*ii*).

$$H = \frac{v_0^2 \sin^2\theta}{2g}$$

$\therefore$ $$v_0^2 = \frac{2gH}{\sin^2\theta} = \frac{2 \times 9.8 \times 10}{\sin^2 26.56°} = 980 \text{ or } v_0 = \sqrt{980} = \mathbf{31.3\ ms^{-1}}$$

Example 9.15. A body is projected with a velocity of 40 ms^{-1}. After 2 s, it crosses a vertical pole of height 20.4 m. Calculate the angle of projection and the horizontal range.

Solution. Let θ be the angle of projection. For vertical motion,

$$h = (v_0 \sin\theta)\, t - \frac{1}{2} g\, t^2$$

or $$20.4 = (40 \sin\theta) \times 2 - \frac{1}{2} \times 9.8 \times (2)^2$$

or $$20.4 = 80 \sin\theta - 19.6$$

$\therefore$ $$\sin\theta = \frac{20.4 + 19.6}{80} = \frac{40}{80} = \frac{1}{2} \quad \therefore \quad \theta = \mathbf{30°}$$

Horizontal range, $R = \frac{v_0^2 \sin 2\theta}{g} = \frac{(40)^2 \times \sin 60°}{9.8} = \mathbf{141.4\ m}$

PROBLEMS FOR PRACTICE

1. A gun has a muzzle velocity of 60 ms^{-1}. At what angle should it be aimed to strike a target 320 m away? Ignore air resistance. **[30.3° or 59.7°]**
2. A tiger leaps horizontally from a 16 m high rock with a speed of 7 ms^{-1}. How far from the base of the rock will he land ? **[12.65 m]**
3. What is the least velocity with which a cricket ball can be thrown through a distance of 100 m? **[31.3 m s^{-1}]**
4. An aeroplane travelling at 150 km/h wants to drop supplies to flood victims isolated on a patch of land 250 m below. The supplies should be dropped how many seconds before the plane is directly overhead ? **[7.1 s]**
5. A ball thrown horizontally at 22 ms^{-1} from the roof of a building lands 36 m from the base of the building. How high is the building ? **[12.9 m]**
6. A football is kicked at an angle of 37° with a velocity of 20 m s^{-1}. Calculate (*i*) the maximum height (*ii*) the time of flight. (*iii*) How far away it hits the ground ? **[(*i*) 7.35 m *(ii)* 2.45 s *(iii)* 39.2 m]**
7. An athlete executing a long jump leaves the ground at an angle of 30° and travels 8.9 m. What was the take off speed? **[10 m s^{-1}]**

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9.9. NON-UNIFORM MOTION IN A PLANE

An object is said to have *non-uniform motion* in a plane if its velocity (and hence acceleration) changes from instant to instant. In such a case, we have to deal with *instantaneous velocity* and *instantaneous acceleration*. Obviously, the path of the object will not be a straight line (as for uniform motion) rather the object will follow a curved path.

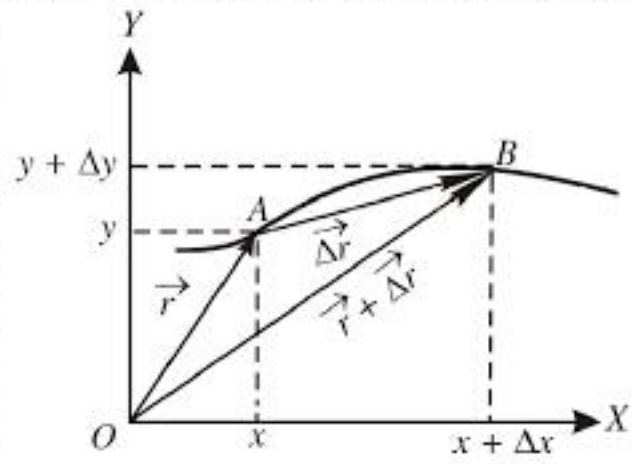


Fig. 9.16

Consider an object moving in the XY plane with a non-uniform velocity. Clearly, the object will follow a curved path as shown in Fig. 9.16. Suppose at time t, the object is at point $A(x, y)$ and its position vector is $\vec{r}$. Let at time $(t + \Delta t)$ its position be $B(x + \Delta x, y + \Delta y)$ and the position vector $\vec{r} + \vec{\Delta r}$. The displacement of the object in the small time interval Δt is $\vec{\Delta r} = (\Delta x\,\hat{i} + \Delta y\,\hat{j})$.

1. Instantaneous velocity vector. The average velocity of the object between points A and B is

$$\vec{v}_{av} = \frac{\vec{\Delta r}}{\Delta t}$$

The instantaneous velocity $\vec{v}$ of the object at point A is the limit of the average velocity as the time interval Δt approaches zero *i.e.*,

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\vec{\Delta r}}{\Delta t} = \frac{\vec{dr}}{dt}$$

$$\therefore \quad \vec{v} = \frac{\vec{dr}}{dt}$$

We can also express instantaneous velocity vector $\vec{v}$ in terms of its rectangular components as under :

$$\vec{v} = \frac{\vec{dr}}{dt} = \frac{d}{dt}(x\hat{i} + y\hat{j}) = \frac{*dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

or $$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

or $$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

The coefficients are the scalar components of $\vec{v}$.

$$v_x = \frac{dx}{dt}; \quad v_y = \frac{dy}{dt}$$

The following points may be noted :

(*i*) The magnitude of the velocity is $v = \sqrt{v_x^2 + v_y^2}$.

(*ii*) The direction in which the object is heading at any time may be described in terms of angle θ between velocity vector and X-axis as :

$$\tan\theta = \frac{v_y}{v_x}$$

---

* Note that $\frac{d\hat{i}}{dt} = \frac{d\hat{j}}{dt} = 0$ since these are unit vectors and are constant in magnitude and direction.

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2. Instantaneous acceleration vector. If $\vec{v}$ and $\vec{v} + \overrightarrow{\Delta v}$ are the velocities of the object at times t and $t + \Delta t$, then its average acceleration between points A and B is

$$\vec{v}_{av} = \frac{\overrightarrow{\Delta v}}{\Delta t}$$

The instantaneous acceleration $\vec{a}$ is given by ;

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\therefore \quad \vec{a} = \frac{d\vec{v}}{dt}$$

We can also express the instantaneous acceleration vector $\vec{a}$ in terms of its rectangular components as under :

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x \hat{i} + v_y \hat{j}) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$\therefore \quad \vec{a} = a_x \hat{i} + a_y \hat{j}$$

The two scalar components of $\vec{a}$ are :

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}; \quad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$$

9.10. TERMS RELATING TO CIRCULAR MOTION

When an object moves in a circular path, it is said to have circular motion. The circular motion differs from linear motion in one very important aspect. In circular motion, the direction of motion changes continuously unlike the linear motion. Therefore, circular motion is described in terms of angular displacement.

(*i*) Angular displacement. *The angle turned by an object moving along a circular path in a given time is called the angular displacement of the object during that time interval.*

Consider an object moving along a circular path with centre O as shown in Fig. 9.17. Let us consider O as the origin of our coordinate system. Suppose initially (*i.e.*, at $t = 0$), the object is at point P. At time t_1, the object is at point P_1 and its angular position is θ_1. At time t_2, the object is at point P_2 and its angular position is θ_2. During the time interval $t_2 - t_1$ $(= \Delta t)$, the angular displacement is $\theta_2 - \theta_1$ $(= \Delta\theta)$.

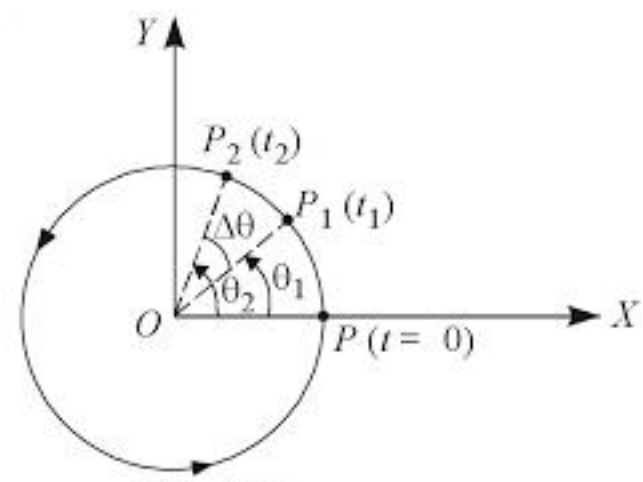


Fig. 9.17

$\therefore$ Angular displacement, $\Delta\theta = \theta_2 - \theta_1$ ...during $\Delta t (= t_2 - t_1)$

The SI unit of angular displacement is radian (rad).

(*ii*) Average angular velocity ($\bar{\omega}$). *The average angular velocity of an object moving along a circular path is defined as the time rate of change of its angular displacement.*

If the angular displacement of an object is $\Delta\theta$ $(= \theta_2 - \theta_1)$ during the time interval Δt $(= t_2 - t_1)$, then average angular velocity of the object is

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$$\text{Average angular velocity, } \bar{\omega} = \frac{\text{Angle turned}}{\text{Time taken}} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

The SI unit of angular velocity is rad/s and its dimensional formula is $[M^0L^0T^{-1}]$.

Instantaneous angular velocity (ω). *The angular velocity of an object moving along a circular path at any instant of time is called its instantaneous angular velocity.* It is denoted by ω. It is given by the limit of $\Delta\theta/\Delta t$ as Δt approaches zero *i.e.*,

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

The linear counterpart of angular velocity (ω) is the linear velocity (v).

***(iii)* Average angular acceleration ($\bar{\alpha}$).** *The average angular acceleration of an object moving along a circular path is defined as the time rate of change of its angular velocity.*

$$\text{Average angular acceleration, } \bar{\alpha} = \frac{\text{Change in angular velocity}}{\text{Time taken}}$$

If the change in the angular velocity of an object is $\Delta\omega(=\omega_2 - \omega_1)$ during the time interval Δt $(=t_2 - t_1)$, then average angular acceleration $\bar{\alpha}$ of the object is

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

The SI unit of angular acceleration is rad/s^2 and its dimensional formula is $[M^0L^0T^{-2}]$.

Instantaneous angular acceleration (α). *The angular acceleration of an object moving along a circular path at any instant of time is called its instantaneous angular acceleration.* It is denoted by α. It is given by the limit of $\Delta\omega/\Delta t$ as Δt approaches zero *i.e.*,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

The linear counterpart of angular acceleration (α) is the linear acceleration (a).

***(iv)* Time period.** *The time period of an object moving along a circular path is defined as the time taken by it to complete one revolution.* It is denoted by T and its unit is second.

For example, if an object completes 120 revolutions in 30 seconds, its time period is $T = 30/120 = 0.25$ s. It means that the object will complete one cycle in 0.25 s.

***(v)* Frequency.** *The frequency of an object moving along a circular path is defined as the number of revolutions completed by it in one second.* It is denoted by f and its SI unit is s^{-1} or Hertz (Hz).

Thus in the above case, the object completes 120 revolutions in 30 seconds. Therefore, the frequency of the object is $f = 120/30 = 4$ Hz. It means that the object will complete 4 revolutions in one second.

***(vi)* Relation between T and f.** Suppose an object executing circular motion has frequency f. It means that the object completes f revolutions in 1 second. Therefore, the time taken to complete one revolution is $1/f$. But this is the time period of the object.

$$\therefore \qquad T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T} \quad \text{or} \quad fT = 1$$

All these three equations are equivalent.

***(vii)* Relation between ω, f and T.** When an object executing circular motion completes one revolution, angular displacement $\theta = 2\pi$ radians and time taken is T(= time period; by definition).

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$$\therefore \qquad \text{Angular velocity, } \omega = \frac{\theta}{T} = \frac{2\pi}{T} = 2\pi f \qquad \left(\because f = \frac{1}{T}\right)$$

$$\therefore \qquad \omega = \frac{2\pi}{T} = 2\pi f$$

9.11. RELATION BETWEEN LINEAR AND ANGULAR QUANTITIES

As an object moves in a circle, it not only sweeps out an angle θ but it also covers an arc of length s as shown in Fig. 9.18. We know that angle in radians is related to arc length s and radius r as :

$$s = r\,\theta \qquad \ldots\theta \text{ in radians}$$

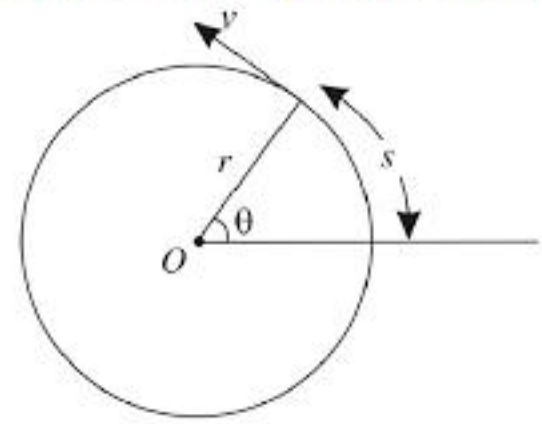


Fig. 9.18

Thus there is a definite relation between linear and angular quantities.

(*i*) Relation between linear and angular velocity. As an object moves along the circumference of the circle, it has linear velocity v which is always tangent to the circular path at every instant as shown in Fig. 9.18. The relation between the linear velocity v and angular velocity ω of the object can be found as under :

$$v = \frac{ds}{dt} = \frac{d}{dt}(s) = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt} = r\omega$$

$$\therefore \qquad v = r\omega \qquad \ldots(i) \ [\omega \text{ in rad/s}]$$

Eq. (*i*) is an important relationship between the circular motion of an object and the linear motion that results from rotation.

(*ii*) Relation between linear and angular acceleration. The relation between linear acceleration a and angular acceleration α can be found as under:

We know that : $v = \omega r$

$$\text{Now,} \qquad a = \frac{dv}{dt} = \frac{d}{dt}(\omega r) = r\frac{d\omega}{dt} = r\alpha$$

$$\therefore \qquad a = r\,\alpha \qquad \ldots(ii)$$

Eq. (*ii*) is again a useful result.

Note. In vector form : $\vec{v} = \vec{\omega} \times \vec{r}$; $\vec{a} = \vec{\alpha} \times \vec{r}$. Note that these are the cross products of two vectors.

9.12. UNIFORM CIRCULAR MOTION

An important case of two dimensional motion (*i.e.*, motion in a plane) is that of an object moving in a circular path at constant speed. This is called uniform circular motion.

When an object moves in a circular path at constant speed, the object is said to be in uniform circular motion.

The word "uniform" refers to the constant speed. It means that in uniform circular motion, the object covers equal distances along the circumference in equal intervals of time *i.e.*, the speed is constant. Although the magnitude of velocity (*i.e.*, speed) remains constant, the direction of the velocity is changing continuously. Therefore, the object is undergoing acceleration. This is called *centripetal acceleration* and is directed radially towards the centre of the circle as shown in Fig. 9.19. Note that centripetal means "centre-seeking". If an object moves in a circle of radius r with constant speed v, then it can be proved that the magnitude of centripetal acceleration $a_c = v^2/r$.

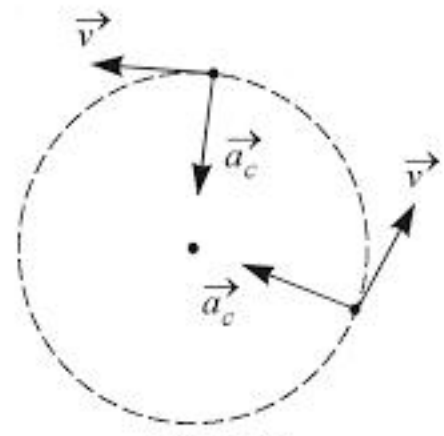


Fig. 9.19

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9.13 CENTRIPETAL ACCELERATION

The acceleration acting on the object executing uniform circular motion is called ***centripetal acceleration***. It is denoted by a_c.

In uniform circular motion, the magnitude of velocity (*i.e.*, speed) remains constant but direction of velocity changes continuously. Therefore, centripetal acceleration acting on the object describes the rate of turning *i.e.*, it represents the rate of change of the *direction* of the motion. We now determine the direction and magnitude of the centripetal acceleration.

Consider an object moving with a constant speed v around a circle of radius r as shown in Fig. 9.20(*i*). At point A, its velocity $\vec{v}_A$ is in the direction of the tangent to the path at point A. After a short time Δt, the object is at point B and its velocity $\vec{v}_B$ is in the direction of the tangent to the path at point B. During the time interval Δt, the object moves from A to B, covering a small distance Δl along the arc which subtends a small angle $\Delta\theta$ at the centre O. Note that magnitudes of velocity at points A and B are the same (since speed is constant). However, directions of velocity differ at points A and B. Hence the object has acceleration. The change in velocity vector ($\overrightarrow{\Delta v}$) is

$$\overrightarrow{\Delta v} = \vec{v}_B - \vec{v}_A$$

The change in velocity ($\overrightarrow{\Delta v}$) is shown in Fig. 9.20(*ii*). The acceleration of the object is given by :

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta v}}{\Delta t} = \frac{\overrightarrow{dv}}{dt}$$

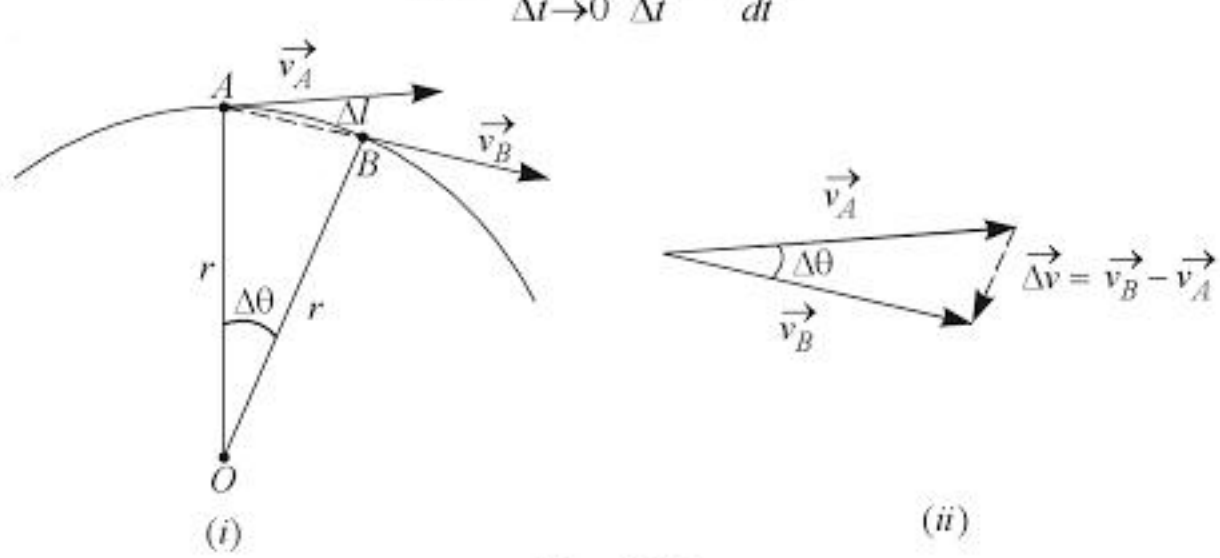


Fig. 9.20

(*i*) Direction of acceleration. In Fig. 9.20(*ii*), we find that when Δt is very small (approaching zero), Δl and $\Delta\theta$ are also very small. Consequently, $\vec{v}_B$ will be almost parallel to $\vec{v}_A$ and $\overrightarrow{\Delta v}$ will be essentially perpendicular to them. In the limit $\Delta t \to 0$, $\overrightarrow{\Delta v}$ will be exactly perpendicular to $\vec{v}_A$. Thus $\overrightarrow{\Delta v}$ is along the radius of the circle. By definition, $\vec{a}$ is always in the direction of $\overrightarrow{\Delta v}$. Therefore, acceleration 'a' must point towards the centre of the circle. Therefore, this acceleration is called *centripetal acceleration* (a_c).

Hence when an object moves with a constant speed around a circle, the centripetal acceleration is always directed radially towards the centre of the circle.

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(*ii*) Magnitude of a_c. The vectors $\vec{v}_B, \vec{v}_A$ and $\vec{\Delta v}$ in Fig. 9.20(*ii*) form a triangle that is geometrically *similar to the triangle ABO in Fig. 9.20(*i*). Thus we can write

$$\frac{\Delta v}{v} \quad \frac{\Delta l}{r}$$

or $$\Delta v \quad \frac{v}{r}\Delta l$$

This becomes an exact equality when $\Delta t \to 0$ for then Δl equals the chord AB. In order to get the magnitude of centripetal acceleration a_c, we use the above relation for Δv and write

$$a_c = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{v \Delta l}{r \Delta t}$$

$$= \frac{v}{r} \lim_{\Delta t \to 0} \frac{\Delta l}{\Delta t} = \frac{v}{r} \times v \qquad \left(\because \lim_{\Delta t \to 0} \frac{\Delta l}{\Delta t} = v\right)$$

$\therefore$ Centripetal acceleration, $a_c = \dfrac{v^2}{r}$

Thus when an object moves in a circle of radius r with a constant speed v, the magnitude of the centripetal acceleration is v^2/r and it is directed radially towards the centre of the circle.

Since v and r are constant, the magnitude of centripetal acceleration a_c $(= v^2/r)$ is constant. However, the direction of a_c is not constant; it is changing continuously as shown in Fig. 9.21. Note that linear velocity $\vec{v}$ and $\vec{a}_c$ are perpendicular to each other at every instant. Further, the directions of $\vec{v}$ and $\vec{a}_c$ are changing continuously.

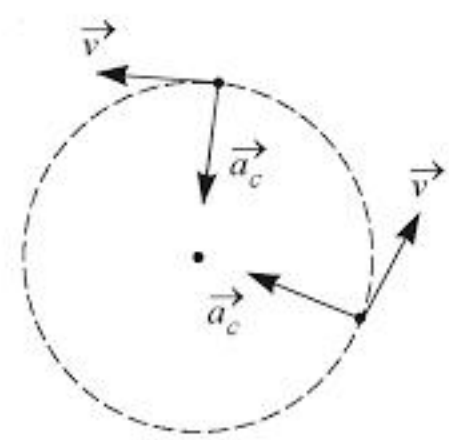


Fig. 9.21

Since centripetal acceleration always acts radially towards the centre of the circle, it is sometimes called *radial acceleration*. The magnitude of a_c can be given in the following equivalent formulas :

Centripetal acceleration, $a_c = \dfrac{v^2}{r} = \dfrac{(\omega r)^2}{r} = \omega^2 r = (2\pi f)^2 r = 4\pi^2 f^2 r$

9.14. PROJECTILE MOTION VERSUS UNIFORM CIRCULAR MOTION

(*i*) Both are two dimensional motions.

(*ii*) In projectile motion, the acceleration is constant in both magnitude and direction but the velocity changes in both magnitude and direction. However, in uniform circular motion, both velocity and acceleration are constant in magnitude but both change their directions continuously.

(*iii*) In projectile motion, $\vec{a}$ $(= \vec{g})$ makes various angles with $\vec{v}$ during the course of the motion. However, in case of uniform circular motion, $\vec{a}_c$ is perpendicular to $\vec{v}$ at every instant.

(*iv*) For projectile motion, $\vec{a}$ $(= \vec{g})$ is constant in both magnitude and direction. Therefore, kinematic equations for constant acceleration can be used for this type of motion. However, these equations *cannot* be used for uniform circular motion because the acceleration is not constant in direction.

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* $OA = OB = r$ and $v_A = v_B = v$, the velocity-vector triangle and ΔABO are isosceles (two sides are equal). Also since $\vec{v}_A \perp OA$ and $\vec{v}_B \perp OB$, the angle between $\vec{v}_A$ and $\vec{v}_B$ is $\Delta\theta$. Hence the two triangles are similar.

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$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

The three scalar components of $\vec{v}$ are :

$$v_x = \frac{dx}{dt} ; v_y = \frac{dy}{dt} ; v_z = \frac{dz}{dt}$$

(*iii*) The instantaneous acceleration $\vec{a}$ of an object moving in space is given by ;

$$\vec{a} = \frac{\vec{dv}}{dt} = \frac{d}{dt}(\vec{v})$$

$$= \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$$

$$\therefore \quad \vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

The three scalar components of $\vec{a}$ are :

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} ; \quad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2} ; \quad a_z = \frac{dv_z}{dt} = \frac{d^2z}{dt^2}$$

CONCEPTUAL QUESTIONS

Q.1. Give two practical examples of two dimensional motion.

Ans. The first example is the projectile motion. Two coordinates are required to describe projectile's motion since it moves horizontally as well as vertically. The second example is the uniform circular motion *i.e.*, motion in a circle at constant speed.

Q.2. In projectile motion, what is the relationship between the directions of acceleration and velocity?

Ans. In projectile motion (non-vertical), $\vec{a}$ (= g) is constant in both magnitude and direction (downward) and makes various angles with $\vec{v}$ during the course of motion.

Q.3. In the uniform circular motion, what is the relationship between the directions of acceleration and velocity?

Ans. In uniform circular motion, $\vec{a}_c$ and $\vec{v}$ are perpendicular to each other since $\vec{v}$ is tangential to the circle and $\vec{a}_c$ points toward the centre. Note that directions of both $\vec{a}_c$ and $\vec{v}$ change continuously but at every instant $\vec{v} \perp \vec{a}_c$.

Q.4. Can you cite any practical example of uniformly accelerated motion in two dimensions.

Ans. The projectile motion is an example of uniformly accelerated motion in two dimensions. In this motion, acceleration is constant in both magnitude and direction.

Q.5. What assumptions are made in the treatment of projectile motion ?

Ans. To treat projectile motion, we make the following two assumptions :

(*i*) We neglect the variation in the direction or magnitude of gravitational acceleration (g). This assumption is equivalent to neglecting the curvature of the earth and is valid for projectiles whose horizontal and vertical motions are small compared with earth's radius.

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(*ii*) We neglect air resistance. For dense objects moving relatively slowly, it is almost negligible. But for object whose surface area to mass is large (*e.g.*, ping-pong ball) or objects moving at high speeds, air resistance may dramatically alter the motion.

Q.6. Give two practical examples of uniform circular motion.

Ans. Driving around a curve at constant speed, you temporarily describe a circular arc and are, therefore, temporarily in uniform circular motion.As you swing a hockey stick, you produce motion that is circular and that may be approximately uniform.

Q.7. In uniform circular motion, centripetal acceleration is $a_c = v^2/r$ where v is the constant speed of the body and r is the radius of the circle. How do you explain this relation physically ?

Ans. Centripetal acceleration, $a_c = v^2/r$. An increase in speed v means that the time Δt for a given change in direction of the velocity becomes shorter. Not only that, but the associated change $\overrightarrow{\Delta v}$ in velocity is larger. These two effects combine to give an acceleration that depends on the square of speed. On the other hand, an increase in radius with a fixed speed simply increases the time Δt associated with a given change in velocity. Therefore, acceleration is inversely proportional to the radius.

Note. Although the form of equation does not look familiar, we can check that dimensions of R.H.S. are that of acceleration.

$$[a] = \frac{[v^2]}{[r]} = \frac{[L^2T^{-2}]}{[L]} = [LT^{-2}]$$

Q.8. In uniform circular motion, what is the relation between time period and frequency?

Ans. Consider a particle describing uniform circular motion. Let T be its time period and f be its frequency. The particle completes f revolutions in 1 second. The particle will complete 1 revolution (*i.e.* time period) in $1/f$ second.

$$\therefore \qquad T = \frac{1}{f} \text{ or } f = \frac{1}{T}$$

Q.9. What is the relation between angular velocity (ω), frequency (f) and time period (T) in uniform circular motion ?

Ans. Consider a particle describing uniform circular motion. Let its anugular velocity, time period and frequency be ω, T and f respectively. In one complete revolution, the angular displacement is 2π and time = T (time period).

$$\therefore \qquad \omega = \frac{2\pi}{T} = 2\pi f \qquad \left(\because \frac{1}{T} = f\right)$$

Q.10. A bob of mass 100 g hung from the ceiling of a string 1.5 m long is set into oscillations. The speed of the bob at mean position is 2 ms^{-1}. What is the trajectory of the bob if string is cut when bob is (*i*) at one of its extreme position (*ii*) at its mean position?

Ans. (*i*) If the string is cut when the bob is at one of its extreme position, the bob will fall vertically downward. The reason is simple. When the bob is at the extreme position, it comes to rest for a moment. If the string is now cut, it will fall vertically downward like a freely falling body.

(*ii*) In the second case, the bob will follow a parabolic path. When the bob is at the mean position, it has only horizontal velocity. If the string is now cut, the bob will become a projectile fired horizontally and, therefore, follow a parabolic path.

Q.11. The magnitude and direction of the acceleration of a body are constant. Will the path of the body necessarily be a straight line?

Ans. If the magnitude as well as direction of the acceleration of a body is constant, it is not necessary that the path of the body is a straight line. For example, in projectile motion,

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the projectile is under g which has constant magnitude (9.8 m/s^2) and constant direction (vertically downward). The path of projectile is parabolic and not a straight line.

Q.12. A body is dropped freely from the window of a train. Will the time of the free fall be equal, if (*i*) the train is stationary (*ii*) the train moves with a constant velocity (*iii*) the train moves with an acceleration?

Ans. In all the three cases, the time of free fall of the body will be the same. It is because the motion of train affects only the horizontal motion and not motion along the vertical. For vertically downward motion, the parameters (g and h) are the same in the three cases.

Q.13. A body slides down a smooth inclined plane when released from the top, while another body falls freely from the same point. Which one will strike the ground earlier?

Ans. For a freely falling body, the acceleration due to gravity g acts on it. But when a body slides down a smooth inclined plane, its acceleration is $g \sin \theta$ where θ is the inclination of the plane with the horizontal. Since $g > g \sin \theta$, the body falling freely will reach earlier ($v = v_0 + gt$).

Q.14. A person sitting in a moving train throws a ball vertically upwards. How does the ball appear to move to an observer (*i*) inside the train (*ii*) outside the train?

Ans. (*i*) To the observer sitting inside the train, the ball will appear to move straight vertically upwards and then downwards. It is because in this case the ball has only one velocity acting vertically.

(*ii*) To an observer outside the train (*e.g.*, standing on the ground outside the train), the ball will appear to move along the parabolic path. It is because in this case, the ball is acted upon by vertical as well as horizontal velocities.

Q.15. The velocity at the maximum height of a projectile is half its initial velocity of projection v_0. What is the horizontal range of the projectile?

Ans. It is given that $v_0 \cos \theta = v_0/2$. Therefore, $\theta = 60°$.

$$\therefore \text{Horizontal range, } R = \frac{v_0^2}{g} \sin 2\theta = \frac{v_0^2}{g} \sin 2 \times 60° = \frac{\sqrt{3}\, v_0^2}{2g}$$

VERY SHORT ANSWER QUESTIONS

Q.1. Give three examples of motion in a plane (two dimensional motion).

Ans. (*i*) Crawling of a snake (*ii*) A bag dropped from an aeroplane flying horizontally (*iii*) A gun shot fired at an angle with the horizontal.

Q.2. Give two examples of three dimensional motion.

Ans. (*i*) Flight of an insect (*ii*) Molecular motion.

Q.3. Give three examples of projectile motion.

Ans. (*i*) A piece of stone thrown in any direction (*ii*) A body dropped from the window of a moving train (*iii*) A bomb released from a flying aeroplane.

Q.4. What is the nature of projectile motion?

Ans. It is a case of two dimensional motion (*i.e.*, motion in a plane) with constant acceleration.

Q.5. For a given projection angle (θ), what will be the effect on the horizontal range if initial velocity is doubled?

Ans. It will be four times the initial horizontal range ($\because R = v_0^2 \sin 2\theta/g$).

Q.6. What is a projectile?

Ans. A projectile is an object that is thrown into air and then moves under the influence of gravity alone (for simplicity, we neglect air resistance).

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Q.7. A particle is projected at an angle of 45° with the horizontal with a velocity of 12 ms^{-1}. What will be its horizontal range?

Ans. Since projection angle $\theta = 45°$, the horizontal range will be maximum and $= v_0^2/g = 12^2/9.8 = 14.7$ m.

Q.8. Is projectile motion two dimensional motion?

Ans. Yes. Since the projectile moves horizontally as well as vertically, it is a case of two dimensional motion *i.e.*, two coordinates are required to specify the position of the projectile at any time.

Q.9. Is rocket flight an example of projectile?

Ans. No. A rocket is driven by engines into space and does not move under gravity alone.

Q.10. What is the characteristic of projectile motion?

Ans. A projectile motion consists of two perpendicular motions *viz.* (*i*) a horizontal motion with constant velocity (*ii*) motion along vertical under gravity *i.e.*, with constant acceleration g (= 9.8 ms^{-2}).

Q.11. What is the angle of projection for a projectile thrown parallel to the horizontal?

Ans. It is 0°.

Q.12. Is there any point on the projectile path at which the acceleration is perpendicular to the velocity?

Ans. Yes. At the highest point of the parabolic path, the projectile has only horizontal velocity because the vertical component of velocity is zero. However, acceleration due to gravity is acting vertically downward.

Q.13. In projectile motion, when is the kinetic energy of projectile maximum and minimum?

Ans. The K.E. of the projectile is maximum at the point of projection or point of reaching the ground and is minimum at the highest point.

Q.14. Do the time of flight, horizontal range and maximum height of a projectile depend on its mass?

Ans. No because there is no mass term in their equations.

Q.15. For a given initial velocity v_0 of the projectile when is its (*i*) horizontal range maximum (*ii*) height attained maximum?

Ans. (*i*) The horizontal range is maximum when projection angle $\theta = 45°$ because $R = v_0^2 \sin 2\theta/g$. (*ii*) The height attained is maximum when $\theta = 90°$ because $H_{max} = v_0^2 \sin^2 \theta/2g$.

Q.16. What is the angular velocity of the hour hand of a watch?

Ans. For hour hand of watch, the time taken to complete one revolution is 12 hours. Therefore, its angular velocity $= 2\pi/12 = \pi/6$ rad/hour.

Q.17. What is the angular velocity of earth about its own axis?

Ans. The time taken by earth to complete one revolution about its own axis is 24 hours. Therefore, its angular velocity $= 2\pi/24 = \pi/12$ rad/hour.

Q.18. In uniform circular motion, which quantities (*i*) are constant (*ii*) not constant?

Ans. (*i*) In uniform circular motion, speed and magnitude of acceleration are constant (*ii*) Velocity and acceleration are not constant in uniform circular motion.

Q.19. What does the word "uniform" indicate in uniform circular motion?

Ans. In uniform circular motion, the word "uniform" means constant speed. Although the magnitude of velocity (*i.e.*, speed) remains constant in this type of motion, the direction of velocity is changing continuously.

Q.20. Is uniform circular motion two dimensional motion?

Ans. Yes, because it is a motion in a plane.

Q.21. What is the angle between velocity vector and acceleration vector in uniform circular motion?

Ans. In uniform circular motion, velocity vector and acceleration vector are perpendicular to each other at every instant.

Q.22. What is the direction of velocity vector of a particle in uniform circular motion?

Ans. At every point, the velocity vector is tangent to the circular path.

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SHORT ANSWER QUESTIONS

Q.1. What is the angle of projection for which horizontal range and maximum height are equal?

Ans. $\frac{v_0^2 \sin 2\theta}{g} = \frac{v_0^2 \sin^2 \theta}{2g}$ or $2 \sin\theta \cos\theta = \frac{\sin^2\theta}{2}$ $\therefore \theta = \tan^{-1}(4)$

Q.2. How does projectile motion differ from uniform circular motion?

Ans. In projectile motion, the acceleration ($= g$) is constant in both magnitude and direction but the velocity changes in both magnitude and direction. However, in uniform circular motion, both velocity and acceleration are constant in magnitude but both change their directions continuously.

Q.3. A body is projected with a speed v_0 at an angle to the horizontal to have maximum horizontal range. What is its velocity at the highest point?

Ans. For maximum horizontal range, projection angle $\theta = 45°$. At the highest point, the velocity has only horizontal component $= v_0 \cos\theta = v_0 \cos 45° = v_0/\sqrt{2}$.

Q.4. A bullet is dropped from a certain height and at the same time another bullet is fired horizontally from the same height. Which one will hit the ground earlier and why?

Ans. Both will hit the ground at the same time. It is because in both cases, the initial vertical downward velocity is zero, vertical downward acceleration $g(= 9.8\ ms^{-2})$ is the same and the vertical height is the same.

Q.5. Prove that the maximum horizontal range is four times the maximum height attained by the projectile.

Ans. For maximum horizontal range, projection angle $\theta = 45°$.

$$\therefore \quad R_{max} = \frac{v_0^2 \sin 2\theta}{g} = \frac{v_0^2 \sin 2\times 45°}{g} = \frac{v_0^2}{g}$$

$$\text{Now} \quad H_{max} = \frac{v_0^2 \sin^2\theta}{2g} = \frac{v_0^2 \sin^2 45°}{2g} = \frac{v_0^2}{4g}$$

Therefore, $R_{max}/H_{max} = 4$ or $R_{max} = 4\ H_{max}$.

Q.6. Can we apply constant acceleration kinematic equations to uniform circular motion?

Ans. No. It is because in uniform circular motion, direction of acceleration is not constant, although magnitude of acceleration is constant.

Q.7. A projectile with kinetic energy of 500 J is fired for maximum horizontal range. What is the K.E. of the projectile at the highest point?

Ans. Initial K.E., $K = mv_0^2/2 = 500$ J. For maximum horizontal range, projection angle $\theta = 45°$. We know that at the highest point, the projectile has only horizontal velocity $= v_0 \cos\theta = v_0 \cos 45° = v_0/\sqrt{2}$.

$$\therefore \quad \text{K.E. at highest point} = \frac{1}{2}m(v_0/\sqrt{2})^2 = \frac{1}{2}mv_0^2 \times \frac{1}{2} = K/2 = 250 \text{ J}$$

Q.8. In long jumping, what factors determine the span of jump?

Ans. $H_{max} = \frac{v_0^2 \sin^2\theta}{2g}$ and $R = \frac{v_0^2 \sin 2\theta}{g}$ $\therefore \frac{R}{H_{max}} = \frac{2 \sin 2\theta}{\sin^2\theta} = \frac{2 \times 2 \sin\theta \cos\theta}{\sin^2\theta}$

Therefore, $R = 4\ H_{max} \cot\theta$. Now $H_{max} \propto v_0$ (velocity of jump). It follows that the length of jump depends upon v_0 and θ.

Q.9. Two particles are thrown with same velocity at an angle of 30° and 60° with the horizontal. Which parameter (time of flight, horizontal range and maximum height) will remain same?

Ans. The horizontal range will remain the same. It is because horizontal range is the same for projection angles θ and $(90° - \theta)$ [*i.e.*, 30° and $(90° - 30°)$].

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Q.10. At which point of projectile motion (*i*) K.E. is maximum (*ii*) P.E. is maximum (*iii*) K.E. is minimum (*iv*) velocity is minimum (*v*) velocity is maximum.

Ans. (*i*) Place of projection (*ii*) At the highest point (*iii*) At the highest point (*iv*) At the highest point (*v*) Place of projection or at a point where it hits the ground.

Q.11. The greatest height to which a man can throw a stone is *h*. What will be the greatest distance upto which he can through the stone?

Ans. $H_{max} = v_0^2/2g = h$ (for $\sin\theta = 1$). Therefore, maximum horizontal range $= v_0^2/g = 2h$.

Q.12. If both speed of a body and radius of its circular path are doubled, what will happen to centripetal acceleration?

Ans. Centripetal acceleration, $a_c = v^2/r$. If both v and r are doubled, the new centripetal acceleration is $a_c' = (2v)^2/2r = 2 \times v^2/r = 2a_c$.

Q.13. Prove that the horizontal range of a projectile is the same for projection angles of $(45° + \theta)$ and $(45° - \theta)$.

Ans. Horizontal range, $R = v_0^2 \sin 2\theta/g$

For projection angle $(45° + \theta)$, $R_1 = \dfrac{v_0^2 \sin(90° + 2\theta)}{g} = \dfrac{v_0^2 \cos 2\theta}{g}$

For projection angle $(45° - \theta)$, $R_2 = \dfrac{v_0^2 \sin(90° - 2\theta)}{g} = \dfrac{v_0^2 \cos 2\theta}{g}$

Therefore, $R_1 = R_2$ *i.e.,* horizontal range is the same for projection angles $(45° + \theta)$ and $(45° - \theta)$.

Q.14. A particle moves with a constant angular speed of certain radius *r* and is acted upon by a certain centripetal force *F*. If the angular speed is doubled, keeping the radius the same, what will be the new centripetal force?

Ans. Centripetal force, $F_c = \dfrac{mv^2}{r} = \dfrac{m(\omega r)^2}{r} = m\omega^2 r$

If ω is doubled, $F_c' = m(2\omega)^2 r$ $\therefore$ $F_c'/F_c = 4$ or $F_c' = 4F_c$

Q.15. A particle of mass *m* is moving in a circular path of constant radius *r* such that its centripetal acceleration a_c is varying with time as $a_c = k^2 r t^2$ where *k* is a constant. What is the power delivered to the particle by the force acting on it?

Ans. $a_c = k^2 r t^2$ or $\dfrac{v^2}{r} = k^2 r t^2$ $\therefore$ $v = krt$

Tangential force, $F = m\dfrac{dv}{dt} = m\dfrac{d}{dt}(krt) = mkr$

$\therefore$ Power, $P = F \times v = (m\ k\ r)\ (k\ r\ t) = m\ k^2\ r^2\ t$

LONG ANSWER QUESTIONS

1. Derive an expression for displacement in two dimensions with uniform acceleration. **[Refer to Art. 9.3]**
2. Derive an expression for displacement in two dimensions with uniform velocity. **[Refer to Art. 9.2]**
3. A projectile is fired with a velocity v_0 making an angle θ with the horizontal. Find expressions for (*i*) time of flight (*ii*) maximum height attained and (*iii*) horizontal range. **[Refer to Art. 9.6]**
4. A projectile is fired with velocity v_0 making an angle θ with the horizontal. Show that its path is parabolic. **[Refer to Art. 9.5]**
5. What is centripetal acceleration? Find its magnitude and direction in case of uniform circular motion. **[Refer to Art. 9.13]**

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COMPETITION SUCCESS MATERIAL

Useful Concepts/Information

1. Projectile motion is two dimensional motion with constant acceleration.
2. It is useful to think of projectile motion as the superposition of two motions: (*i*) uniform motion in the X direction with velocity v_x, which remains constant throughout the motion (*ii*) motion in the vertical direction, subject to a constant downward acceleration of magnitude $g = 9.8\ m/s^2$.
3. Fig. 9.23 shows a projectile fired from the origin at $t = 0$ with an initial velocity v_0 making an angle θ with the positive X-axis.

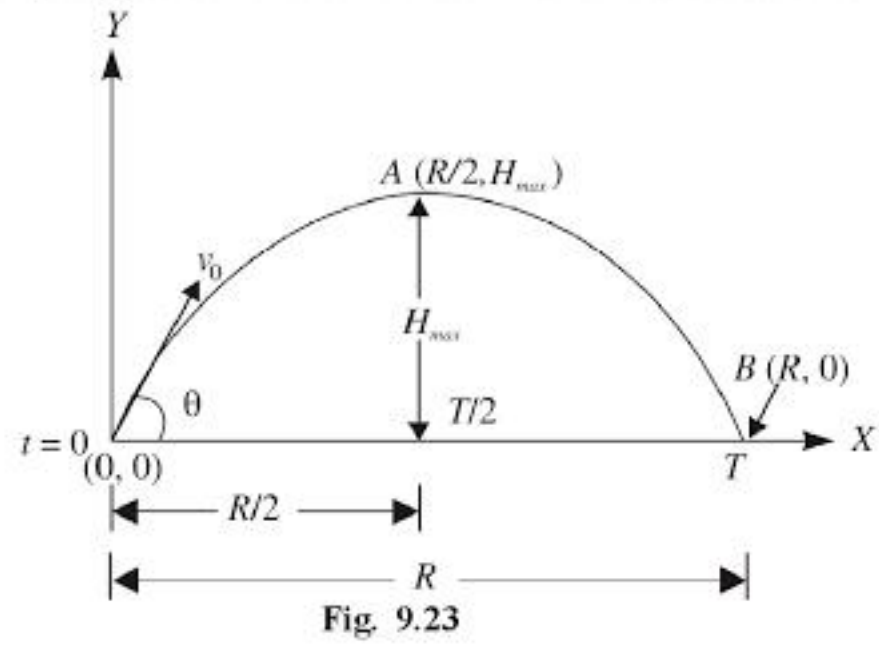


Fig. 9.23

$$\text{Time of flight, } T = \frac{2v_0 \sin\theta}{g}$$

$$\text{Max. height, } H_{max} = \frac{v_0^2 \sin^2\theta}{2g}$$

$$\text{Horizontal range, } R = \frac{v_0^2 \sin 2\theta}{g}$$

(*i*) The highest point is $A\ (R/2, H_{max})$ while point $B\ (R, 0)$ represents the condition when projectile returns to ground.

(*ii*) During the entire motion of the projectile, it is subjected to a constant acceleration $g\ (= 9.8\ ms^{-2})$ acting vertically downward.

(*iii*) At any time t during projectile motion, the horizontal and vertical components of velocity are :

$$v_x = v_0 \cos\theta = \text{constant} \ ; \quad v_y = v_0 \sin\theta - gt$$

(*iv*) The velocity of the projectile is maximum (v_0) at the starting point (*i.e.* origin) and goes on decreasing (component v_y decreases but v_x is constant) till it becomes minimum $(v_0 \cos\theta)$ at the highest point A. At point A, $v_y = 0$ and the velocity of the projectile is $v_0 \cos\theta$ acting horizontally.

(*v*) At any time t during projectile motion, the velocity v of the projectile is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(v_0\cos\theta)^2 + (v_0\sin\theta - gt)^2}$$

$$= \sqrt{v_0^2 + g^2t^2 - 2v_0 gt\sin\theta}$$

The angle β which v makes with horizontal is

$$\tan\beta = \frac{v_y}{v_x} = \frac{v_0\sin\theta - gt}{v_0\cos\theta} = \tan\theta - \frac{gt}{v_0\cos\theta}$$

(*vi*) The linear momentum of the projectile at the highest point is $mv_0 \cos\theta$ and it is mv_0 at the lowest point.

(*vii*) The K.E. of the projectile is maximum at the starting point (*i.e.* origin) and lowest at the highest point.

4. The horizontal range of the projectile is maximum when the angle of projection $\theta = 45°$.

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5. The horizontal range of the projectile is the same for projection angles of θ and $(90° - \theta)$.
6. The projectile attains maximum height when the projection angle $\theta = 90°$.
7. When the projectile is fired for maximum horizontal range R_{max} ($\theta = 45°$), the maximum height attained by the projectile is $R_{max}/4$.
8. Consider a projectile fired horizontally with velocity v_0 at a height H above the ground. Let us choose starting point as the origin. Initially, the vertical velocity is zero. If T is the time of flight, then,

$$-H = -\frac{1}{2}gT^2 \text{ or } T = \sqrt{\frac{2H}{g}}$$

Horizontal range, $R = v_0 \times T = v_0\sqrt{\frac{2H}{g}}$

At any time t, $v_x = v_0$ and $v_y = gt$. Therefore, velocity of projectile at time t is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + g^2t^2}$$

The angle β which $\vec{v}$ makes with horizontal is $\tan\beta = \frac{v_y}{v_x} = \frac{gt}{v_0}$

9. If a projectile of mass m is fired with velocity v_0 making an angle θ with the horizontal ground, then change in its linear momentum at the point of landing on the ground is $2mv_0 \sin\theta$ acting vertically downward.
10. When a body moves in a circular path, it has tangential velocity as well as angular velocity. The magnitudes of tangential velocity and angular velocity are related as :

$$v = \omega r$$

11. When a body is moving in a circular path, the rate of change of angular velocity (ω) is called angular accelerations α *i.e.* $\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}$

where θ is the angular displacement.

If the body moves with constant angular velocity, then angular acceleration is zero.

12. If a body moves in a circle of radius r with constant speed v, it has centripetal acceleration a_c that acts along the radius towards the centre of the circle.

$$a_c = \frac{v^2}{r}$$

13. If the speed of a body revolving in a circle is changing, then there will be tangential acceleration a_t as well as centripetal acceleration a_c.

$$a_t = \frac{dv}{dt} \text{ and } a_c = \frac{v^2}{r}$$

(*i*) The tangential acceleration arises from the change in magnitude of the velocity whereas the centripetal acceleration arises from the change in direction of the velocity.

(*ii*) The a_t and a_c are always perpendicular to each other and their directions change continuously. The total vector acceleration $\vec{a}$ is the vector sum of $\vec{a_t}$ and $\vec{a_c}$.

$$\therefore \quad \vec{a} = \vec{a_t} + \vec{a_c}$$

The magnitude of $\vec{a}$ is given by ; $a = \sqrt{a_t^2 + a_c^2}$

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Clearly, second law tells us that mass of a body is a measure of its inertia. For given net force, an object with a larger mass will have smaller acceleration *i.e.*, smaller change in the velocity of the object. Therefore, the mass of an object is that property of the object which causes it to resist any change in its velocity. Since inertia means resistance to change, the mass of a body is a measure of its inertia.

(*iv*) $$F = \frac{dp}{dt}$$

It means that slope of linear momentum-time graph gives the magnitude of force. Referring to Fig. 10.2,

$$F = \frac{dp}{dt} = \tan\theta = \text{Slope of } p\text{-}t \text{ graph}$$

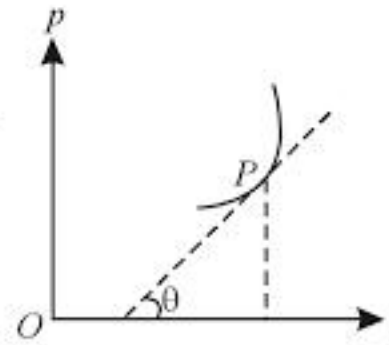


Fig. 10.2

Note. Forces acting simultaneously at the same point on a body are called **concurrent forces** and forces acting in the same plane are called **coplanar forces.** Forces acting parallel to each other are called **collinear forces**.

Example 10.1. A constant force acts for 10 seconds on a stationary body of mass 10 kg after which the force ceases to act. The body moves a distance of 50 m in next 5 seconds. Find the magnitude of force.

Solution. Initial velocity, $v_0 = 0$

When the force ceases to act, the body moves with a constant velocity. Since the body moves a distance of 50 m in 5 seconds, constant velocity $v = 50/5 = 10$ m s^{-1}. Note that this is the velocity of the body at the instant the force was removed.

$$\text{Acceleration, } a = \frac{v - v_0}{t} = \frac{10-0}{10} = 1 \text{ ms}^{-2}$$

$\therefore$ Magnitude of force, $F = m\,a = 10 \times 1 = \mathbf{10\ N}$

Example 10.2. A ship of mass 3×10^7 kg initially at rest is pulled by a force of 5×10^4 N through a distance of 3 m. Assuming the resistance due to water to be negligible, what is the speed of the ship?

Solution. Acceleration of ship, $a = \frac{F}{m} = \frac{5\times10^4}{3\times10^7} = \frac{5}{3}\times10^{-3}$ ms^{-2}

Let v be the speed of the ship. Since ship starts from rest, its initial speed $v_0 = 0$.

$\therefore$ $$v^2 - v_0^2 = 2aS \quad \text{or} \quad v^2 - 0^2 = 2\times\frac{5}{3}\times10^{-3}\times3$$

$\therefore$ $$v^2 = 10^{-2} \text{ or } v = \sqrt{10^{-2}} = \mathbf{0.1\ ms^{-1}}$$

Example 10.3. A 100 kg man jumps into swimming pool from a height of 5 m. It takes 0.4 second for the water to reduce his velocity to zero. What is the average force exerted by water on the man ? Take $g = 10$ ms^{-2}.

Solution. Let v be the velocity of the man just before entering the water.

$\therefore$ $$v^2 - 0^2 = 2gh \quad \text{or} \quad v = \sqrt{2gh} = \sqrt{2\times10\times5} = 10 \text{ ms}^{-1}$$

Now $$F = \frac{mv - 0}{t} = \frac{mv}{t} = \frac{100\times10}{0.4} = \mathbf{2500\ N}$$

Example 10.4. A girl pulls a box of mass 10.4 kg with a force of 25 N inclined to the horizontal at an angle of 37°. As a result, the box accelerates horizontally. What is the magnitude of acceleration?

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Solution. Fig. 10.3 shows the conditions of the problem. The horizontal component of force accelerates the box in the horizontal direction.

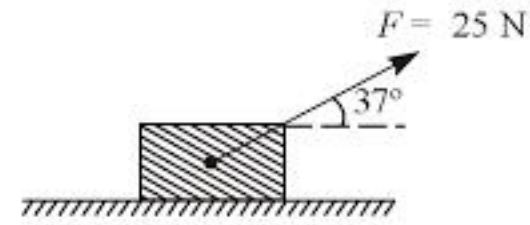


Fig. 10.3

Horizontal component of force, $F_x = F \cos 37°$

$= 25 \cos 37° = 20$ N

$\therefore$ Acceleration, $a_x = \dfrac{F_x}{m} = \dfrac{20}{10.4} = \mathbf{1.92\ ms^{-2}}$

Example 10.5. A bullet of mass 0.04 kg moving with a speed of 90 ms^{-1} enters a heavy wooden block and is stopped after a distance of 60 cm. What is the average resistive force exerted by the block?

Solution. The initial speed of the bullet is $v_0 = 90$ m s^{-1}. As the bullet enters the wooden block, its velocity decreases to zero (*i.e.*, $v = 0$) after covering a distance of 60 cm (*i.e.*, $S = 60$ cm).

Now $v^2 - v_0^2 = 2aS$

or $a = \dfrac{v^2 - v_0^2}{2S} = \dfrac{(0)^2 - (90)^2}{2 \times 0.6} = -6750\,ms^{-2}$

$\therefore$ Average retarding force, $F = m\,a = 0.04 \times 6750 = \mathbf{270\ N}$

Example 10.6. Sand drops vertically at the rate of 100 gs^{-1} on to a horizontal belt conveyor moving at a steady velocity of 5 cm s^{-1}. What is the force required to keep the belt moving?

Solution. Fig. 10.4 shows the conditions of the problem. The initial horizontal velocity of the sand is zero ($v_0 = 0$). The final horizontal velocity of sand is 5 cm s^{-1} ($v = 5$ cm $s^{-1} = 0.05$ m s^{-1}).

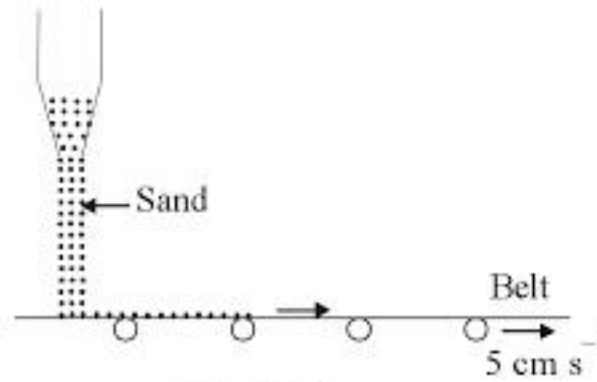


Fig. 10.4

Force required to keep belt moving, F = Rate of increase of horizontal linear momentum

$\therefore$ $F = \dfrac{mv - mv_0}{t} = \dfrac{0.1 \times 0.05 - 0}{1} = \mathbf{5 \times 10^{-3}\ N}$

Example 10.7. The mass of an elevator (lift) is 500 kg. Calculate the tension in the cable of the elevator when the elevator is (*i*) stationary (*ii*) ascending with an acceleration of 2 ms^{-2} (*iii*) descending with the same acceleration. Take $g = 9.8\ ms^{-2}$.

Solution. (*i*) When the elevator is stationary [See Fig. 10.5 (*i*)], its acceleration is zero. Therefore, net force on it is zero.

Net force $= T - mg$

or $0 = T - mg$

$\therefore$ $T = m\,g = 500 \times 9.8 = \mathbf{4900\ N}$

(*ii*) When the elevator is ascending [See Fig. 10.5 (*ii*)], the net force on it is upward. Therefore, tension T in the cable will be greater than gravity-force mg.

Net force $= T - mg$

or $ma = T - mg$

$\therefore$ $T = mg + ma = m\,(g+a) = 500\,(9.8 + 2) = \mathbf{5900\ N}$

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Note that when the lift is ascending with an acceleration, its apparent weight is increased. This is indicated by the increased tension in the cable.

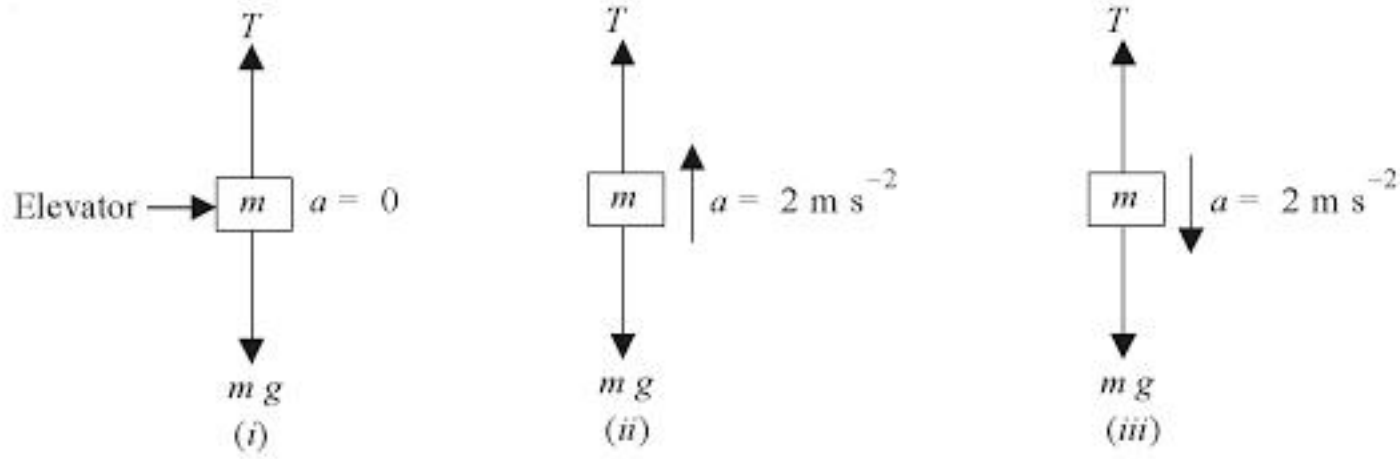


Fig. 10.5

(*iii*) When the elevator is descending [See Fig. 10.5 (*iii*)], the net force is in the downward direction. Therefore, tension T in the cable will be less than mg.

$$\text{Net force} = mg - T$$

or $$ma = mg - T$$

$$\therefore \quad T = mg - ma = m\,(g - a) = 500\,(9.8 - 2) = \mathbf{3900\ N}$$

Note that when the lift is descending with an acceleration, its apparent weight is decreased. This is indicated by the decreased tension in the cable.

Example 10.8. A hydrogen gas filled balloon having a mass of 25 g is released up in the air. The gas starts leaking from the balloon at a uniform rate with a velocity of 5 cm/s and the balloon shrinks completely in 5 s. Find the average force acting on the balloon.

Solution. Here $m = 25\ \text{g} = 0.025\ \text{kg}$; $v = 5\ \text{cm/s} = 0.05\ \text{m/s}$

$$\text{Force, } F = \frac{d}{dt}(mv) = v \times \frac{dm}{dt} \quad \ldots v \text{ remains constant}$$

$$= 0.05 \times \frac{0.025}{5} = \mathbf{25 \times 10^{-5}\ N}$$

Example 10.9. A car starts from rest and accelerates uniformly with 2 ms^{-2}. At $t = 10$ s, a stone is dropped out of the window of the car 2 m high. What are the (*i*) velocity and (*ii*) acceleration of the stone at $t = 10.2$ s? Take $g = 10\ \text{ms}^{-2}$.

Solution. (*i*) Initial velocity of car is $v_0 = 0$.

Velocity of car after 10 s is $\quad v_x = v_0 + at = 0 + 2 \times 10 = 20\ \text{m/s}$

Once the stone is dropped at $t = 10$ s, its acceleration along the horizontal direction is zero. Therefore, horizontal velocity remains unchanged *i.e.*, $v_x = 20$ m/s. But at $t = 10$ s, the stone also comes under the accelerated motion due to gravity.

$\therefore$ At $t = 10.2$ s : Horizontal velocity of stone $= v_x = 20$ m/s

Also At $t = 10.2$ s : Vertical downward velocity of stone is

$$v_y = 0 + gt = 0 + 10 \times 0.2 = 2\ \text{ms}^{-1} \quad (\because\ t = 10.2 - 10 = 0.2\ \text{s})$$

$\therefore$ Resultant velocity of stone at $t = 10.2$ s is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20)^2 + (2)^2} = \mathbf{20.1\ m/s}$$

(*ii*) Once the stone is dropped, the horizontal acceleration ceases to act on the stone and it is now only under acceleration due to gravity. Therefore, at $t = 10.2$ s, acceleration of stone $= g = \mathbf{10\ m/s^2}$.

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PROBLEMS FOR PRACTICE

1. Find the force required to give a mass of 70 kg an acceleration of 4.5 m/s^2 (*i*) horizontally (*ii*) vertically. **[(*i*) 315 N (*ii*) 1001 N]**
2. A force acts for 1 second on a body of mass 12 kg. The force is then withdrawn and the body moves through 2 m in the next second. Find the magnitude of force. **[24 N]**
3. A cricket ball of mass 100 g is moving with a speed of 25 ms^{-1} and is brought to rest in 0.025 s. Find the average force applied. **[100 N]**
4. A bullet of mass 6 g travelling at 126 m s^{-1} penetrates into a fixed target and is brought to rest in 0.01 second. Determine (*i*) the distance of penetration in the target (*ii*) average retarding force exerted on the bullet. **[(*i*) 0.6 m (*ii*) 72 N]**
5. Sand drops vertically at the rate of 2 kg s^{-1} onto a conveyor belt moving horizontally with a velocity of 0.1 m s^{-1}. Find the force required to keep the belt moving. **[0.2 N]**
6. A man of mass 50 kg is standing in a lift. Calculate the apparent weight of man when the lift is (*i*) stationary (*ii*) moving upward with an acceleration of 2 m s^{-2} (*iii*) moving downward with an acceleration of 2 m s^{-2}. **[(*i*) 490 N (*ii*) 590 N (*iii*) 390 N]**
7. A constant force acting on a body of mass 3 kg changes its speed from 2 m s^{-1} to 3.5 ms^{-1} in 25 s. The direction of motion of the body remains unchanged. What is the magnitude and direction of the force? **[0.18 N in the direction of motion]**
8. In an X-ray machine, an electron is subjected to a force of 10^{-23} N. What is its acceleration? In how much time the electron will cover a distance of 0.1 m? Take the mass of electron as 10^{-30} kg. **[10^7 ms^{-2} ; 1.4×10^{-4} s]**
9. A body of mass 5 kg is acted upon by two perpendicular forces 8 N and 6 N. Find the magnitude and direction of the resultant force. **[10 N; 36.87° with 8 N force]**

10.8. BASIC FORCES IN NATURE

There are four basic forces that operate in nature. These are :

1. The gravitational force **2.** The electromagnetic force **3.** The strong nuclear force **4.** The weak nuclear force.

1. The gravitational force. All objects exert a force of attraction upon each other. The gravitational force of attraction between two objects is governed by Newton's law of gravitation. If two objects of masses m_1 and m_2 are held at a distance r, then according to Newton's law of gravitation, the force of attraction between the two objects is given by ;

$$F = \frac{G \times m_1 m_2}{r^2} \qquad ...(i)$$

where G ($= 6.67 \times 10^{-11}$ Nm^2 kg^{-2}) is called *universal gravitational constant*. The mass m_1 exerts an attractive force on mass m_2 given by eq. (*i*). Similarly, the mass m_2 exerts an attractive force on m_1 given by eq. (*i*).

Properties. The gravitational force has the following properties :

(*i*) It is always attractive*.

* Unlike electrostatic force which can be attractive or repulsive. It is because mass is of one kind whereas charge is of two kinds (positive and negative).

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(*ii*) It is a central force *i.e.*, gravitational force between two objects acts along the line joining the centres of the objects.

(*iii*) It obeys inverse square law *i.e.*, gravitational force between two objects is inversely proportional to the square of distance between them.

(*iv*) Of all the four forces operating in nature, the gravitational force is the weakest.

(*v*) It is a long range force *i.e.*, it operates over very long distances.

(*vi*) It is a conservative force *i.e.*, work done by the gravitational force is recoverable.

Examples. A few examples of gravitational force in everyday life are :

(*i*) A stone dropped from a height falls because earth exerts gravitational force on the stone.

(*ii*) The rotation of earth around the sun is due to gravitational pull of sun on earth.

(*iii*) The moon revolves around the earth due to the gravitational force on it by the earth.

2. The electromagnetic force. The electromagnetic force includes both electric and magnetic forces and is relatively strong.

(*a*) The force between two static (at rest) electric charges is given by Coulomb's law in electrostatics. If two charges q_1 and q_2 are at a distance r apart, then according to Coulomb's law, the electrostatic force between them is given by :

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \quad \text{.... in vacuum}$$

where $\varepsilon_0 \left(= 8.854 \times 10^{-12}\ \mathrm{C^2N^{-1}m^{-2}}\right)$ is called *absolute permittivity of free space.*

(*b*) A static charge does not produce magnetic field. However, a moving charge produces a magnetic field. When two charges are in motion, each charge experiences a force called electromagnetic force.

Properties. The electromagnetic force has the following properties :

(*i*) It may be attractive or repulsive. It is because electric charges are of two kinds (positive and negative).

(*ii*) It is a central force.

(*iii*) It obeys inverse square law.

(*iv*) It is also a long range force.

(*v*) It is also a conservative force.

(*vi*) It is about 10^{36} times stronger than the gravitational force.

Examples. A few examples of electromagnetic force in everyday life are :

(*i*) If two current carrying parallel conductors are placed near each other, then force (repulsive or attractive) acts on them. This is the electromagnetic force. If the conductors carry currents in the same direction, then they attract each other *i.e.*, electromagnetic force is attractive. If the conductors carry currents in the opposite direction, then they repel each other.

(*ii*) In atoms, the electrons are bound by the electromagnetic forces acting on them due to protons.

(*iii*) Atoms combine to form molecules due to electromagnetic forces.

3. The strong nuclear force. The strong nuclear force is the attractive force that holds the constituents of the atomic nucleus *viz.*, protons and neutrons. The stability of the nucleus suggests that attractive nuclear forces must be very strong. For example, consider the helium nucleus ($_2He^4$) consisting of two protons and two neutrons and contained in a volume with a radius of about 10^{-15} m. The protons are so close together that repulsive electric force is very large. The gravita-

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tional force between them is negligible. Yet $_2He^4$ is very stable. It is because the strong attractive nuclear force acting between all the constituents of the nucleus overpowers the repulsive electric force between protons.

Properties. The strong nuclear force has the following properties :

(*i*) It is basically an attractive force.

(*ii*) It is a short-range force and operates only upto distances of the order of 10^{-14}m.

(*iii*) It does not depend upon the charge on the nucleons.

(*iv*) It is a non-central force.

(*v*) It does not obey inverse square law.

(*vi*) It is a non-conservative force.

Examples. A few examples of the strong nuclear force in everyday life are :

(*i*) It is the strong nuclear force that binds the neutrons and protons together in a nucleus.

(*ii*) The nuclear energy available from the processes of nuclear fission and fusion uses the concept of strong nuclear force.

4. The weak nuclear force. The weak nuclear force appears only in certain nuclear processes such as β-decay of a nucleus. In β-decay, electrons are emitted from a radioactive nucleus. The weak nuclear force is much stronger than the gravitational force.

10.9. IMPULSIVE FORCE AND IMPULSE

1. Impulsive forces. *The forces which act on bodies for a short time are called impulsive forces.* The examples of impulsive forces are :

(*i*) A bat hitting the ball.

(*ii*) The collision of two billiard balls.

(*iii*) The firing of a gun.

The impulsive force is not constant but varies with time. For example, when a bat hits the ball, the impulsive force jumps from zero at the moment of contact to a very large value within a very short time and then abruptly returns to zero again as shown in Fig. 10.6. Note that during the short time interval Δt ($= t_2 - t_1$), the impulsive force is varying continuously. Therefore, it is not easy to measure the impulsive force. In such cases, we measure the total effect of the impulsive force, called *impulse*.

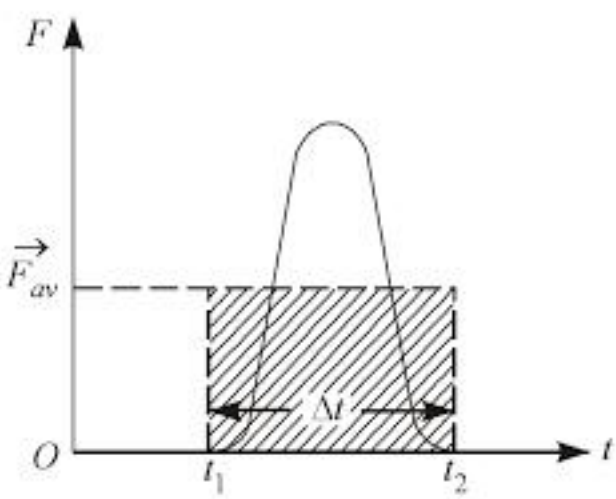


Fig. 10.6

2. Impulse. If $\vec{F}_{av}$ is the average force [See Fig. 10.6] exerted by the bat on the ball during the time interval Δt and the change in linear momentum during this time interval is $\vec{\Delta p}$, then according to Newton's second law of motion,

$$\vec{F}_{av} = \frac{\vec{\Delta p}}{\Delta t}$$

or $$\vec{F}_{av}\,\Delta t = \vec{\Delta p} = \text{Change in linear momentum}$$

The product $\vec{F}_{av}\,\Delta t$ is called *impulse* of the force.

Hence impulse *of a force is the product of average force during the impact and the time for which the impact lasts i.e.,*

$$\text{Impulse} = F_{av} \times \Delta t \qquad \text{...in magnitude}$$

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Mathematical analysis. If $\vec{F}$ is the force at any time t during the collision, then according to Newton's second law of motion,

$$\vec{F} = \frac{\overrightarrow{dp}}{dt}$$

or $$\overrightarrow{dp} = \vec{F}\,dt$$

If we integrate it over the time interval $\Delta t (= t_2 - t_1)$, we get the total change in linear momentum $\overrightarrow{\Delta p}$ during that time interval.

$$\therefore \quad \overrightarrow{\Delta p} = \int_{t_1}^{t_2} \vec{F}\,dt \qquad ...(i)$$

If $\vec{F}_{av}$ is the average force during the collision, then,

$$\overrightarrow{\Delta p} = \vec{F}_{av}\,\Delta t \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), $\int_{t_1}^{t_2} \vec{F}\,dt = \vec{F}_{av}\,\Delta t$ = Change in linear momentum.

The quantity $\int_{t_1}^{t_2} \vec{F}\,dt$ is known as impulse of the force $\vec{F}$ during the time interval t_1 to t_2 and is equal to the change in the linear momentum of the body on which it acts (= area under the $F - t$ graph during the time interval t_1 to t_2).

Units of Impulse. The impulse has the same SI units as that of linear momentum *i.e.*, kg m/s or Ns. Therefore, the dimensional formula of impulse is also the same as that of linear momentum *i.e.*, $[MLT^{-1}]$. *Impulse is a vector quantity and its direction is the same as the direction of change in linear momentum.*

10.10. IMPULSE-LINEAR MOMENTUM THEOREM

Impulse = $F \times t$ = Change in linear momentum

Thus a given change in linear momentum can be produced by applying a larger force for a smaller time or by applying a smaller force for a larger time. This is called *impulse-linear momentum theorem.*

10.11. EXAMPLES OF IMPULSE–LINEAR MOMENTUM RELATIONSHIP

Average impulsive force, $\vec{F}_{av} = \dfrac{\text{Change in linear momentum}}{\text{Time}}$

Therefore, whenever you wish the force of impact to be small, extend the time of impact. On the other hand, if the time of impact is small, the impact of force will be large. There are a large number of practical examples where impulse-linear momentum relationship plays an important role. A few cases are being discussed below by way of illustration :

(*i*) **While catching a ball, a cricket player extends his hands forward** so that he has plenty of room to let his hands move backward after making contact with the ball. This extends the time of impact and thus reduces the force of impact.

(*ii*) **A person is better off falling on a wooden floor than a concrete floor.** The wooden floor allows for a longer time of impact and, therefore, a lesser force of impact than a concrete floor.

(*iii*) **A person jumping from an elevated position on a floor below bends his knees upon making contact.** This extends the time of impact. Therefore, the force of impact is reduced.

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(*iv*) **China wares are wrapped in a paper or straw before packing** to avoid breakage. This increases the time of impact between various articles during jerks, thereby decreasing the force of impact on the articles.

(*v*) **While catching a cricket ball, if you do not move your hands away upon contact, you may be hurt.** It is because the time of impact will be small so that the impact force will be large.

(*vi*) **When the car goes out of control while driving, you would prefer to hit something soft** (say haystack) than something hard (say concrete wall). It is because by hitting haystack you extend your time of impact, thereby reducing the impact force.

Example 10.10. A 3 kg hammer is used to drive a nail into a block of hard wood. If at the time of impact, hammer's speed is 5 ms^{-1} and it drives the nail 1 cm into the block, find (*i*) the acceleration (*ii*) the force exerted (*iii*) the time of impact (*iv*) impulse.

Solution. The initial velocity of nail is $v_0 = 5\ ms^{-1}$ and its final velocity is $v = 0$. The distance covered by the nail is $S = 1\ cm = 10^{-2}\ m$.

(*i*) $$v^2 - v_0^2 = 2\ a\ S$$

$\therefore$ $$a = \frac{v^2 - v_0^2}{2S} = \frac{(0)^2 - (5)^2}{2 \times 10^{-2}} = \mathbf{-1.25 \times 10^3\ ms^{-2}}$$

(*ii*) Force exerted, $F = m\ a = 3 \times 1.25 \times 10^3 = \mathbf{3.75 \times 10^3\ N}$

(*iii*) Time of impact, $t = \frac{v - v_0}{a} = \frac{0 - 5}{-1.25 \times 10^3} = \mathbf{4 \times 10^{-3} s}$

(*iv*) Impulse $= F\ t = (3.75 \times 10^3) \times 4 \times 10^{-3} = \mathbf{15\ Ns}$

Alternatively : Impulse $= m\ (v - v_0) = 3\ (0 - 5) = 15$ Ns .... in magnitude

Example 10.11. A body of mass 0.25 kg moving with a velocity of 12 ms^{-1} is stopped by applying a force of 0.6 N. Calculate the time taken to stop the body. Also calculate the impulse of this force.

Solution. Mass of body, $m = 0.25$ kg ; Retarding force, $F = -0.6$ N

$\therefore$ Retardation, $a = \frac{F}{m} = \frac{-0.6}{0.25} = -2.4\ ms^{-2}$

Now $v = v_0 + at$ or $0 = 12 + (-2.4) \times t$ $\therefore$ $t = \mathbf{5\ s}$

Impulse $= F \times t = -0.6 \times 5 = \mathbf{-3.0\ Ns}$

Example 10.12. A ball of mass 10 g hits a hard surface vertically with a speed of 5 m/s and rebounds with the same speed. The ball remains in contact with the surface for 0.01 second. Find the average force exerted by the surface on the ball.

Solution. $F \times t$ = Change in linear momentum

Now change in linear momentum $= mv - (-mv) = 2\ mv$

$\therefore$ $F \times t = 2\ mv$ or $F \times 0.01 = 2 \times \frac{10}{1000} \times 5$ $\therefore$ $F = \mathbf{10\ N}$

Example 10.13. A 1200 kg car initially moving at 20 ms^{-1} strikes a tree and comes to rest in a distance of 1.5 m. Find the average stopping force the tree exerts on the car.

Solution. $m = 1200$ kg ; $v_0 = 20\ ms^{-1}$; $v = 0$; $S = 1.5$ m

Change in car's linear momentum, $\Delta p = m\ (v - v_0) = 1200 \times (0 - 20) = -24000\ kg\ ms^{-1}$

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Average speed during impact, $\bar{v} = \frac{v_0 + v}{2} = \frac{20+0}{2} = 10 \text{ m s}^{-1}$

Time of impact, $t = \frac{S}{\bar{v}} = \frac{1.5}{10} = 0.15\text{s}$

If $\vec{F}_{av}$ is the average force, then,

$F_{av} \times t$ = Change in linear momentum

$\therefore \quad F_{av} = \frac{\text{Change in linear momentum}}{t} = \frac{24000}{0.15} = \mathbf{1.6 \times 10^5\ N}$

Example 10.14. A ball moving with a momentum of 5 kg m s^{-1} strikes against a wall at an angle of 45° and is reflected at the same angle. Find the magnitude of change in momentum.

Solution. Fig. 10.7 shows the conditions of the problem. Suppose $\vec{P}_1$ is the initial linear momentum and $\vec{P}_2$ is the final linear momentum.

$\therefore$ Change in linear momentum = $\vec{P}_2 - \vec{P}_1 = \vec{P}_2 + (-\vec{P}_1)$

Therefore, in order to find the change in linear momentum, we should add $\vec{P}_2$ to $-\vec{P}_1$ as shown in Fig. 10.7. The change in linear momentum is $\vec{AB}$.

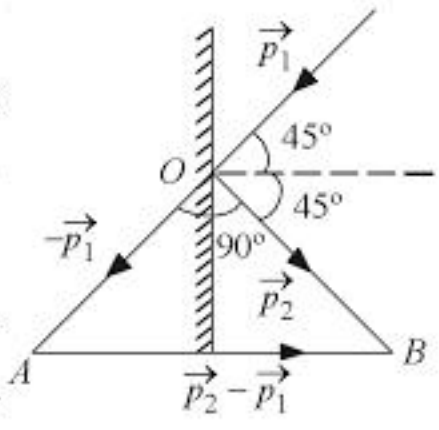


Fig. 10.7

$\therefore \quad AB = \sqrt{P_1^2 + P_2^2} = \sqrt{5^2 + 5^2} = \sqrt{50} = \mathbf{7.07 kg\ ms^{-1}}$

Note that linear momentum is a vector quantity i.e. it has magnitude as well as direction.

PROBLEMS FOR PRACTICE

1. A cricket ball of mass 100 g is moving with a speed of 25 ms^{-1} and is brought to rest in 0.025 s. Find the average force applied and the impulse. **[100 N ; 2.5 Ns]**
2. A force of 10 N acts on a body for 3 microsecond. Calculate the impulse. If the mass of the body is 5 g, what is the change in velocity? **[3 × 10^{-5} Ns ; 6 × 10^{-3} m s^{-1}]**
3. Determine the average force needed to change the velocity of 22,000 kg bus from rest to 13.6 m s^{-1} in 10.5 s. **[28500 N]**
4. A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 km/h. What is the impulse imparted to the ball ? Mass of the ball is 0.15 kg. **[4.16 Ns]**
5. A 12.5 g bullet moving at 235 ms^{-1} passes through a sheet of plastic 3.4 cm thick and emerges with a speed of 125 ms^{-1}. The time taken by the bullet to pass through the sheet is 1.9 × 10^{-4} s. Find the average stopping force exerted on the bullet. **[7240 N]**

10.12. NEWTON'S THIRD LAW OF MOTION

According to Newton's third law of motion: *To every action, there is an equal and opposite reaction.* It simply means that if one body exerts a force (*action*) on the second body, then second body exerts an equal and opposite force (*reaction*) on the first body. We can restate Newton's third law of motion in more quantitative terms as under :

If body A exerts a force $\vec{F}_{AB}$ on body B (action) and body B exerts a force $\vec{F}_{BA}$ on body A (reaction), then according to Newton's third law of motion,

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$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Note that $\vec{F}_{AB}$ (*action*) and $-\vec{F}_{BA}$ (*reaction*) are equal in magnitude but opposite in direction. Two things are worth noting about Newton's third law of motion. First, *action and reaction act on different bodies and, therefore, they cannot cancel each other.* Thus in the above case, action ($\vec{F}_{AB}$) is on body B and reaction ($-\vec{F}_{BA}$) is on body A. Secondly, *action and reaction involve a pair of bodies.* It does not matter which force in such a pair is called the action and which the reaction.

Proof. We now prove Newton's third law of motion. If $\vec{F}_{AB}$ is the force exerted by body A on body B (action) and $\vec{F}_{BA}$ is the force exerted by body B on body A (reaction), then according to Newton's third law of motion,

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Consider an isolated system (*i.e.,* no net external force acts on the system) consisting of bodies A and B moving along the same straight line. Suppose the two bodies collide. As a result of collision, their velocities will change and hence their linear momenta will also change. Suppose the two bodies interact for time t.

Change in linear momentum of body A is

$$\overrightarrow{\Delta p_A} = \vec{F}_{BA} \times t$$

Change in linear momentum of body B is

$$\overrightarrow{\Delta p_B} = \vec{F}_{AB} \times t$$

Total change in linear momentum of bodies A and B is

$$\overrightarrow{\Delta p} = \overrightarrow{\Delta p_A} + \overrightarrow{\Delta p_B} = \overrightarrow{F_{BA}} \times t + \vec{F}_{AB} \times t$$

Since no net external force is acting on the system, according to Newton's second law of motion, the change in linear momentum is zero.

$$\therefore \quad \overrightarrow{\Delta p} = 0$$

or $$\vec{F}_{AB} \times t + \vec{F}_{BA} \times t = 0$$

or $$\vec{F}_{AB} = -\vec{F}_{BA}$$

Thus action and reaction are equal and opposite. This proves Newton's third law of motion.

10.13. EXAMPLES OF NEWTON'S THIRD LAW OF MOTION

In everyday life, there are a large number of practical examples of the applications of Newtons's third law of motion. However, we shall discuss only a few by way of illustration.

(*i*) Block resting on a table. Fig. 10.8 shows a block resting on a table. The force acting downward on the block is its weight W ($= mg$) due to the attraction of the earth *i.e.,* earth is exerting a force W ($= mg$) on the block. An equal and opposite force W' ($= mg$) is exerted by the block *on the earth.* Here W and W' are the action and reaction forces. Note that action and reaction act on different bodies. Therefore, they cannot cancel each other. Let us now turn to another action-reaction pair. The table is in contact with the block and exerts an upward force N on it. The block also exerts a

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force N' ($= N$) downward on the table. The forces N and N' are also action-reaction pair. Note that N and N' act on different bodies and therefore cannot cancel each other.

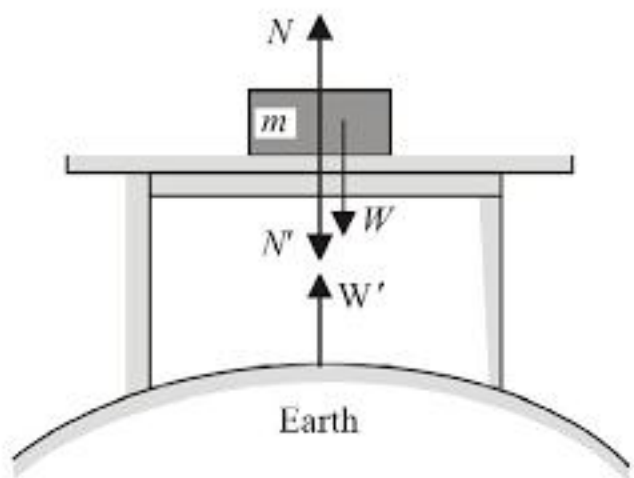


Fig. 10.8



Fig. 10.9

(*ii*) **Walking.** When a person walks on the ground, he or she exerts a force on the ground and in turn ground exerts an equal force on the person. It is this force on the person which moves him or her forward. Thus is illustrated in Fig. 10.9. Similarly, when a man jumps, he exerts downward force on the ground. The ground exerts an equal upward force on the man.

(*iii*) **When a person throws a package out of the boat, the boat moves (atleast a little) in the opposite direction.** The person exerts a force on the package; the package exerts an equal and opposite force back on the person.

(*iv*) **The working of rockets is based on Newton's third law.** The rocket exerts a strong force on the gases, expelling them. The gases exert an equal and opposite force *on the rocket* and it is this force which moves the rocket forward.

(*v*) **A falling object exerts upward force on the earth as much as the earth exerts downward force on it.** This fact may seem puzzling. The downward force on the object is less puzzling because the acceleration of 9.8 m s^{-2} is quite noticeable. The same amount of force acting upward on the huge mass of the earth, however, produces an acceleration ($F = m\ a$) so small that it cannot be measured. But it is there.

(*vi*) **When a small car collides head-on with a huge truck, which is hit with a greater force?** Most people will conclude that the car is certainly hit with a greater force. Yet Newton's third law states that force exerted by the car on the truck is exactly equal in magnitude (and in the opposite direction) to the force exerted by the truck on the car. This fact seems puzzling because the car is damaged more. Note that $F = m\ a$; for the same force, larger mass will undergo less acceleration than the smaller mass. Thus the truck's velocity undergoes a relatively small change during collision, slowing down but continuing in the same direction. The much less massive car, *struck with the same force,* has a much larger change in its velocity. Thus the car is not only stopped but is compelled to move in the reverse direction. The larger acceleration puts a much greater stress on the structural frame of the car. This causes more damage to the car than the truck.

(*vii*) **If a car is accelerating forward, it is because its tyres are pushing backward on the road and the road is pushing forward on the tyres.** But if the car is moving forward and slowing down, the tyres push forward and the road pushes backward.

(*viii*) **A bird flies forward by exerting a force on the air** but it is the air pushing back on the bird's wings that moves bird forward.

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(*ix*) **When a gun is fired, the force exerted on the bullet is equal to the reaction force exerted on the gun.** Since the forces are equal in magnitude, why does not gun recoil with the same speed as that of bullet. Recall that $a = F/m$. Since the gun is much more massive than the bullet, the change in its acceleration is very small as compared to that of the bullet.

(*x*) **Using Newton's third law, we can explain how a helicopter gets its lifting force.** The whirling blades of helicopter are so shaped that they exert a downward force on air. The air exerts an equal upward force on the helicopter. This upward reaction force is called *lift*. When lift equals the weight of the helicopter, it hovers in mid-air. When the lift is greater, the helicopter climbs upward.

Third law is contained in First law. Newton's third law is contained in first law of motion. As the system in Fig. 10.8 is at rest, according to Newton's first law of motion, the net force on the system must be zero. The resultant of two forces of action will be zero only when they are equal and opposite. This is third law of motion. Therefore, Newton's third law is contained in first law of motion.

10.14. NEWTON'S SECOND LAW IS THE REAL LAW OF MOTION

Newton's second law of motion is the real law of motion because Newton's first law and third law are contained in it.

(*i*) Newton's first law is contained in second law

According to Newton's second law of motion, the net external force F applied on a body of mass m produces an acceleration a given by ;

$$a = \frac{F}{m}$$

If the net external force is zero (*i.e.* $F = 0$), then, $a = 0/m = 0$. The zero acceleration means either the body is at rest or it is moving with constant velocity. Thus when the net external force on a body is zero, the body is either at rest or it is moving with a constant velocity. This is Newton's first law of motion. Thus Newton's first law of motion is contained in Newton's second law of motion.

(*ii*) Newton's third law is contained in the second law

If $\vec{F}_{AB}$ is the force exerted by body A on body B (action) and $\vec{F}_{BA}$ is the force exerted by body B on body A (reaction), then according to Newton's third law of motion,

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

We shall now prove it by using Newton's second law of motion. Consider an isolated system (*i.e,* no net external force acts on the system) consisting of bodies A and B moving along the same straight line. Suppose the two bodies collide. As a result of collision, their velocities will change and hence their linear momenta will also change. Suppose the two bodies interact for time t.

Change in linear momentum of body A is

$$\vec{\Delta p}_A = \vec{F}_{BA} \times t$$

Change in linear momentum of body B is

$$\vec{\Delta p}_B = \vec{F}_{BA} \times t$$

Total change in linear momentum of bodies A and B is

$$\vec{\Delta p} = \vec{\Delta p}_A + \vec{\Delta p}_B = \vec{F}_{BA} \times t + \vec{F}_{AB} \times t$$

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Solution. Mass of gun, $M = 5$ kg ; Mass of bullet, $m = 10$ g $= 0.01$ kg
Velocity of gun, $V = ?$; Velocity of bullet, $v = 400$ ms^{-1}

The gun and bullet form an isolated system. Therefore, the linear momentum of the system remains constant. Before firing, the linear momentum of the system is zero because both gun and bullet are at rest. Therefore, after firing, the linear momentum of the system should also be zero.

$$\therefore \quad MV + mv = 0 \quad \text{or} \quad V = -\frac{mv}{M} = -\frac{0.01 \times 400}{5} = \mathbf{-0.8\ ms^{-1}}$$

The negative sign shows that velocity of the gun will be in a direction opposite to that of the bullet.

Since the hunter is standing on ice, on firing the bullet, he himself along with the gun will move backward with a velocity V' (say).

Linear momentum of gun plus man $= (M + 75)\ V' = (5 + 75)\ V' = 80V'$

Linear momentum of bullet $= m\ v = 0.01 \times 400 = 4$

The whole system consists of man + gun + bullet.

$$\therefore \quad 80\ V' + 4 = 0 \quad \text{or} \quad V' = -\frac{4}{80} = \mathbf{-0.05\ ms^{-1}}$$

Example 10.19. A 30 kg shell is flying at 48 ms^{-1}. When it explodes, its one part of 18 kg stops while the remaining part flies on. What is the velocity of the latter?

Solution. Mass of shell, $M = 30$ kg ; Mass of first part, $m_1 = 18$ kg
Velocity of shell, $V = 48$ ms^{-1} ; Mass of second part, $m_2 = 12$ kg
Velocity of first part, $v_1 = 0$; Velocity of second part, $v_2 = ?$
Initial linear momentum $= MV$; Final linear momentum $= m_1 v_1 + m_2 v_2$

According to principle of conservation of linear momentum,

$$MV = m_1 v_1 + m_2 v_2$$

or $$30 \times 48 = 18 \times 0 + 12 \times v_2 \quad \therefore \quad v_2 = \frac{30 \times 48}{12} = \mathbf{120\ ms^{-1}}$$

Example 10.20. A hunter has a machine gun that can fire 50 g bullets with a velocity of 150 m/s. A 60 kg tiger springs at him with a velocity of 10 ms^{-1}. How many bullets must the hunter fire per second into the tiger in order to stop him in his track?

Solution. Let n bullets be fired per second at the tiger to stop him in his track. Then,

Linear momentum of bullets = Linear momentum of tiger

or $$n \times (50 \times 10^{-3}) \times 150 = 60 \times 10$$

$$\therefore \quad n = \frac{60 \times 10}{(50 \times 10^{-3}) \times 150} = \mathbf{80}$$

Example 10.21. A cart of mass 500 kg is standing at rest on the rails. A man weighing 70 kg and running parallel to rail track with a velocity of 10 ms^{-1} jumps onto the cart on approaching it. Find the velocity with which the cart will start moving.

Solution. $m_1 = 500$ kg ; $v_1 = 0$; $m_2 = 70$ kg ; $v_2 = 10$ ms^{-1}

Let v be the velocity with which the cart and man move when the man jumps onto the cart.

According to principle of conservation of linear momentum,

$$(m_1 + m_2)v = m_1 v_1 + m_2 v_2$$

or $$(500 + 70)v = 500 \times 0 + 70 \times 10 \quad \therefore\ v = \mathbf{1.23\ ms^{-1}}$$

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PROBLEMS FOR PRACTICE

1. What is the thrust of a rocket that burns fuel at a rate of 1.3×10^4 kgs^{-1} if the exhaust gases have a velocity of 2.5×10^3 m s^{-1} *w.r.t.* the rocket ? **[3.3×10^7 N]**
2. For a certain rocket, the velocity of the exhaust gas is 50,000 ms^{-1} and gas is expelled at the rate of 1300 kg s^{-1}. Find rocket's thrust. **[6.5×10^7 N]**
3. Sand drops vertically at the rate of 2 kg s^{-1} onto a conveyor belt moving horizontally with a velocity of 0.1 m s^{-1}. Calculate the force required to keep the belt moving. **[0.2 N]**

10.20. EQUILIBRIUM OF CONCURRENT FORCES

When forces act at the same point, they are called *concurrent forces.*

A number of concurrent forces are said to be in equilibrium if their vector sum is zero.

When the vector sum of the concurrent forces acting on a body is zero, it means that the net force on the body is zero. According to Newton's second law, the body will have zero acceleration. As a result, either the body is at rest (*i.e.,* zero velocity) or it is moving with a constant velocity (*i.e.,* with constant speed in a straight line). If the body is at rest, it is said to be in *static equilibrium.* If the body is in motion, it is said to be in *dynamic equilibrium.* Note that in either case, the acceleration of the body is zero.

Suppose three concurrent forces $\vec{A}, \vec{B}$ and $\vec{C}$ acting at point O are in equilibrium as shown in Fig. 10.20. Then,

$$\vec{A} + \vec{B} + \vec{C} = 0$$

or

$$\vec{A} + \vec{B} = -\vec{C}$$

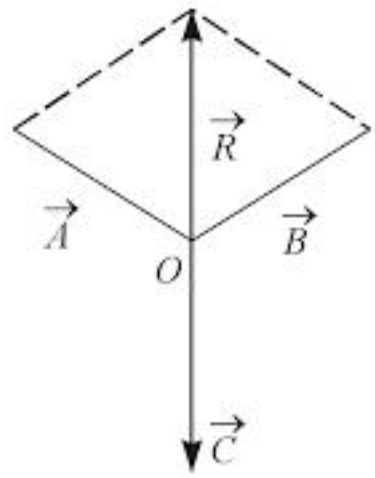


Fig. 10.20

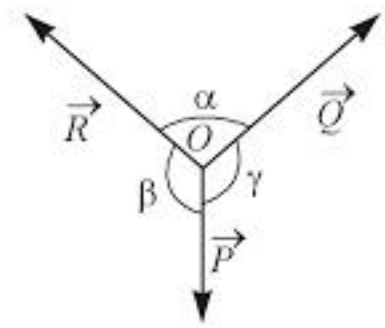


Fig. 10.21

It means that the vector sum of $\vec{A}$ and $\vec{B}$ is equal in magnitude and opposite in direction to force $\vec{C}$. Thus in Fig. 10.20, $\vec{R}$ is equal in magnitude and opposite in direction to $\vec{C}$. Therefore, point O is in equilibrium. Indeed when three concurrent forces are in equilibrium, the resultant of any two forces is equal in magnitude and opposite in direction to the third force. Lami gave his theorem regarding the three concurrent forces in equilibrium and is called Lami's theorem.

Lami's theorem. *It states that if three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces.* Thus referring to Fig. 10.21, suppose the three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting at point O are in equilibrium. Let α, β and γ be the angles between forces $\vec{Q}$ and $\vec{R}$, $\vec{R}$ and $\vec{P}$ and $\vec{P}$ and $\vec{Q}$ respectively. Then according to Lami's theorem :

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$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

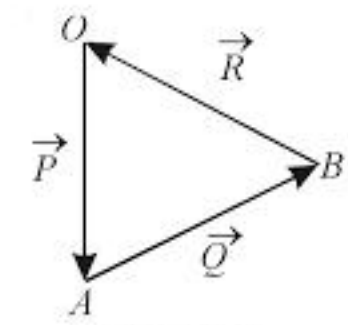


Fig. 10.22

Proof. The three forces $\vec{P}, \vec{Q}$ and $\vec{R}$ acting at point O in Fig. 10.21 are in equilibrium and can be represented by the three sides of a triangle OAB taken in the same order as shown in Fig. 10.22.

$$\therefore \quad \frac{P}{OA} = \frac{Q}{AB} = \frac{R}{BO} \qquad ...(i)$$

From triangle OAB, by sine formula, we have,

$$\frac{OA}{\sin\angle ABO} = \frac{AB}{\sin\angle BOA} = \frac{BO}{\sin\angle OAB} \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), $\frac{P}{\sin\angle ABO} = \frac{Q}{\sin\angle BOA} = \frac{R}{\sin\angle OAB}$

or $\frac{P}{\sin(180^\circ - \alpha)} = \frac{Q}{\sin(180^\circ - \beta)} = \frac{R}{\sin(180^\circ - \gamma)}$

or $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$

Thus Lami's theorem stands proved.

10.21. CONNECTED MOTION

Consider two masses m_1 and m_2 tied at the ends of massless rope passing over a frictionless and massless pulley [See Fig.10.23 (*i*)]. Suppose $m_2 > m_1$. The heavier body (m_2) will move downward and the lighter body (m_1) will move upward. Let a be the common acceleration and T be the *tension in the rope. We shall find the values of a and T by applying Newton's second law of motion.

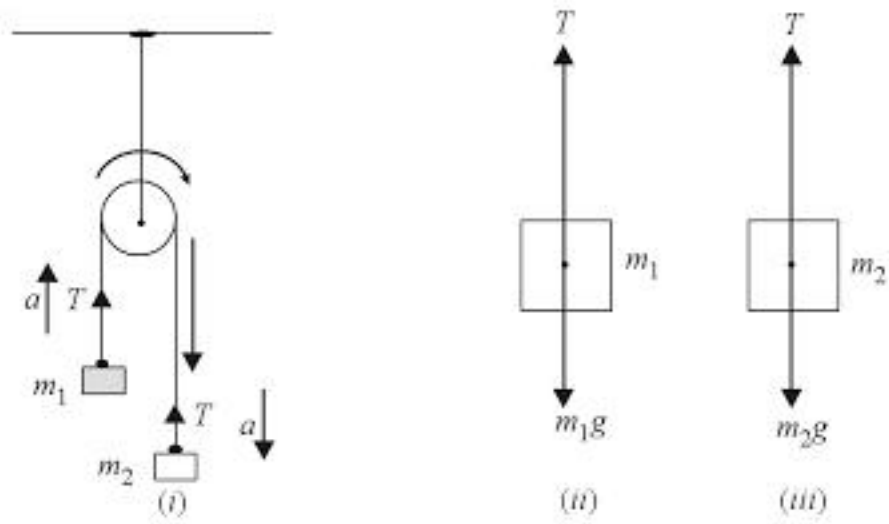


Fig. 10.23

Mass m_2. Fig. 10.23 (*iii*) shows the free-body diagram of m_2. The forces acting on m_2 are (*i*) a downward force m_2g due to gravity and (*ii*) upward force T provided by the rope.

$$\text{Net force on } m_2 = m_2g - T$$

or $$m_2 a = m_2 g - T \qquad ...(i)$$

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* Note that we assume that rope and pulley are massless so that we can neglect their inertias. Because the pulley is both massless and frictionless, the tension in the rope is the same on both sides of the pulley.

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Mass m_1. Fig. 10.23 (*ii*) shows the free-body diagram of m_1.

$$\text{Net force on } m_1 = T - m_1 g$$

or $$m_1 a = T - m_1 g \quad \ldots(ii)$$

Adding eqs. (*i*) and (*ii*), we get,

$$(m_1 + m_2)\, a = m_2 g - m_1 g$$

$$\therefore \quad a = \frac{(m_2 - m_1)g}{m_1 + m_2} \quad \ldots(iii)$$

Note that $a < g$ *i.e.* acceleration a of a system of two connected masses is always less than acceleration due to gravity g.

Dividing eq. (*i*) by eq.(*ii*), we get,

$$\frac{(m_2 g - T)}{T - m_1 g} = \frac{m_2 a}{m_1 a} = \frac{m_2}{m_1}$$

or $$m_1 m_2 g - m_1 T = m_2 T - m_1 m_2 g$$

or $$(m_1 + m_2)T = 2 m_1 m_2 g$$

$$\therefore \quad T = \frac{2 m_1 m_2 g}{m_1 + m_2} \quad \ldots(iv)$$

Note that when $m_1 = m_2 = (= m)$, then tension T equals the weight mg and there is no acceleration.

10.22. SOLVING PROBLEMS IN MECHANICS

We now have the necessary background for applying Newton's laws of motion to a variety of situations. Before giving a number of examples, we will list a series of helpful steps for problem solving :

(*i*) Sketch the picture of the problem.

(*ii*) Isolate the body whose motion is to be analysed. Show the various forces that act on the body. Do *not* include forces that do not act directly on the body you have isolated. Such a diagram is called a *free-body diagram* or *force diagram.*

A diagram showing all the forces acting on a given body (and only at that body) is called a **free-body diagram.**

(*iii*) If necessary, resolve the forces in the free-body diagram into rectangular components.

(*iv*) Write $F = m\,a$ equations in the component form for the forces in the free body diagram.

(*v*) Solve the component equations for the unknown.

(*vi*) Check the reasonableness of the result you obtain.

Example 10.26. Two blocks having masses $m_1 = 1$ kg and $m_2 = 2$ kg are in contact with each other on a horizontal table as shown in Fig. 10.24 (*i*). A horizontal force $\vec{F}$ is applied to m_1, causing the blocks to accelerate to the right with an acceleration of 3 ms^{-2}. Calculate (*i*) the magnitude of F and (*ii*) the forces of compression between the blocks. Neglect friction.

Solution. *Fig. 10.24 (*ii*) shows the free-body diagram for m_2 whereas Fig. 10.24 (*iii*) shows the free-body diagram for m_1. Note that m_1 exerts a force $\vec{F}_N$ on m_2 and according to Newton's third law, m_2 exerts a force $\vec{F}_N$ on m_1 in the opposite direction. Since $\vec{F}$ is not applied to m_2,

* $m_2 g$ is the force with which m_2 is attracted downward due to gravity. R is the upward force which the table exerts on m_2. Since there is no vertical motion, $R = m_2 g$. Similarly, $m_1 g = R'$.

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5. Consider three blocks of masses m_1, m_2 and m_3 in contact with one another on a smooth horizontal surface as shown in Fig. 10.35 (*i*). When force F is applied on m_1, the three blocks move with a common acceleration 'a' given by ;

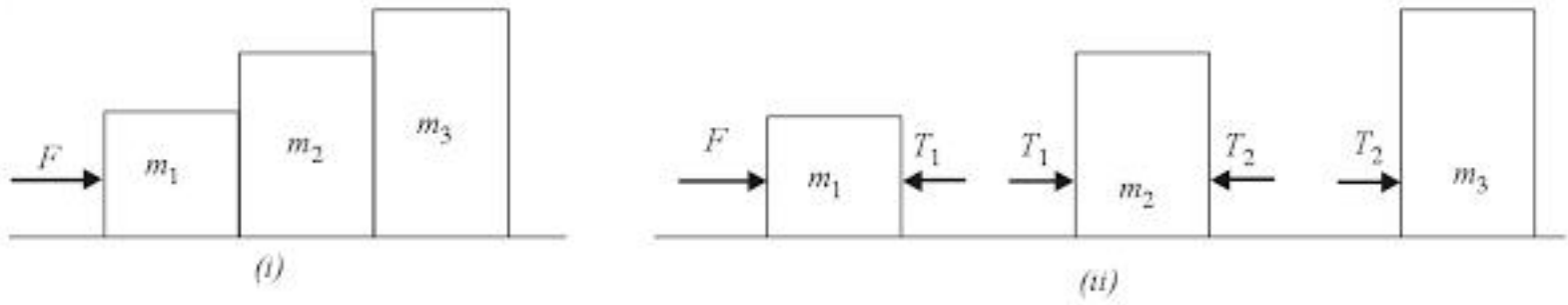


Fig. 10.35

Common acceleration, $a = \dfrac{F}{m_1 + m_2 + m_3}$

Fig. 10.35 (*ii*) shows the action-reaction forces on the three blocks. We can easily find the equation of motion for each block.

For block 1, $F - T_1 = m_1 a$; For block 2, $T_1 - T_2 = m_2 a$; For block 3, $T_2 = m_3 a$

6. Fig. 10.36 shows mass m_1 on a smooth horizontal table and connected to mass m_2 by a light string passing over a frictionless pulley of negligible mass. If 'a'is the acceleration of the system in the direction shown, then equations of motion for the two masses are :

$$T = m_1 a \; ; \quad m_2 g - T = m_2 a$$

On solving, $a = \dfrac{m_2}{m_1 + m_2} \times g \; ; \; T = \dfrac{m_1 m_2}{m_1 + m_2} \times g$

Note that T is always less than $m_2 g$ and common acceleration 'a' of the system is always less than g.

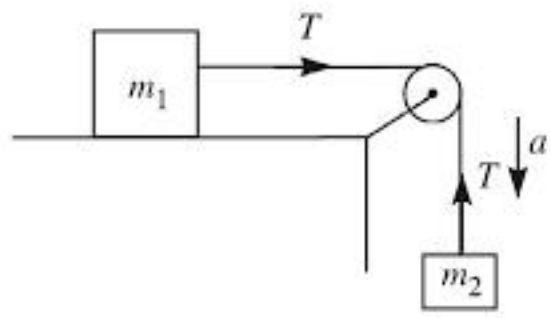


Fig. 10.36

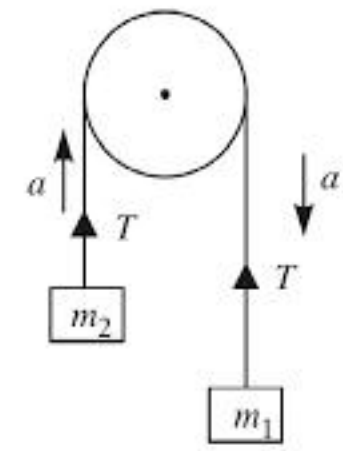


Fig. 10.37

7. Consider two masses m_1 and m_2 connected to the opposite ends of a light string passing over a frictionless and massless pulley as shown in Fig. 10.37. Suppose $m_1 > m_2$. As a result, mass m_1 moves downward while mass m_2 moves upward. Let the common acceleration of the system be 'a'. Then equations of motion for the two masses are :

$$m_1 g - T = m_1 a \quad \text{and} \quad T - m_2 g = m_2 a$$

$$\text{On solving, } a = \frac{m_1 - m_2}{m_1 + m_2} \times g \; ; \; T = \frac{2 m_1 m_2}{m_1 + m_2} \times g$$

Note that when $m_1 = m_2$, then, $a = 0$ and $T = m_1 g$.

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the man jumps outside the platform, then reading of the spring balance will **[AIIMS 2000]**

(*a*) increase (*b*) remain same

(*c*) become zero (*d*) first (*a*) then (*c*)

4. A body is moving with a constant speed in a straight line path. A force is not required to **[AFMC 2001]**

(*a*) increase its speed

(*b*) decrease the momentum

(*c*) change the direction

(*d*) keep it moving with uniform velocity

5. If a body is in equilibrium under a set of non-linear forces, then the minimum number of forces has to be **[Pb PMT 2001]**

(*a*) four (*b*) three

(*c*) two (*d*) five

6. A force vector applied on a mass is represented as $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$ and accelerates with $1 m/s^2$. What will be the mass of the body? **[CBSE PMT 1996]**

(*a*) $10\sqrt{2}$ kg (*b*) $2\sqrt{10}$ kg

(*c*) 10 kg (*d*) 20 kg

7. The rocket engines lift a rocket from the earth because hot gases with high velocity **[AIIMS 1998]**

(*a*) push it against the earth

(*b*) push it against the air

(*c*) react against rocket and push it up

(*d*) heat up the air which lifts the rocket

8. The motion of rocket is based on the principle of conservation of **[AFMC-Pune 2000]**

(*a*) mass (*b*) kinetic energy

(*c*) linear momentum

(*d*) angular momentum

9. If the force on a rocket moving with a velocity of 300 m/s is 210 N, then rate of combustion of fuel is **[AIIMS 1997]**

(*a*) 0.7 kg/s (*b*) 1.4 kg/s

(*c*) 7 kg/s (*d*) 10.7kg/s

10. A 5000 kg rocket is set for vertical firing. The exhaust speed is 800 m/s and acceleration (a) of the rocket is 20 m/s^2. The acceleration due to gravity is 10 m/s^2. The amount of gas ejected per second is **[Pb PMT 2001]**

(*a*) 127.5 kg/s (*b*) 137.5 kg/s

(*c*) 185.5 kg/s (*d*) 187.5 kg/s

11. A body of mass 2 kg is moving with a velocity of 8 m/s on a smooth surface. If it is to be brought to rest in 4 seconds, then force to be applied is **[Pb PMT 2000]**

(*a*) 8 N (*b*) 4 N

(*c*) 2 N (*d*) 1 N

12. The forces which meet at one point but their lines of action do not lie in one plane are called **[AIIMS 1996]**

(*a*) coplanar concurrent forces

(*b*) coplanar non-concurrent forces

(*c*) non-planar concurrent forces

(*d*) non-planar non-concurrent forces

13. A boy of mass 50 kg is standing in a lift moving down with an acceleration 9.8 m/s^2. The apparent weight of the boy is **[Pb PMT 1999]**

(*a*) zero (*b*) 50 N

(*c*) (50 × 9.8)N (*d*) 50/9.8 N

14. A monkey is descending from the branch of a tree with a constant acceleration. If the breaking strength of the branch is 75% of the weight of the monkey, the minimum acceleration with which the monkey can slide down without breaking the branch is **[CBSE PMT 1993]**

(*a*) g (*b*) $3g/4$

(*c*) $g/2$ (*d*) $g/4$

15. The force acting on a body of mass 10 kg is $(2\hat{i} + \hat{j} - \hat{k})N$. If the body is initially at rest, the velocity at the end of 20 sec will be (m/s) **[Pb PMT 1997]**

(*a*) $2\sqrt{6}$ (*b*) $6\sqrt{2}$

(*c*) $3\sqrt{2}$ (*d*) $2\sqrt{3}$

16. A cricket ball of mass 250 g collides with a bat with velocity of 10 m/s and returns with same velocity within 0.01 sec. The force acted on the bat is **[CPMT-1997]**

(*a*) 25 N (*b*) 50 N

(*c*) 250 N (*d*) 500 N

17. A mass of 1 kg is thrown up with a velocity of 100 m/s. After 5 seconds, it explodes into two parts. One part of mass 400 g comes down with a velocity of 25 m/s. Calculate the velocity of other part. **[CBSE PMT 2000]**

(*a*) 40 m/s ↑ (*b*) 40 m/s ↓

(*c*) 100 m/s ↑ (*d*) 60 m/s ↑

18. Two bodies having same mass 40 kg are moving in opposite directions, one with a velocity of 10 m/s and other with a velocity of

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4. A ship of mass 2×10^7 kg is at rest. On applying a force of 25×10^5 N, it is displaced through 25 m. The speed of the ship after this displacement is **[CET Karnataka 1998]**

(*a*) 3.5 m/s (*b*) 5 m/s

(*c*) 12.5 m/s (*d*) 2.5 m/s

5. A ball of mass 10 g hits a hard surface vertically with a speed of 5 m/s and rebounds with the same speed. The ball remains in contact with the surface for 0.01 sec. The average force exerted by the surface on the ball is **[Roorkee 1993]**

(*a*) 100 N (*b*) 10 N (*c*) 1N (*d*) 0.1N

6. A block of mass M is pulled along a horizontal surface by a rope of mass m by applying a force F at one end of the rope. The force which the rope exerts on the block is **[CET Haryana 2000]**

(*a*) $\frac{mF}{m+M}$ (*b*) $\frac{mF}{M-m}$

(*c*) $\frac{MF}{M-m}$ (*d*) $\frac{FM}{m+M}$

7. A light string passes over a frictionless pulley. To one of its ends, a mass of 6 kg is attached. To its other end, a mass of 10 kg is attached

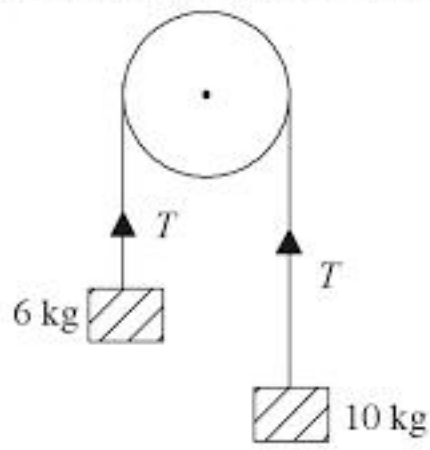


Fig. 10.44

(See Fig. (10.44). The tension in the string will be **[Rajasthan PET 1996]**

(*a*) 24.5 N (*b*) 2.45 N

(*c*) 79 N (*d*) 73.5 N

8. A mass M of 100 kg is suspended by strings A, B and C as shown in Fig. 10.45 where W is the vertical wall and R is a rigid horizontal rod. The tension in string B is **[BHU 1994]**

(*a*) 100g N (*b*) 0

(*c*) $100\sqrt{2}\ g$ N (*d*) $\frac{100}{\sqrt{2}}\ g$ N

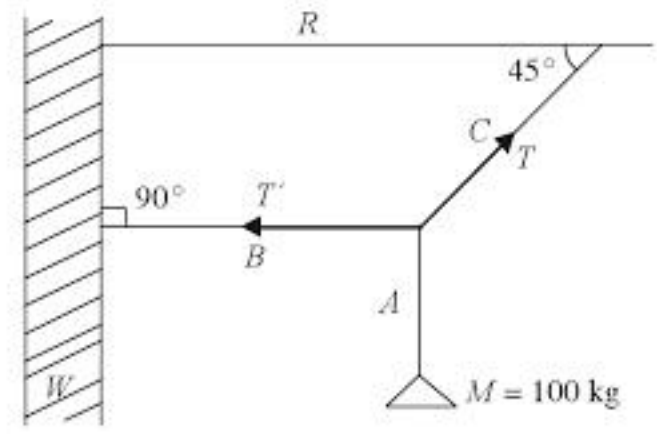


Fig. 10.45

9. A person is sitting in a travelling train and facing engine. He tosses up a coin and coin falls behind him. It can be concluded that train is **[SCRA 1994]**

(*a*) moving forward and gaining speed

(*b*) moving forward and losing speed

(*c*) moving forward with uniform speed

(*d*) moving backward with uniform speed

10. The engine of a car produces acceleration of 4 m/s^2 in car. If this car pulls another car of same mass, what will be the acceleration produced? **[Rajasthan PET 1996]**

(*a*) 8 m/s^2 (*b*) 4 m/s^2

(*c*) 2 m/s^2 (*d*) 0.5 m/s^2

11. A 100 kg man jumps into swimming pool from a height of 5 m. It takes 0.4 sec for the water to reduce his velocity to zero. The average force exerted by water on the man is **[CET 1998]**

(*a*) 25 N (*b*) 250 N

(*c*) 2500 N (*d*) 5000 N

12. A ship of mass 3×10^7 kg initially at rest is pulled by a force of 5×10^4 N through a distance of 3m. Assuming the resistance due to water to be negligible, the speed of the ship is **[I.I.T. 1990]**

(*a*) 0.1 m/s (*b*) 60 m/s

(*c*) 1.5 m/s (*d*) 5 m/s

13. A man of mass 75 kg is standing on spring balance inside a lift. If the lift falls freely downward, the reading of the spring balance will be **[CET Karnataka 1993]**

(*a*) 75 kgf (*b*) >75 kgf

(*c*) <75 kg f (*d*) zero

14. A particle of mass 0.3 kg is subjected to a force $F = -kx$ where $k = 15$ N/m. What will be its initial acceleration, if it is released from a point 20 cm away from the origin? **[AIEEE 2005]**

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(*a*) 3 ms^{-2} (*b*) 15 ms^{-2}

(*c*) 5 ms^{-2} (*d*) 10 ms^{-2}

15. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most? **[AIEEE 2004]**

(*a*) One (*b*) Four

(*c*) Two (*d*) Three

16. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2:1. The ratio of their nuclear sizes will be **[AIEEE 2004]**

(*a*) $2^{1/3} : 1$ (*b*) $1 : 3^{1/2}$

(*c*) $3^{1/2} : 1$ (*d*) $1 : 2^{1/3}$

17. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then the true statement about the scale reading is **[AIEEE 2003]**

(*a*) both the scales read M kg each

(*b*) the scale of the lower one reads M kg and of the upper one zero

(*c*) the reading of the two scales can be anything but the sum of the readings will be M kg

(*d*) both the scales read $M/2$ kg

18. Two blocks A and B of masses $2m$ and m, respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in Fig. 10.46. The magnitudes of acceleration of A and B, immediately after the string is cut, are respectively. **[IIT 2006]**

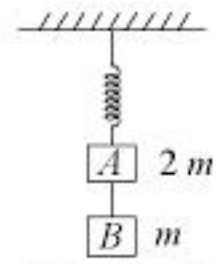


Fig. 10.46

(*a*) $g, g/2$ (*b*) $g/2, g$

(*c*) g, g (*d*) $g/2, g/2$

19. A block of mass m is connected to another block of mass M by a massless spring of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then, a constant force F starts acting on the block of mass M to pull it. Find the force on block of mass m. **[AIEEE 2007]**

(*a*) $\frac{mF}{M}$ (*b*) $\frac{(M+m)F}{M}$

(*c*) $\frac{mF}{(M+m)}$ (*d*) $\frac{MF}{(M+m)}$

20. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary (See Fig. 10.47). Then, a is equal to **[AIEEE 2005]**

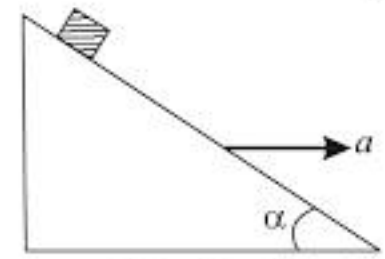


Fig. 10.47

(*a*) $g/\tan\alpha$ (*b*) $g\ \mathrm{cosec}\ \alpha$

(*c*) g (*d*) $g\tan\alpha$

ANSWERS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. (*a*)	**2.** (*a*)	**3.** (*b*)	**4.** (*d*)	**5.** (*b*)
6. (*d*)	**7.** (*d*)	**8.** (*a*)	**9.** (*a*)	**10.** (*c*)
11. (*c*)	**12.** (*a*)	**13.** (*d*)	**14.** (*d*)	**15.** (*d*)
16. (*d*)	**17.** (*a*)	**18.** (*b*)	**19.** (*c*)	**20.** (*d*)

HINTS TO MCQ FROM ENGINEERING ENTRANCE EXAMINATIONS

1. Retardation, $a = \frac{v - v_0}{t} = \frac{0 - 10}{0.1} = -100\ \mathrm{m/s^2}$

Force exerted by the player is $F = ma = 0.1 \times 100 = \mathbf{10\ N}$

Negative sign is omitted because only magnitude is required.

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2. Impulse and change in linear momentum are in the same direction. Therefore, angle between the two is **zero degree**.

3. The two blocks will move with common acceleration 'a' given by ;

$$a = \frac{F}{m_1 + m_2}$$

$\therefore$ Force exerted by A on $B = m_2 a = \boldsymbol{\frac{m_2 F}{m_1 + m_2}}$

4. Acceleration of the ship, $a = \frac{F}{m} = \frac{25 \times 10^5}{2 \times 10^7} = 12.5 \times 10^{-2}\ \text{m/s}^2$

Initial velocity, $v_0 = 0$; Final velocity, $v = ?$; Displacement, $S = 25$ m

$\therefore \quad v^2 - v_0^2 = 2\ aS$

or $v^2 - 0^2 = 2 \times (12.5 \times 10^{-2}) \times 25 = 6.25$ or $v = \sqrt{6.25} =$ **2.5 m/s**

5. $F \times t =$ Change in linear momentum

or $F \times t = m(v + v)$ or $F \times 0.01 = \frac{10}{1000}(5 + 5) = \frac{1}{10}$

$\therefore \quad F = \frac{1}{10 \times 0.01} =$ **10 N**

6. Acceleration, $a = \frac{F}{m + M}$ $\therefore$ Force on block $= Ma = \boldsymbol{\frac{FM}{m + M}}$

7. Let 'a' be the acceleration of the system (The mass 10 kg will move downward while 6 kg will move upward).

$$a = \frac{\text{Resultant unbalanced force}}{\text{Total mass of system}} = \frac{(10-6)9.8}{10+6} = 2.45\ \text{m/s}^2$$

The equation of motion of 10 kg mass is

$10 \times 9.8 - T = 10a$ or $98 - T = 10 \times 2.45$ $\therefore\ T =$ **73.5 N**

8. Since the system is stationary,

$T \sin 45° = 100$ g and $T \cos 45° = T'$

$\therefore$ Tension in B, $T' = T \cos 45° = \frac{100g}{\sin 45°} \times \cos 45° =$ **100g N**

9. Since the coin falls behind the person, the train must be **moving forward and gaining speed.**

10. Acceleration $= \frac{\text{Force}}{\text{Mass}}$. For the same force, if mass is doubled, the acceleration is halved.

$\therefore$ New acceleration $= 4/2 =$ **2 m/s^2**

11. Velocity v of the man just before entering water is

$v^2 - (0)^2 = 2\ gh$ or $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10$ m/s

Now $F \times t = mv - 0$ or $F = \frac{mv}{t} = \frac{100 \times 10}{0.4} =$ **2500 N**

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Let F be the average force exerted by the muddy ground. Then,

$$mg - F = ma \text{ or } F = mg - ma = 10\ [10 - (-666.7)] = \mathbf{6767\ N}$$

6. Masses of 50 g and 40 g are connected by a string passing over a smooth pulley. If the system travels 2.18 m in the first two seconds, find the value of g.

Hint. Fig. 10.49 shows the conditions of the problem. The acceleration a of the masses is given by ;

$$S = v_0 t + \frac{1}{2}at^2 \text{ or } 2.18 = 0 + \frac{1}{2} \times a \times (2)^2$$

$$\therefore \quad a = 1.09\ \text{ms}^{-2} \qquad \ldots(i)$$

$$\text{Net force} = (m_2 g - m_1 g) = g(m_2 - m_1)$$

$$= g\ (0.05 - 0.04) = 0.01\ g \text{ newton}$$

$$\text{Acceleration, } a = \frac{\text{Net force}}{m_1 + m_2} = \frac{0.01g}{0.04 + 0.05} = \frac{0.01g}{0.09} \qquad \ldots(ii)$$

From eqs. (i) and (ii), $1.09 = \dfrac{0.01g}{0.09}$

$$\therefore \quad g = 1.09 \times \frac{0.09}{0.01} = \mathbf{9.81 ms^{-2}}$$

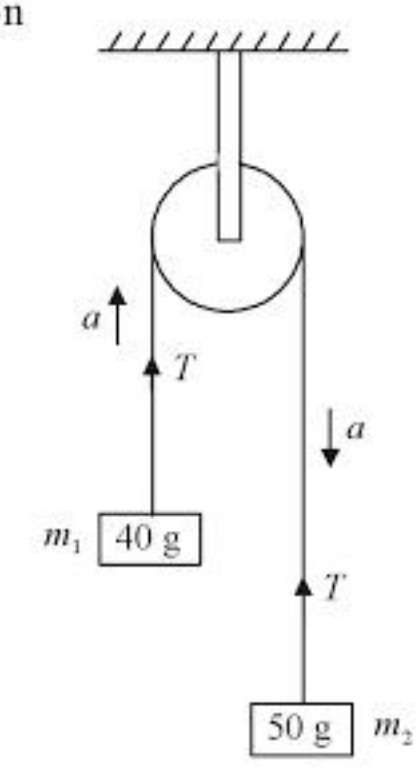


Fig. 10.49

7. A lift is going up. The total mass of the lift and the passengers is 1500 kg. The variation of the speed of the lift is given by graph shown in Fig. 10.50.

(i) What will be the tension in the rope at t = 1 sec; at t = 6 sec and at t = 11 sec?

(ii) What is the height to which the lift takes the passengers?

(iii) What will be the average velocity and average acceleration during the course of entire motion ?

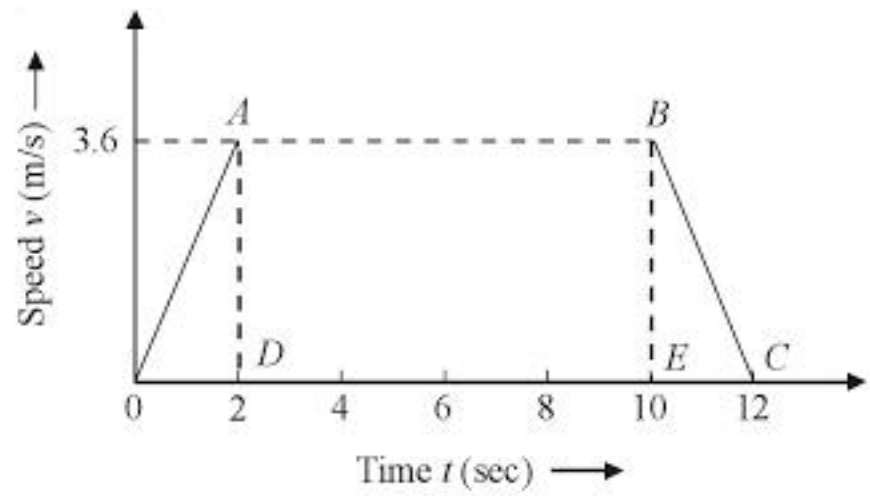


Fig. 10.50

Hint. From 0 – 2 sec, the lift has uniform acceleration given by ;

$$a = \frac{\Delta v}{\Delta t} = \frac{3.6 - 0}{2} = 1.8\ \text{ms}^{-2}$$

From t = 2 sec to t = 10 sec, the speed of the lift is constant so that during this period, a = 0. From t = 10 sec to t = 12 sec, the lift has uniform retardation given by ;

$$a = \frac{0 - 3.6}{12 - 10} = -1.8 \text{ms}^{-2}$$

(i) Suppose m is the mass of lift and passengers. If a is the upward acceleration and T is the tension in the rope, then,

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$$T - mg = ma \text{ or } T = m(g + a)$$

At $t = 1$ sec, $a = 1.8$ ms^{-2} $\quad \therefore\ T = 1500(9.8 + 1.8) = \mathbf{17400\ N}$

At $t = 6$ sec, $a = 0$ $\quad \therefore\ T = 1500(9.8 + 0) = \mathbf{14700\ N}$

At $t = 11$ sec, $a = -1.8$ ms^{-2} $\quad \therefore\ T = 1500(9.8 - 1.8) = \mathbf{12000\ N}$

(*ii*) Height reached by the lift is

h = Area under speed-time graph

= Area of trapezium $OABC$

$$= \frac{(AB + OC)}{2} \times AD = \frac{(8+12)}{2} \times 3.6 = \mathbf{36\ m}$$

(*iii*) $$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{36}{12} = \mathbf{3\ ms^{-1}}$$

$$\text{Average acceleration} = \frac{\text{Total change in velocity}}{\text{Total time}} = \frac{0}{12} = \mathbf{0}$$

8. A train is moving along a straight horizontal track. A pendulum suspended from the roof makes an angle of 5° with the vertical. Find the acceleration of the train. Take $g = 10$ ms^{-2}.

Hint. Fig. 10.51 shows the conditions of the problem. The forces acting on the bob are :

(*i*) Weight mg of the bob.

(*ii*) Tension T developed in the string. Resolving T along vertical and horizontal directions, we have, $T \cos 5°$ and $T \sin 5°$.

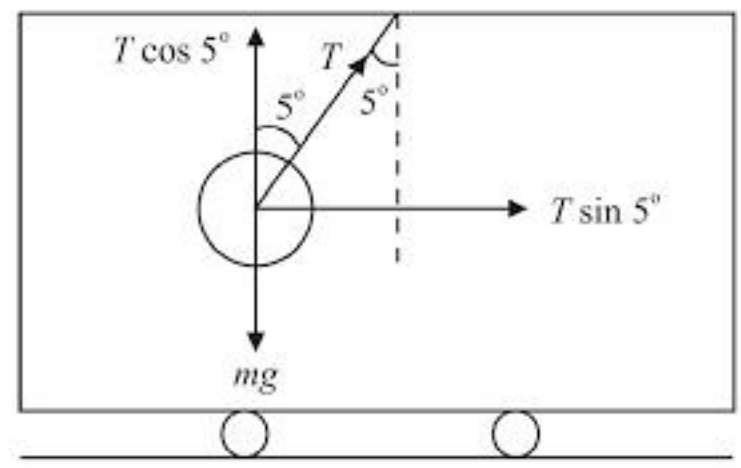


Fig. 10.51

$\therefore \quad T \cos 5° = mg \quad \ldots (i)$

Now $T \sin 5°$ is the unbalanced force and causes the acceleration a in the pendulum.

$\therefore \quad T \sin 5° = ma \quad \ldots (ii)$

From eqs. (*i*) and (*ii*), we have, $\dfrac{T \sin 5°}{T \cos 5°} = \dfrac{ma}{mg}$

or $\quad \tan 5° = \dfrac{a}{g} \quad \therefore\ a = g \tan 5° = 10 \tan 5° = \mathbf{0.87\ ms^{-2}}$

9. The masses m_1, m_2 and m_3 of the three bodies shown in Fig. 10.52 are 5 kg, 2 kg and 3 kg respectively. Calculate the values of tensions T_1, T_2 and T_3 when (*i*) the whole system is going upward with an acceleration of 2 ms^{-2} (*ii*) the whole system is stationary.

Hint. (*i*) In this case, all the bodies are moving upward together. The upward force on the system is T_1 and the downward force is $(m_1 + m_2 + m_3)\, g$.

$\therefore \quad$ Net upward force $= T_1 - (m_1 + m_2 + m_3)\, g$

If a is the upward acceleration in the system, then,

$$(m_1 + m_2 + m_3)\, a = T_1 - (m_1 + m_2 + m_3)\, g \quad \ldots (i)$$

or $$T_1 = (m_1 + m_2 + m_3)\,(a + g) = (5 + 2 + 3)\,(2 + 9.8) = \mathbf{118\ N}$$

The force pulling m_2 and m_3 in the upward direction is T_2 and the downward force on them is $(m_2 + m_3)g$. Hence, the net upward force $= T_2 - (m_2 + m_3)\, g$.

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$\therefore \quad (m_2 + m_3)\, a = T_2 - (m_2 + m_3)\, g$ ...(*ii*)

or $T_2 = (m_2 + m_3)\,(a + g) = (2 + 3)\,(2 + 9.8) = \mathbf{59\ N}$

The net upward force on $m_3 = T_3 - m_3 g$

$\therefore \quad m_3\, a = T_3 - m_3\, g$ ...(*iii*)

or $T_3 = m_3\,(a + g) = 3\,(2 + 9.8) = \mathbf{35.4\ N}$

(*ii*) When the whole system is stationary or moving with a constant velocity, $a = 0$. Putting $a = 0$ in eqs. (*i*), (*ii*) and (*iii*), we get,

$T_1 = (m_1 + m_2 + m_3)\, g = (5 + 2 + 3)\ 9.8 = \mathbf{98\ N}$

$T_2 = (m_2 + m_3)\, g = (2 + 3)\ 9.8 = \mathbf{49\ N}$

$T_3 = m_3 g = 3 \times 9.8 = \mathbf{29.4\ N}$

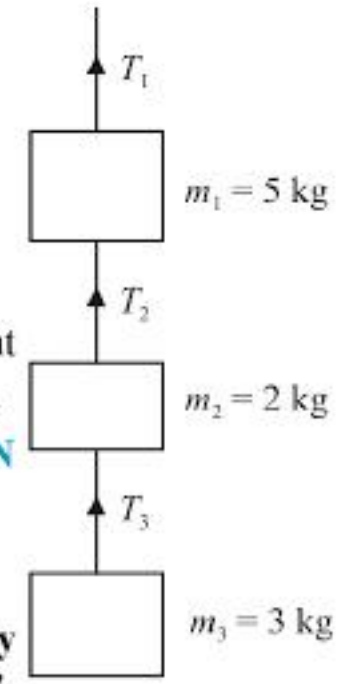


Fig. 10.52

10. Four blocks of the same mass *m* connected by cords are pulled by a force *F* on a smooth horizontal surface as shown in Fig. 10.53. Determine the tensions T_1, T_2 and T_3 in the cords.

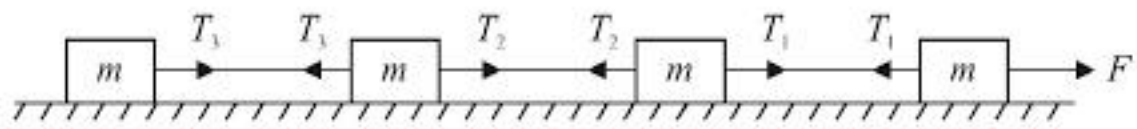


Fig. 10.53

Hint. Acceleration of the system, $a = \dfrac{F}{4m}$

Considering each block separately, net force = mass × acceleration.

$\therefore \quad F - T_1 = m\, a$ or $F - T_1 = m \times \dfrac{F}{4m} \quad \therefore T_1 = \mathbf{\dfrac{3}{4}F}$

$T_1 - T_2 = m\, a$ or $\dfrac{3}{4}F - T_2 = m \times \dfrac{F}{4m} \quad \therefore T_2 = \mathbf{\dfrac{1}{2}F}$

$T_2 - T_3 = m\, a$ or $\dfrac{1}{2}F - T_3 = m \times \dfrac{F}{4m} \quad \therefore T_3 = \mathbf{\dfrac{1}{4}F}$

11. Two masses 10 kg and 20 kg are connected by a massless spring as shown in Fig. 10.54. A force of 200 N acts on the 20 kg mass. At the instant shown, the 10 kg mass has an acceleration of 12 ms^{-2}. What is the acceleration of 20 kg mass ?

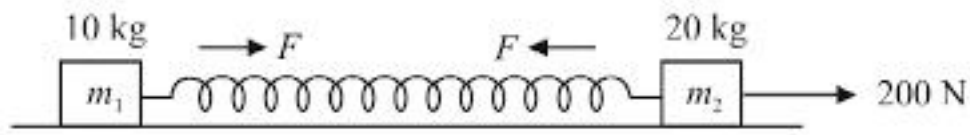


Fig. 10.54

Hint. For mass m_1; $F = m_1\, a_1 = 10 \times 12 = 120$ N

For mass m_2; $200 - F = m_2\, a_2$

or $200 - 120 = 20 \times a_2 \quad \therefore a_2 = 80/20 = \mathbf{4\ ms^{-2}}$

12. Three blocks are connected by strings as shown in Fig. 10.55. They are pulled by a force *F* on a frictionless horizontal surface. The tension *P* in the first string is 16 N. Find (*i*) acceleration of blocks (*ii*) Force *F* (*iii*) tension *Q* in the second string.

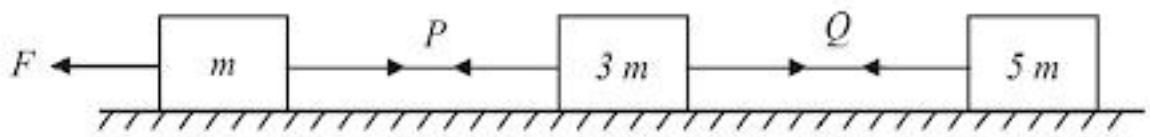


Fig. 10.55

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Hint. The blocks will have the same acceleration a. The force F is pulling all the three blocks; force P pulling blocks 3 m and 5 m; force Q pulling only block 5 m.

(*i*) $\therefore \quad P = (3\ m + 5\ m)\ a = 8\ m\ a$

or $\quad a = \dfrac{P}{8m} = \dfrac{16}{8m} = \mathbf{\dfrac{2}{m}}$ **metre/sec^2**

(*ii*) $\quad F = (m + 3m + 5m)a = 9\ ma = 9m \times \dfrac{2}{m} = \mathbf{18\ N}$

(*iii*) $\quad Q = 5\,m\,a = 5\,m \times \dfrac{2}{m} = \mathbf{10\ N}$

13. A massless string passes over a frictionless pulley and carries a mass m_1 hanging at one end and mass m_2 connected by another massless string to mass m_3 at the other end as shown in Fig. 10.56. Calculate the tension in the string joining masses m_2 and m_3.

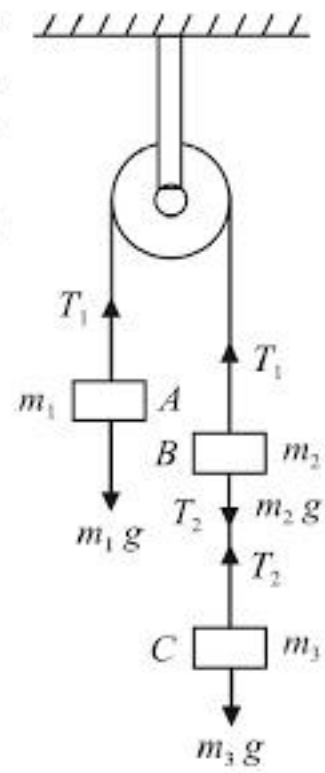


Fig. 10.56

Hint. Let a be the acceleration of the masses. The net force causing this acceleration $= (m_2 + m_3 - m_1)\ g$.

$$\therefore \quad \text{Acceleration, } a = \frac{\text{Net force}}{\text{Total mass}} = \frac{(m_2 + m_3 - m_1)g}{m_1 + m_2 + m_3}$$

Applying Newton's second law to mass m_3, we have,

$$m_3\ g - T_2 = m_3\ a$$

or $$m_3 g - T_2 = \frac{m_3(m_2 + m_3 - m_1)g}{m_1 + m_2 + m_3}$$

$\therefore$ $$T_2 = m_3 g - \frac{m_3(m_2 + m_3 - m_1)g}{m_1 + m_2 + m_3}$$

or $$T_2 = \frac{\mathbf{2m_1 m_3 g}}{\mathbf{m_1 + m_2 + m_3}}$$

14. A metal plate of mass 200 g is balanced in mid-air by throwing 40 balls per second, each of mass 2g vertically upwards from below. The balls get rebounded with the same speed with which they strike the plate. Find the speed with which the balls strike the plate.

Hint. Mass of plate, $M = 200\ \text{g} = 200 \times 10^{-3}\ \text{kg}$

Mass of ball, $m = 2\ \text{g} = 2 \times 10^{-3}\ \text{kg}$

In order that the plate remains balanced in air, the weight Mg of the plate must be balanced by the upward impulsive force imparted by the balls to the plate. Let the required striking speed of the balls be v metres/sec. It is given that the ball rebounds with the same speed after striking the plate.

$\therefore$ Change of linear momentum of ball in one collision

$$= -\ m\ v - (m\ v) = -2\ m\ v$$

Change of linear momentum of plate per collision of each ball is

$$\Delta p = 2\ mv$$

Time for one collision, $\Delta t = 1/40$ sec

$\therefore$ Rate of change of linear momentum of plate is

$$\frac{\Delta p}{\Delta t} = \frac{2mv}{1/40} = 80\ mv$$

This is the force imparted to the plate by the balls.

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For the equilibrium of the plate, we have,

$$Mg = 80\ mv$$

or $$v = \frac{Mg}{80m} = \frac{\left(200\times10^{-3}\right)\times9.8}{80\times2\times10^{-3}} = \mathbf{12.25\ ms^{-1}}$$

15. A rifle man, who together with his rifle has a mass of 100 kg, stands on a smooth surface and fires 10 shots horizontally. Each bullet has a mass of 10 g with a muzzle velocity of 800 ms^{-1}.

(*i*) What velocity does the rifle man acquire at the end of 10 shots ?

(*ii*) If the shots were fired in 5 s, what was the average force exerted on him ?

Hint. Mass of bullet, $m = 10 \times 10^{-3}$ kg ; Mass of rifle and man, $M = 100$ kg

Velocity of bullet, $v = 800\ ms^{-1}$; Velocity of rifle and man, $V = ?$

(*i*) Initially the linear momentum of the system is zero because the bullet and the rifle man are at rest. Therefore, the linear momentum of the system remains constant after firing the bullet *i.e.*,

$$mv + MV = 0$$

or $$V = -\frac{mv}{M} = -\frac{\left(10\times10^{-3}\right)\times800}{100} = -0.08\,ms^{-1}$$

$\therefore$ Velocity acquired after 10 shots $= 10 \times (-0.08) = \mathbf{-0.8\ ms^{-1}}$

Note that negative sign shows that velocity of rifle man is in a direction opposite to that of bullet.

(*ii*) The linear momentum acquired by the rifle man is $\Delta p = MV = 100 \times 0.8 = 80$ kg ms^{-1}.

This change in linear momentum occurs in 5 sec. Therefore, average force exerted on rifle man is

$$F = \frac{\Delta p}{\Delta t} = \frac{80}{5} = \mathbf{16\ N}$$

16. An 8000 kg rocket is set for a vertical firing. If the exhaust speed is 800 ms^{-1}, how much gas must be ejected per second to supply thrust needed (*i*) to overcome the weight of the rocket (*ii*) to give the rocket an initial upward acceleration of 19.6 ms^{-2}?

Hint. The upward thrust F of the rocket is given by ;

$$F = v\frac{dm}{dt}$$

where v = velocity of exhaust = 800 ms^{-1}

dm/dt = time rate of mass of gas ejected

(*i*) The weight of the rocket acting downward = mg.

The upthrust needed to overcome the weight of rocket (*i.e.*, acceleration of rocket = 0),

$$v\frac{dm}{dt} = mg \quad \therefore \quad \frac{dm}{dt} = \frac{mg}{v} = \frac{8000\times9.8}{800} = \mathbf{98\ kg/sec}$$

(*ii*) Net force on rocket = Upthrust – Weight of rocket

$$= v\frac{dm}{dt} - mg$$

If a is the upward acceleration of the rocket, then,

$$v\frac{dm}{dt} - mg = ma \quad \text{or} \quad v\frac{dm}{dt} = (ma + mg) = m(a+g)$$

$\therefore$ $$\frac{dm}{dt} = \frac{m(a+g)}{v} = \frac{8000(19.6+9.8)}{800} = \mathbf{294\ kg/sec}$$

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11

Friction and Dynamics of Circular Motion

INTRODUCTION

When an object (*e.g.,* wooden block) moves over the surface of another object (*e.g.,* floor), its motion is always opposed by a retarding force that resists motion. This retarding force is called *friction* and always acts in a direction opposite to motion. For example, an object sliding down an inclined plane experiences a frictional force directed up the inclined plane. Similarly, an object that moves to the right on a level floor experiences the frictional force to the left. Friction can operate between two solids, between a solid and a fluid or between two layers of a fluid. However, in this chapter, we shall discuss only the friction between solid surfaces and its role in motion of vehicles on circular roads.

11.1. FRICTION

When a solid object moves over the surface of another solid object, its motion is always opposed by a retarding force, called friction.

Hence friction is an opposing force that comes into play when one body moves or attempts to move over the surface of another body. The frictional force is always parallel to the surfaces in contact.

Let us discuss *how frictional force behaves.* Consider a wooden block of mass m placed on a horizontal surface as shown in Fig. 11.1. The block pushes down on the horizontal surface with a force equal to block's weight ($= mg$). The supporting horizontal surface pushes back with an equal and opposite force R so that $R = mg$. The force R is perpendicular to the surface and is called **normal reaction** (normal means perpendicular).

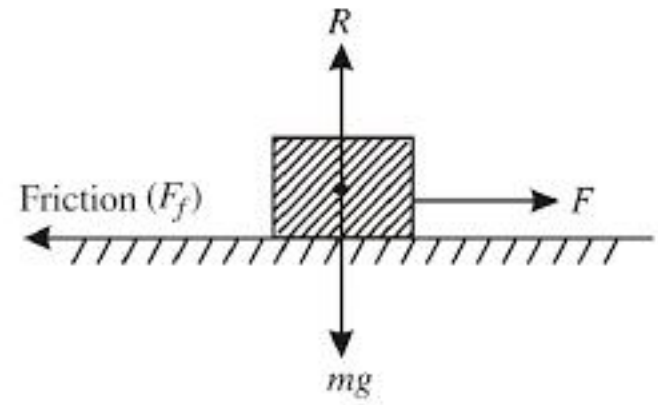


Fig. 11.1

Suppose a small horizontal force F is applied on the block to the right but the block does not move. It means that a force has come into play to oppose the motion of the block. This is the force of friction and is called **static friction** F_S because the block remains at rest or static even though we have applied force. As we increase the applied force, the static friction F_S also increases till a stage is reached when the block is about to move. The value of static friction F_S is now maximum and is called **limiting friction** or **critical friction** F_C. Once the limiting friction (*i.e.,* maximum

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static friction) is overcome, the block begins to move and frictional force decreases. The friction is now called **kinetic friction** F_k. Note that the word *kinetic* means that the block is moving. Thus the behaviour of friction can be summed up as under :

(*i*) Before the body begins to move, the static friction plays the role ; the static friction increases as the applied force increases upto the value of limiting friction (*i.e.*, maximum static friction).

(*ii*) Once limiting friction is overcome by the applied force, the role of static friction is over. Now smaller frictional force F_k (= kinetic friction) opposes the motion.

(*iii*) Both limiting friction (F_C) and kinetic friction (F_k) are important. The limiting friction must be overcome to start the body moving and then smaller frictional force F_k to oppose the motion.

11.2. CAUSE OF FRICTION

Friction is a complicated phenomenon. Many aspects of friction even today are not completely understood. We now give the old view and modern view about the cause of friction.

Old view. According to the old view, friction comes into play due to the irregularities in the surfaces in contact. These irregularities are in the form of projections and depressions in the surfaces as shown in Fig. 11.2. When two surfaces come in contact, these irregularities get interlocked. This interlocking opposes the sliding of one surface over the other. Once sliding has begun, the surfaces do not have time to "settle down" onto each other completely. Therefore, the friction decreases. This explains why kinetic friction is less than limiting friction (maximum static friction).

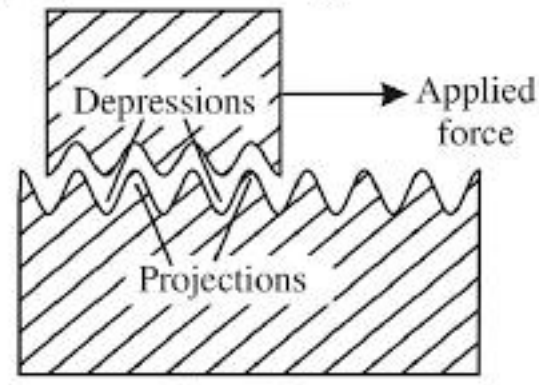


Fig. 11.2

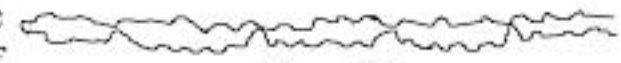
Drawbacks. The old view has the following drawbacks :

(*i*) According to old view, the greater the area of contact between the surfaces, the greater should be the force of friction between them. However, experiments show that force of friction is independent of area of contact of the two surfaces.

(*ii*) According to the old view, when the surfaces are made smooth, the friction decreases because the size of projections and depressions in the surfaces is decreased. However, experiments show that when surfaces are highly polished, the force of friction becomes very large.

Modern view. According to modern view, *the friction is due to the atomic and molecular forces of attraction between the two surfaces at the point of their actual contact.*

On a microscopic level, even the most polished surface is far from plain *i.e.*, every surface has roughness. Therefore, when two surfaces are put together, the actual area of contact is very much less than the apparent area of contact (See Fig. 11.3).


Fig. 11.3

For example, it has been found that for steel on steel, the actual contact area can be as little as one ten-thousandth of the apparent area. The pressures at the contact points are very high and the molecules are pushed into such close proximity that the attractive forces between them weld the surfaces together at contact points. In other words, the contact points are *cold-welded* together. When we try to move one body over another body, these tiny welds oppose motion. This gives rise to the opposing force of friction.

Explanation of friction on modern view. We now explain many experimentally observed facts about friction on the basis of modern view.

(*i*) The tiny welds formed have to be broken before one surface can move over the other. Therefore, no matter in which direction the motion occurs, there is a force which opposes it. *This explains why force of friction always acts in a direction opposite to motion.*

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(*ii*) When the apparent area of contact of the body is *decreased, the number of contact points is decreased. Since the weight of the body is not changed, there is increased pressure at the contact points. As a result, contact points flatten so that total contact area and pressure return to their original values. *This explains why force of friction is independent of the area of contact of the surfaces.*

(*iii*) The actual area of contact between two surfaces depends upon the force with which the surfaces are pressed together. In other words, actual area of contact will depend upon the normal force exerted by the supporting surface on the body. *This explains why force of friction (static or kinetic) is directly proportional to the normal reaction.*

11.3. TYPES OF FRICTION

There are two common types of friction *viz.* **1.** Static friction **2.** Kinetic friction.

1. Static friction. *The opposing force that comes into play when an object does not move over the surface of another even though force is applied to make it move is called* **static friction.**

Consider a wooden block lying on a horizontal surface. If we apply a small horizontal force (say 1 N) on the block and the block does not move, it means that a force (= 1 N) has come into play to oppose the motion. This is the force of friction and is called *static friction* F_S. As the applied force is increased, the static friction also increases (being always equal and opposite to the applied force) till a stage is reached when the block is about to move. The value of static friction is now maximum and is called *limiting friction* or *critical friction* F_C. Note that static friction always adjusts itself equal to the applied force upto the value of limiting friction.

Thus **limiting friction** *is the maximum static friction that comes into play when one body is just at the verge of moving over the surface of another body.*

Once the limiting friction is overcome by the applied force, the block begins to move and limiting friction F_C is converted into kinetic friction F_k. The subsequent motion of the block is determined by the applied force (F) and kinetic friction F_k. It is reminded that $F_k < F_C$.

Discussion. The following points may be noted :

(*i*) Static friction comes into play *only* when the applied force tends to move a body on the surface of another body. If the applied force is zero, static friction is also zero.

(*ii*) Static friction is a *self-adjusting force*. As the applied force increases, static friction also increases (being always equal and opposite to the applied force) upto limiting friction.

2. Kinetic Friction. *Kinetic friction is the opposing force that comes into play when one body moves over the surface of another body.* It is sometimes called *dynamic friction.*

Once the limiting friction is overcome by the applied force, the body begins to move and the limiting friction is converted into kinetic friction. Fig. 11.4 shows the graph between the applied force and force of friction. The part OA of the graph is a straight line showing that static friction increases as the applied force increases upto limiting friction F_C. The body will remain at rest till the applied force does not exceed OC. Once the applied force exceeds OC, the body starts moving and friction decreases to F_k (kinetic friction). Thus the limiting friction must be overcome to start the body moving and then a smaller kinetic frictional force F_k acts on the body to oppose the motion of the body.

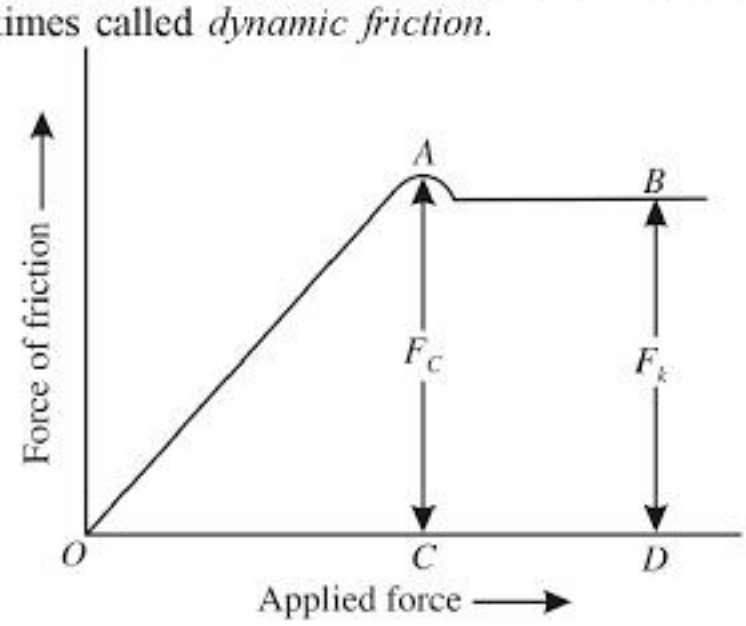


Fig. 11.4

* This can be done by turning the body so that it rests on one of its smaller faces.

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Types of kinetic friction. Kinetic friction is of two types *viz.* (*a*) **Sliding friction** (*b*) **Rolling friction**.

When one body slides over the surface of another body, then opposing force that comes into play is called the *sliding friction*. For example, when a rectangular wooden block is moved on a level floor, the opposing force is the sliding friction.

When one body rolls over the surface of another body, then opposing force that comes into play is called the *rolling friction*. Thus when a ball rolls over a surface, the opposing force is the rolling friction. We shall discuss the cause of rolling friction in Art. 11.10.

11.4. LAWS OF FRICTION

Static friction as well as kinetic friction is a complicated phenomenon. Many aspects of friction even today are not completely understood. However, experiments show that to a good approximation, the force of friction obeys the following laws :

(*i*) *The frictional force (static as well as kinetic) depends upon the nature of the two surfaces in contact and their state of roughness.*

(*ii*) *The frictional force (static as well as kinetic) always acts parallel to the surfaces in contact and its direction is opposite to motion or attempted motion.*

(*iii*) *The frictional force (static as well as kinetic) is independent of the area of contact of the two surfaces.*

(*iv*) ***The magnitude of frictional force (kinetic and static) is directly proportional to the perpendicular force (normal reaction R) between the two surfaces in contact.***

(*v*) *The frictional force is independent of the speed (applicable to kinetic friction only) of one surface relative to the other surface.*

11.5. MAGNITUDE OF LIMITING FRICTION

Consider a block of mass m lying on a horizontal surface as shown in Fig. 11.5. The normal reaction R is the perpendicular force exerted by the surface on the block. Clearly, $R = mg$. If F_C is the limiting friction (*i.e.*, maximum static friction), then according to laws of friction, F_C is directly proportional to the normal reaction R *i.e.*,

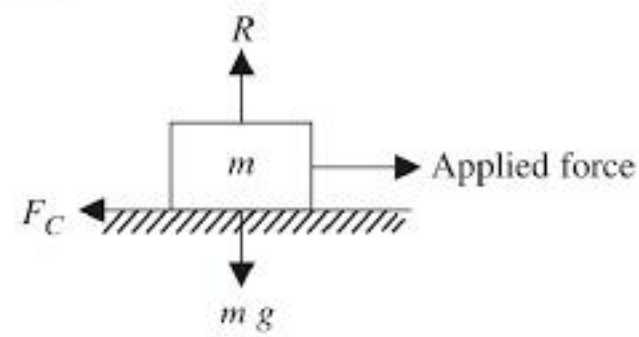


Fig. 11.5

$$F_C \propto R$$

or $$F_C = \mu_S R$$

where μ_S is a constant of proportionality and is called *coefficient of static friction.* Its value depends upon the nature and condition of the surfaces in contact.

$$\mu_S = \frac{F_C}{R} = \frac{\text{Limiting friction}}{\text{Normal reaction}}$$

Therefore, ***co-efficient of static friction*** *between two surfaces in contact is equal to the ratio of limiting friction and the normal reaction.* Since μ_S is a ratio of two forces, it has no units *i.e.* it is a pure number.

Illustration. Let us find the magnitude of limiting friction in the three cases shown in Fig. 11.6. Suppose that co-efficient of static friction between the block and the surface is μ_S in

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each case. The normal reaction R is the perpendicular force that the surface exerts on the block. Its value is different in the three cases shown in Fig. 11.6.

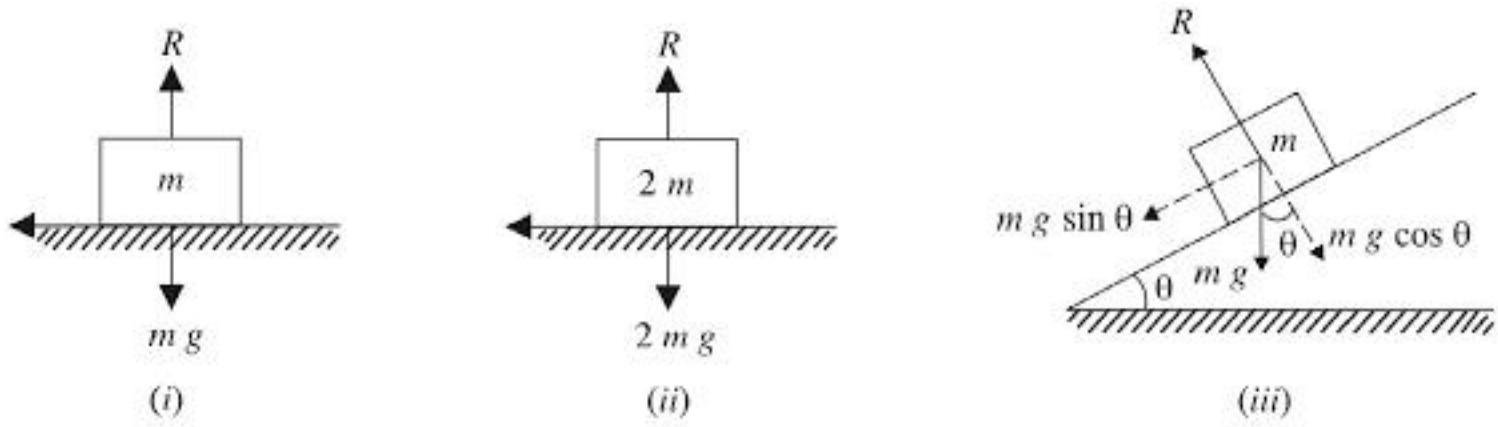


Fig. 11.6

(*i*) For the first case, $R = m\,g$ [See Fig. 11.6 (*i*)]. Therefore, $F_c = \mu_s R = \mu_s\, mg$. This is the value of maximum static friction that the surface can exert on the block.

(*ii*) For the second case [See Fig. 11.6 (*ii*)], $R = 2\,m\,g$ so that $F_C = \mu_S R = 2\mu_S\, mg$. Note that now normal force is double its previous value. Therefore, the magnitude of limiting friction becomes two times the previous value.

(*iii*) For the third case [See Fig. 11.6 (*iii*)], $R = m\,g\cos\theta$ so that $F_c = \mu_s R = \mu_s mg\cos\theta$.

11.6. MAGNITUDE OF KINETIC FRICTION

Like static friction, the magnitude of kinetic friction is directly proportional to the perpendicular force between the surfaces in contact (i.e. normal reaction between the surfaces). Consider a block of mass m moving with a constant velocity $\vec{v}$ on a horizontal surface as shown in Fig. 11.7. The normal reaction R is the perpendicular force exerted by the surface on the block. Clearly, $R = m\,g$. If F_k is the kinetic friction exerted by the surface on the block, then,

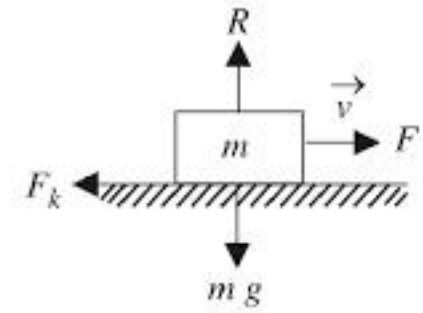


Fig. 11.7

$$F_k \propto R$$

or

$$F_k = \mu_k R$$

where μ_k is a constant of proportionality and is called *co-efficient of kinetic friction.* Its value depends upon the nature and condition of the surfaces in contact. To a good approximation, value of F_k (or μ_k) is independent of relative speed between the surfaces and their area of contact.

$$\mu_k = \frac{F_k}{R} = \frac{\text{Kinetic friction}}{\text{Normal reaction}}$$

Therefore, ***co-efficient of kinetic friction*** *between two surfaces in contact is equal to the ratio of kinetic friction and the normal reaction.* Since μ_k is a ratio of two forces, it has no units; it is a pure number. *Note that for a given pair of surfaces,* $\mu_k < \mu_s$. Therefore, we require a larger force to set an object in motion than to keep it in motion with constant velocity.

Illustration. Suppose a box of mass 3 kg is being pulled horizontally on a level surface by a force of 20 N [See Fig. 11.8]. If the co-efficient of kinetic friction between the box and surface is $\mu_k = 0.5$ and $g = 10\ \text{ms}^{-2}$, we are to find the acceleration of the box. The normal reaction R is the perpendicular force exerted by the surface on the box. Clearly, $R = mg$.

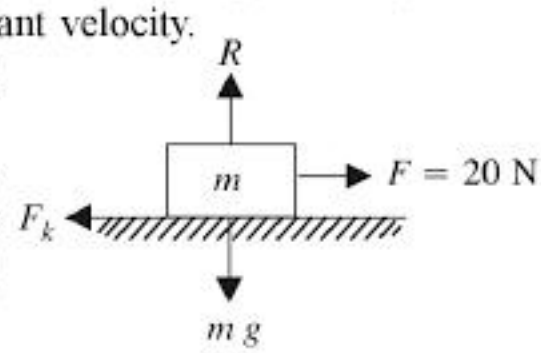


Fig. 11.8

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Kinetic friction, $F_k = \mu_k R = \mu_k\, mg = 0.5 \times 3 \times 10 = 15$ N

Net force $= F - F_k = 20 - 15 = 5$ N

According to Newton's second law, the acceleration a of the box is given by ;

$$a = \frac{\text{Net force}}{\text{Mass}} = \frac{5}{3} = 1.67\ \text{ms}^{-2}$$

Note that role of limiting friction is only confined to set the body moving. Once the body is in motion, its acceleration is determined by the applied force and kinetic friction between the surfaces in contact.

Example 11.1. A block of 10 kg is pulled at a constant speed on a rough horizontal surface by a force of 20 N. Find the co-efficient of kinetic friction. Given that $g = 10\ \text{ms}^{-2}$.

Solution. Since the block is moving with a constant speed on the horizontal surface, kinetic friction exerted by the surface on the block is equal and opposite to the applied force.

$$F_k = 20\ \text{N};\ R = mg = 10 \times 10 = 100\ \text{N};\ \mu_k = ?$$

$$\therefore \quad \mu_k = \frac{F_k}{R} = \frac{20}{100} = \mathbf{0.2}$$

Example 11.2. A box is lying on the floor of a moving train and the coefficient of friction between the floor and the box is 0.15. Find the maximum acceleration of the train for which the box will lie stationary on the floor. Given that $g = 10\ \text{ms}^{-2}$.

Solution. Suppose m is the mass of the box. Let a_{max} be the maximum acceleration of the train at which the box can remain stationary on the floor.

The limiting friction between floor and box is

$$F_C = m\, a_{max}$$

Now $$\mu_S = \frac{F_C}{R} = \frac{m\, a_{max}}{mg} = \frac{a_{max}}{g}$$

$$\therefore \quad a_{max} = \mu_S\, g = 0.15 \times 10 = \mathbf{1.5\ ms^{-2}}$$

Example 11.3. Find the distance travelled by a body before coming to rest if it is moving with a velocity of 15 ms^{-1} and the coefficient of friction between the ground and the body is 0.45. Take $g = 10\ \text{ms}^{-2}$.

Solution. Let a be the retardation of the body.

Now $$\mu_S = \frac{F_C}{R} = \frac{ma}{mg} = \frac{a}{g}$$

$$\therefore \quad \text{Retardation, } a = \mu_S \times g = 0.45 \times 10 = 4.5\ \text{ms}^{-2}$$

Now $$v^2 - v_0^2 = 2\, aS$$

or $$0^2 - 15^2 = 2 \times (-4.5) \times S \quad \therefore S = \mathbf{25\ m}$$

Example 11.4. A 3 kg box rests on a horizontal table. The coefficients of static and kinetic friction between the box and table are $\mu_S = 0.6$ and $\mu_k = 0.5$. The box is pulled by a string at an angle of 30° as shown in Fig. 11.9. Find the frictional force and the acceleration of the box if the tension in the string is 10 N.

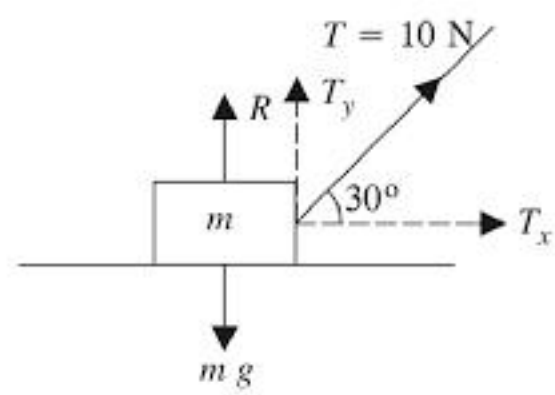


Fig. 11.9

Solution. Fig. 11.9 shows the conditions of the problem. The horizontal and vertical components of tension T are :

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$$T_x = T\cos\theta = 10\cos 30° = 8.66 \text{ N}$$
$$T_y = T\sin\theta = 10\sin 30° = 5 \text{ N}$$

Since there is no acceleration in the vertical direction, the net force in the vertical direction is zero *i.e.*

$$R + T\sin 30° - mg = 0$$
or $$R = mg - T\sin 30° = 3 \times 9.8 - 5 = 24.4 \text{ N}$$

Limiting friction, $F_C = \mu_S R = 0.6 \times 24.4 = 14.7$ N

Since the applied horizontal force T_x (= 8.66 N) is less than the limiting friction (*i.e.* maximum static friction), the box will not accelerate horizontally. The frictional force exerted by the surface on the box is 8.66 N to the left to balance the applied horizontal force T_x. Note that box can only move if limiting friction is overcome.

$\therefore$ Frictional force = **8.66 N**

Example 11.5. A woman pushes a box of mass 20 kg on a horizontal surface with a horizontal force of *F*. The co-efficients of static and kinetic friction are $\mu_S = 0.6$ and $\mu_k = 0.5$.

(*i*) What must *F* be in order that she can make the box start to slide?

(*ii*) If she maintains the same force once the block starts to slide, what will be its acceleration?

Solution. (*i*) $R = m\,g = 20 \times 9.8 = 196$ N ; $\mu_s = 0.6$; $\mu_k = 0.5$

Limiting friction, $F_C = \mu_S R = 0.6 \times 196 = 117.6$ N

Therefore, the applied force must be 117.6 N if she wants to make the box start to slide.

$\therefore$ Applied force, F = **117.6 N**

(*ii*) Once the box starts moving, the frictional force decreases to F_k (kinetic friction).

$$F_k = \mu_k R = 0.5 \times 196 = 98 \text{ N}$$

Net force on box = $F - F_k = 117.6 - 98 = 19.6$ N

According to Newton's second law, acceleration a of the box is given by ;

$$a = \frac{\text{Net force}}{\text{Mass}} = \frac{19.6}{20} = \mathbf{0.98\ ms^{-2}}$$

Example 11.6. A bullet of mass 0.01 kg is fired horizontally into a 4 kg wooden block at rest on a horizontal surface. The co-efficient of kinetic friction between the block and the surface is 0.25. The bullet gets embedded in the block and the combination moves 20 m before coming to rest. With what speed did the bullet strike the block?

Solution. Mass of bullet, $m = 0.01$ kg ; Mass of block, $M = 4$ kg ; $\mu_k = 0.25$

When the bullet hits the block, the block-bullet system gets retarded due to friction between this system and the surface. If a is the retardation produced in block-bullet system, then,

$$a = \frac{F_k}{M+m} = \frac{\mu_k R}{M+m} = \frac{\mu_k (M+m)g}{M+m} = \mu_k g = 0.25 \times 9.8 = 2.45 \text{ ms}^{-2}$$

Suppose V is the velocity of block-bullet system after the bullet hits the block. The system comes to rest after covering a distance $S = 20$ m.

$\therefore$ $$0^2 - V^2 = 2\,a\,S$$

Here $a = -2.45$ ms^{-2} ; $S = 20$ m ; $V = ?$

$\therefore$ $$-V^2 = 2 \times (-2.45) \times 20 \quad \text{or} \quad V = \sqrt{98} = 9.9 \text{ ms}^{-1}$$

Suppose v is the velocity of the bullet before it gets embedded into the block. Therefore, by law of conservation of linear momentum ;

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$$m\,v = (M + m)\,V$$

or $$v = \frac{(M+m)V}{m} = \frac{4+0.01}{0.01} \times 9.9 = \mathbf{3969.9\ ms^{-1}}$$

Example 11.7. Referring to Fig. 11.10, what is the least amount of force F you must exert on the block to keep it in place ? The block has a mass of 2.2 kg and the co-efficient of static friction between the wall and the block is $\mu_S = 0.65$. Take $g = 10\ ms^{-2}$.

Solution. The block is being pressed against the wall by the horizontal force F [See Fig. 11.10]. The wall exerts a normal force R on the block so that $R = F$. You can hold the block in place if you push hard enough horizontally. It is because friction between the block and wall depends upon R and the value of R depends upon the value of F. If the block is to remain stationary, there must be enough static friction upward to balance the downward force $m\,g$.

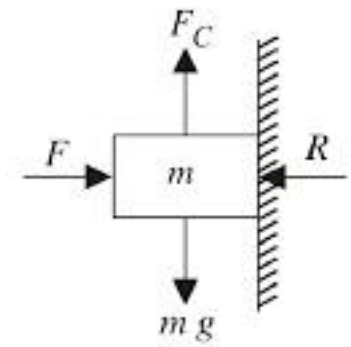


Fig. 11.10

$$\text{Weight of block} = m\,g = 2.2 \times 10 = 22\ \text{N}$$

The static friction must be equal to 22 N.

$$\text{Limiting friction, } F_C = \mu_S R$$

or $$R = \frac{F_C}{\mu_S} = \frac{22}{0.65} = 34\,\text{N}$$

$\therefore$ $$F = R = \mathbf{34\ N}$$

Therefore, the least force F that will keep the block in place is 34 N.

Example 11.8. A 4 kg block A is placed at the top of 8 kg block B, which rests on smooth table. The block A just slips on B when a force of 12 N is applied on block B. What is the maximum horizontal force F required to make both A and B move together?

Solution. Fig. 11.11 shows the conditions of the problem. The maximum force of static friction is $F_S = 12$ N. For A to move together with B, the force on A should not exceed 12 N. If a is the corresponding acceleration, then,

$$12 = M_A \times a$$

or $$a = \frac{12}{M_A} = \frac{12}{4} = 3\,\text{ms}^{-2}$$

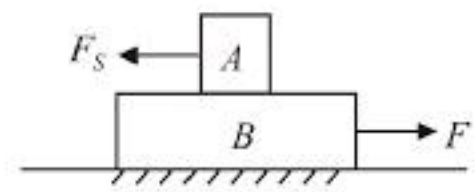


Fig. 11.11

The equation of motion for body B is

$$F - F_S = M_B \times a$$

or $$F = F_S + M_B \times a = 12 + 8 \times 3 = \mathbf{36\ N}$$

PROBLEMS FOR PRACTICE

1. In Fig. 11.12, find the normal reaction. **[19.4 N]**

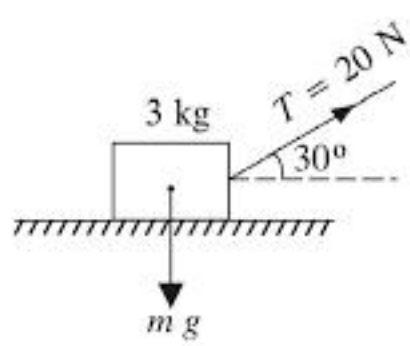


Fig. 11.12

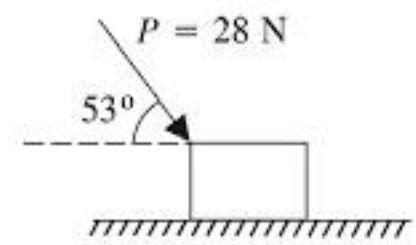


Fig. 11.13

2. In Fig. 11.13, the weight of the block is 47 N. Find the normal reaction. **[69.4 N]**

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3. If the co-efficient of friction between a car's tyres and a roadway is 0.62, what is the least distance in which the car can accelerate from rest to 20.7 ms^{-1} ? **[35.3 m]**

4. A 50 kg block rests on a horizontal surface. If this block is pulled as shown in Fig. 11.14, it will not move unless the tension exceeds 350 N. What is the co-efficient of static friction between the block and the surface ? **[0.714]**



Fig. 11.14

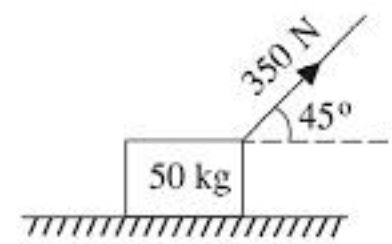


Fig. 11.15

5. Will the block shown in Fig. 11.15 move ? **[Yes]**

6. A 50 kg stone falls into a mud pot from a height of 6 m. It sinks 0.5 m below the mud surface before coming to rest. Assuming that a constant frictional force between the mud and stone brought the stone to rest, determine the magnitude of that force. **[7.84×10^3 N]**

7. The coefficient of static friction between the floor of a truck and a box resting on it is 0.3. The truck is travelling at 80 km/h. What is the least distance in which the truck can stop if the box is not to slide? **[83.9 m]**

8. How long will it take for a wooden block of mass 2 kg to slide from rest a distance of 3 m down a rough plane inclined at 30° to the horizontal ? Given $\mu = 0.20$. **[1.37 s]**

9. The co-efficient of friction between a rubber tyre and a wet concrete road is 0.5. Find the minimum time in which a car whose initial velocity is 15 ms^{-1} can come to a stop on such a road. **[3 s]**

11.7. ANGLE OF FRICTION

The angle which the resultant of the limiting friction and the normal reaction makes with the normal reaction is called the **angle of friction.**

Consider a block of mass m resting on a horizontal surface as shown in Fig. 11.16. The weight mg of the block is balanced by the normal reaction R *(i.e.* $R = m\,g$*)*. The magnitude of applied force is equal to the limiting friction F_C between the surfaces. Under such condition, the block is at the point of sliding. The resultant of R and F_C makes an angle α with R. This angle is the angle of friction. Referring to Fig. 11.16,

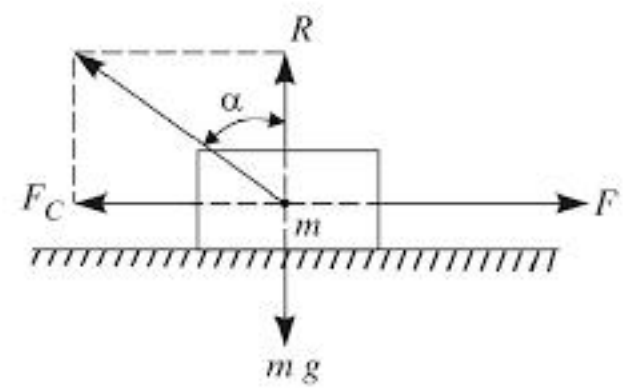


Fig. 11.16

$$\tan\alpha = \frac{F_C}{R} = \frac{\mu_S R}{R} = \mu_S$$

$$\therefore \quad \mu_S = \tan\alpha$$

Thus the coefficient of static friction is equal to the tangent of the angle of friction.

11.8. MEASUREMENT OF μ_S AND μ_k

We can measure μ_s and μ_k for the two surfaces simply by placing a block on a plane surface and incline the plane until the block begins to slide.

(i) **Measurement of μ_S.** Consider a block of mass m placed on a plane surface. If the plane surface is raised gradually above the horizontal, eventually a point is reached when the block just begins to slide. Let θ_c be the critical angle at which sliding starts. In this condition, the block is under the action of the following forces (See Fig. 11.17) :

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(*a*) The weight *mg* of the block acting vertically downward.

(*b*) The limiting friction F_C acting parallel to the surfaces in contact but opposite to the attempted motion. In this case, limiting friction is directed upward along the inclined plane.

We choose *X*-axis parallel to the plane and *Y*-axis perpendicular to the plane. The weight *mg* of the block is resolved into an *x*-component $mg \sin\theta_C$ down the inclined plane and a *y*-component $mg \cos \theta_C$ perpendicular to the inclined plane. The incline exerts a normal force *R* on the block. Since the block is just at the point of sliding, the *x* and *y* components of the net force are zero *i.e.*,

$$R - mg \cos\theta_C = 0 \quad \text{or} \quad R = mg \cos\theta_C \qquad ...(i)$$

and
$$mg \sin\theta_C - F_C = 0 \quad \text{or} \quad F_C = mg \sin\theta_C \qquad ...(ii)$$

Dividing eq. (*ii*) by (*i*), we have,

$$\frac{F_C}{R} = \frac{mg \sin\theta_C}{mg \cos\theta_C} = \tan\theta_C \qquad (\because F_C = \mu_S R)$$

or
$$\frac{\mu_S R}{R} = \tan\theta_C$$

$\therefore$
$$\mu_S = \tan\theta_C$$

Thus the coefficient of static friction (μ_S) *is equal to the tangent of the angle of inclination* (θ_C) *at which the block just begins to slide.*

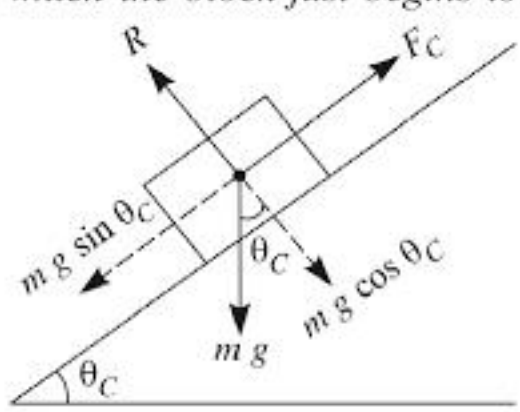


Fig. 11.17

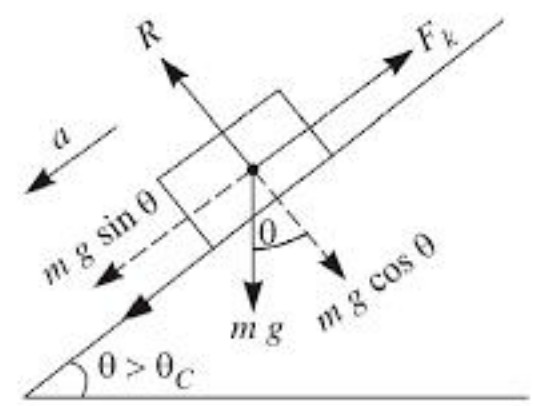


Fig. 11.18

The angle θ_C is called the *angle of repose*. Thus if the angle of repose for a block on a plank is 38°, then co-efficient of static friction between the block and plank is $\mu_S = \tan\theta_C = \tan 38° = 0.78$.

Angle of repose. *The angle of the inclined plane at which a body placed on it just begins to slide is called the* ***angle of repose***. It is also called *angle of sliding*.

Now $\mu_S = \tan\theta_C$ ... θ_C is angle of repose

Also $\mu_S = \tan\alpha$ ... α is angle of friction

$\therefore$ $\alpha = \theta_C$

Hence angle of friction (α) *is equal to the angle of repose* (θ_C).

***(ii)* Measurement of μ_k.** At an angle greater than θ_C (See Fig. 11.18), the block begins to slide down the inclined plane with an acceleration *a*. Note that static friction changes to kinetic friction $F_k (= \mu_k R)$. The net force down the inclined plane is

Net force $= mg \sin\theta - F_k = mg \sin\theta - \mu_k R = mg \sin\theta - \mu_k mg \cos\theta$

$\therefore$ Acceleration of block, $a = \dfrac{\text{Net force}}{\text{Mass}} = \dfrac{mg \sin\theta - \mu_k mg \cos\theta}{m}$

or
$$a = g(\sin\theta - \mu_k \cos\theta)$$

We can measure the acceleration *a* of the block. Thus we can determine μ_k for the two surfaces. Note that mass of the block does not enter into the expression. This means that all masses will have the same acceleration as long as the co-efficients of friction are the same.

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11.9. ACCELERATION OF A BODY DOWN ROUGH INCLINED PLANE

An important type of one dimensional motion is the motion of an object on an inclined plane. Fig. 11.19 shows a block placed on a rough inclined plane; the angle of inclination being θ. If θ is greater than the angle of repose, the static friction changes to kinetic friction and the block slides down with an acceleration a. Referring to Fig. 11.19,

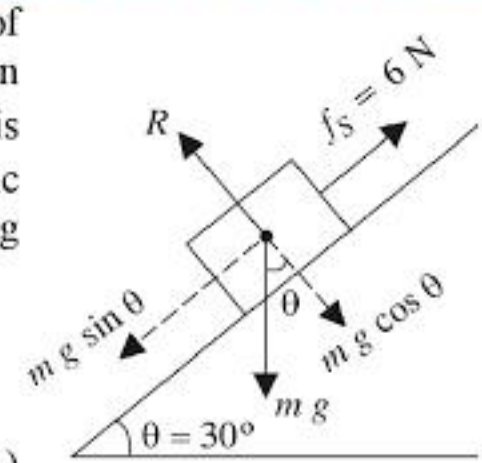


Fig. 11.19

$$R = m\ g\ \cos\theta$$

Net force on block down the plane = $mg\ \sin\theta\ - F_k$

or $\quad m\ a = mg\ \sin\theta\ - \mu_k\ mg\ \cos\theta\ \ (\because\ F_k = \mu_k\ mg\ \cos\theta\ )$

or $\quad a = g\ (\sin\theta - \mu_k\ \cos\theta\ )$

It is clear that $a < g$. *We arrive at a very important conclusion that acceleration of a body down the inclined plane is always less than the acceleration due to gravity.*

Note. If $\theta < \theta_C$ (angle of repose), the body will not slide down and force is required to move the body down the plane.

Minimum force required to move the body down the inclined plane

$$= F_k - mg\ \sin\theta$$

$$= mg\ \mu_k \cos\theta - mg\ \sin\theta$$

$$= mg\ (\mu_k \cos\theta - \sin\theta)$$

Minimum force required to move the body up the inclined plane

$$= mg\ \sin\theta + F_k$$

$$= mg\ \sin\theta + mg\ \mu_k \cos\theta$$

$$= mg\ (\sin\theta + \mu_k \cos\theta)$$

Example 11.9. A box of mass 4 kg is placed on a wooden plank of length 1.5 m which is lying on the ground. The plank is lifted from one end along its length so that it becomes inclined. It is noted that when the vertical height of the top end of the plank from the ground becomes 0.5 m, the box begins to slide. Find the coefficient of friction between the box and the plank.

Solution. Fig. 11.20 shows the conditions of the problem. The angle of the inclined plane at which the box placed on it just begins to slide is called the angle of repose. Therefore, $\angle AOB = \theta_C$ = Angle of repose.

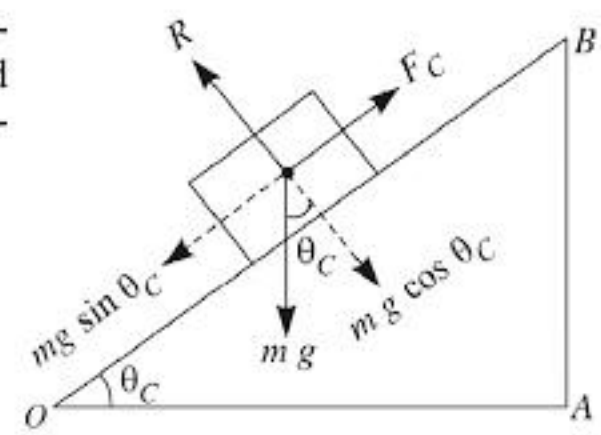


Fig. 11.20

Here $BA = 0.5$ m ; $OB = 1.5$ m

$$\therefore\ OA = \sqrt{OB^2 - BA^2} = \sqrt{(1.5)^2 - (0.5)^2} = \sqrt{2}$$

Therefore, co-efficient of friction μ is given by ;

$$\mu = \tan\theta_C = \frac{BA}{OA} = \frac{0.5}{\sqrt{2}} = \mathbf{0.35}$$

Example 11.10. A book that weighs 12 N is supported by an inclined plane. The coefficient of static friction between the book and the inclined plane is $\mu_S = 0.7$. What is the frictional force that acts on the book if the angle of inclination of the plane is 30° with the horizontal?

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Solution. Fig. 11.21 shows the conditions of the problem. The weight mg (= 12 N) of the book acts vertically downward. The normal force R is exerted by the surface on the book.

$$R = mg \cos\theta = 12 \cos 30° = 10.4 \text{ N}$$

The component of weight down the plane

$$= mg \sin\theta = 12 \sin 30° = 6 \text{ N}$$

The force of static friction can be as great as $\mu_S R$.

$$\therefore \quad \text{Limiting friction, } F_C = \mu_S R = \mu_S mg \cos\theta$$

$$= 0.7 \times 10.4 = 7.28 \text{ N}$$

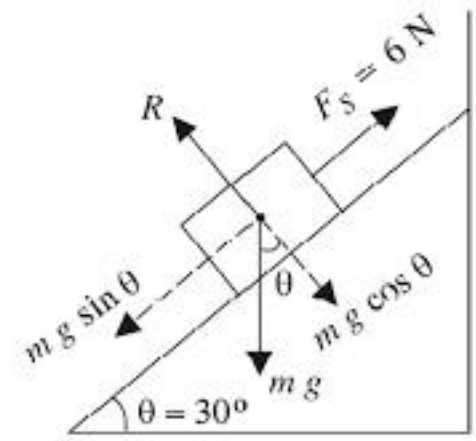


Fig. 11.21

However, the force along incline due to book's weight is only 6 N. Therefore, the force of static friction that acts on the book is also 6 N and acts upward along the incline.

$\therefore$ Frictional force on book = **6 N**

Example 11.11. A block of wood is supported on an inclined plane that makes an angle of 37° with the horizontal. The block is given a push so that it slides up with an initial speed of 5 ms^{-1}. The coefficients of static and kinetic friction between the block and inclined plane are μ_S = 0.8 and μ_k = 0.5.

(*i*) How far along the inclined plane will the block move before it comes to rest ?

(*ii*) Will the block slide back down the plane ?

Solution. Fig. 11.22 (*i*) shows the conditions of the problem.

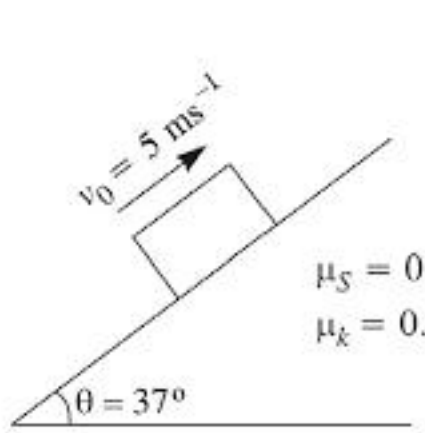
(*i*) Fig. 11.22 (*ii*) shows the conditions of the problem while the block is sliding upward along the plane. Let us take the positive x direction upward along the plane. When the block is moving up along the plane, the component $mg \sin\theta$ as well as kinetic friction F_k act down along the plane. Therefore, net force on the block is

$$\text{Net force on block} = (-mg \sin\theta) + (-F_k)$$

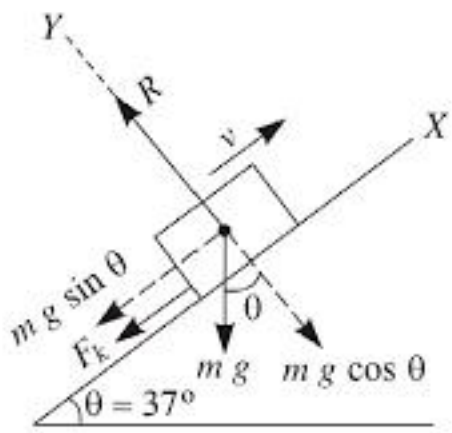
$$= -mg \sin\theta + (-\mu_k mg \cos\theta) \qquad (\because F_k = \mu_k R)$$

$$= -mg (\sin 37° + 0.5 \cos 37°)$$

$$\text{Acceleration of block, } a = \frac{-mg (\sin 37° + 0.5 \cos 37°)}{m} = -g$$



(*i*)



(*ii*)

Fig. 11.22

Now $v^2 - v_0^2 = 2aS$

Here $v_0 = 5 \text{ ms}^{-1}; v = 0 ; a = -g$

$$\therefore \quad S = \frac{0^2 - 5^2}{2 \times (-9.8)} = \mathbf{1.28 \text{ m}}$$

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(*ii*) When the block has reached the highest point, it is momentarily at rest and it is then the static friction that determines whether the block will remain at rest or slide down the plane. We know that maximum angle (θ_C) that an inclined plane can have without causing motion of an object resting on the plane is given by ;

$$\mu_S = \tan\theta_C$$

or $$\theta_C = \tan^{-1}\mu_S = \tan^{-1} 0.8 = 38.7°$$

Since the angle of inclined plane is 37° (*i.e.*, less than θ_C), **the block will not slide down** the plane but will remain at the point where it comes to rest.

Example 11.12. An object of weight W is resting on an inclined plane at an angle θ to the horizontal. The coefficient of static friction is μ. Find the horizontal force needed to just push the object up the plane.

Solution. Figure 11.23 shows the conditions of the problem. Let F be the horizontal force needed to just push the object up the inclined plane. The force F and weight W are resolved into rectangular components along the plane and perpendicular to the plane.

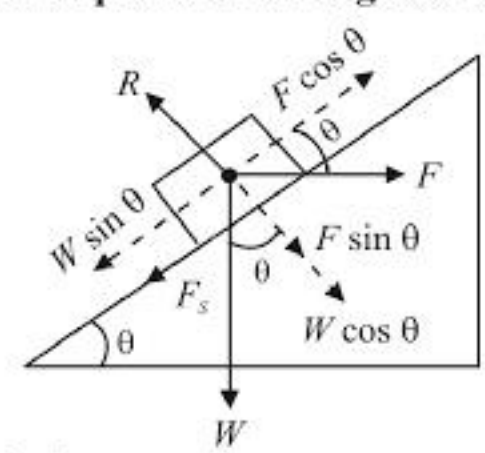


Fig. 11.23

Frictional force, $F_s = \mu R = \mu(W\cos\theta + F\sin\theta)$

Now $$F\cos\theta = W\sin\theta + F_s$$

or $$F\cos\theta = W\sin\theta + \mu(W\cos\theta + F\sin\theta)$$

or $$F\cos\theta - \mu F\sin\theta = W\sin\theta + \mu W\cos\theta$$

or $$F(\cos\theta - \mu\sin\theta) = W(\sin\theta + \mu\cos\theta)$$

$\therefore$ $$F = \frac{W(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}$$

Example 11.13. Find the force required to move a train of 2000 quintals up an incline of 1 in 50 with an acceleration of 2 ms^{-2}, the force of friction being 0.5 N/quintal.

Solution. Here m = 2000 quintals = 2000 × 100 kg ; $\sin\theta$ = 1/50

Force of friction, $F_f = 0.5 \times 2000 = 1000$ N

In moving up the inclined plane, the force required against gravity is

$$F_G = mg\sin\theta = (2000 \times 100) \times 9.8 \times 1/50 = 39200 \text{ N}$$

Force required to produce acceleration a (= 2 ms^{-2}) is

$$F = ma = (2000 \times 100) \times 2 = 400{,}000 \text{ N}$$

$\therefore$ Total force required = $F_f + F_G + F = 1000 + 39200 + 400{,}000 =$ **440,200 N**

Example 11.14. A block of mass 50 kg is placed on a rough inclined plane of angle 30°. If the coefficient of friction between the surface of the plane and the block is $1/\sqrt{3}$, find the values of maximum and minimum force along the plane to keep the mass in equilibrium.

Solution. Here m = 50 kg ; θ = 30° ; $\mu = 1/\sqrt{3}$

Force of friction, $F = \mu R = \mu\, mg\cos\theta$ ($\because R = mg\cos\theta$)

$$= (1/\sqrt{3}) \times 50 \times 9.8 \times \cos 30° = 245 \text{ N}$$

The component of weight acting down the plane is

$$P = mg\sin\theta = 50 \times 9.8 \times \sin 30° = 245 \text{ N}$$

To keep the block in equilibrium, the force required will be *maximum* when the block is moved up the plane and *minimum* when it is moved down the plane.

$\therefore$ Maximum force = $P + F = 245 + 245 =$ **490 N**

Minimum force = $P - F = 245 - 245 =$ **0 N**

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PROBLEMS FOR PRACTICE

1. A block rests on a rough incline whose angle θ with the horizontal can be varied. A trial shows that the block does not begin sliding down the incline until $\theta = 30°$. What is the co-efficient of static friction between the block and the surface of the incline ? **[0.58]**
2. Show that if a block slides at constant velocity down a plank that makes an angle θ_k with the horizontal, then the coefficient of friction between the block and the plank is
$$\mu_k = \tan\theta_k$$
3. When an automobile moving with a speed of 36 km/h reaches an upward inclined road of angle 30°, its engine is switched off. If the co-efficient of friction involved is 0.1, how much distance will the automobile move before coming to rest? Take $g = 10\ ms^{-2}$. **[8.52 m]**
4. A body placed on a rough inclined plane just begins to slide when the slope of the plane is 1 in 4. What is the value of coefficient of static friction? **[0.26]**
5. What is the coefficient of friction between a block and a plane inclined at 30° from the horizontal if the block just slides without accelerating when given an initial push? **[0.58]**

11.10. ROLLING FRICTION

When one body rolls over another, the resistance offered to motion is called rolling friction. It is generally much less than the sliding friction. Rolling friction results from the deformation produced where a wheel pushes against the surface on which it rolls. The deformation of an automobile tyre in contact with the pavement is readily visible. Even in case of a steel wheel rolling on a steel rail, there is some deformation of the surfaces. The deformation of the two surfaces produces rolling friction. The force of rolling friction varies inversely as the radius of the roller and it is less, the more rigid the surface.

Cause. When a wheel rolls on a level surface, the area of contact is very small and the pressure (weight/area) becomes very large. As a result, there is depression in the surface below as shown in Fig. 11.24. In climbing up the depression BC, the wheel encounters some opposition to motion. Further, the wheel also experiences opposition to motion in detaching itself from portion AB against the forces of adhesion. This causes the rolling friction. Experiments show that rolling friction is directly proportional to the normal reaction (R) and inversely proportional to the radius (r) of the rolling wheel.

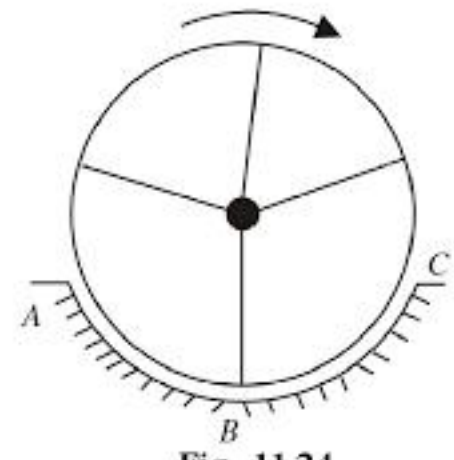


Fig. 11.24

$$\text{Rolling friction, } F_r = \frac{\mu_r R}{r}$$

where μ_r = coefficient of rolling friction

11.11. ADVANTAGES AND DISADVANTAGES OF FRICTION

Friction always opposes motion. It has advantages as well as disadvantages.

Advantages. Although we often view friction as an undesirable evil and go to great pains to reduce it, we would really be in deep trouble without it. The following examples illustrate this point.

(*i*) We would not be able to walk if there had been no friction between the soles of our shoes and the ground. While walking, we push on the ground and the force of friction acts in the opposite direction. In the absence of friction, for example, on icy surface, the person's shoes would slip when placed on the ground. The frictional force always opposes the motion of the shoes.

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