

ADVANCED MATHEMATICS

FOR PAPER 1



FOR FORM FIVE

CALCULATING DEVICES

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To my daughters Gracious and Grace



An electronic calculator is typically a portable electronic device used to perform calculations, ranging from basic arithmetic to complex mathematics.

The first solid-state electronic calculator was created in the early 1960s.

Pocket-sized devices became available in the 1970s, especially after the Intel 4004, the first microprocessor, was developed by Intel for the Japanese calculator company Busicom. They later became used commonly within the petroleum industry (oil and gas).

Modern electronic calculators vary from cheap, give-away, credit-card-sized models to sturdy desktop models with built-in printers. They became popular in the mid-1970s as the incorporation of integrated circuits reduced their size and cost. By the end of that decade, prices had dropped to the point where a basic calculator was affordable to most and they became common in schools.

Computer operating systems as far back as early Unix have included interactive calculator programs such as **dc** and **hoc**, and calculator functions are included in almost all personal digital assistant (PDA) type devices, the exceptions being a few dedicated address book and dictionary devices.

In 1986, calculators still represented an estimated 41% of the world's general-purpose hardware capacity to compute information. By 2007, this had diminished to less than 0.05%

In addition to general purpose calculators, there are those designed for specific markets. For example, there are scientific calculators which include trigonometric and statistical calculations like *Casio fx - 991*. Some calculators even have the ability to do compute algebra problems. Graphing calculators can be used to graph functions defined on the real line, or higher-dimensional Euclidean space such as *TI - 84 Plus CE*. As of 2019, basic calculators cost little, but scientific and graphing models tend to cost more.

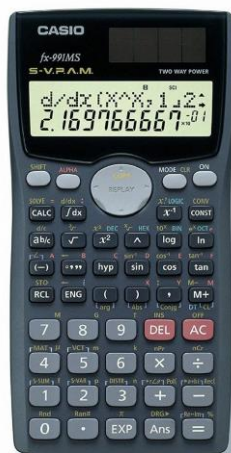
Electronic calculators contain keyboard with buttons for digits and arithmetical operations; some even contain "00" and "000" buttons to make larger or smaller numbers easier to enter. Most basic calculators assign only one digit or operation on each button (key); however, in more specific calculators, a button can perform multi-function working with key combinations.

MC or CM	Clear Memory or Memory Clear
MR, RM or MRC	Memory Recall
M-	Memory Subtraction
M+	Memory Addition
C or AC	Clear or All Clear
CE	Clear (last) Entry; sometimes CE/C: a first press clear the last entry (CE), a second press clear all (C)
± or CHS	Toggle positive/negative number or Change Sign.

Non – programmable scientific calculator

There are various type non – programmable scientific calculators. Some these calculators differ in abilities, some have many built-in functions and some has few functions. In this book, we will discuss Casio $fx - 991MS$ a non – programmable calculator.

The calculator is powered by a battery and the battery is charged by a small solar panel fixed on top of the calculator. If the calculator is original, the solar – panel must be working, otherwise is not original calculator. This calculator has 50 buttons (keys), each button is assign more than one work except the scroll button which is meant for displaying the previous or next results, shift, Alfa and – on – buttons.



The Shift and Alfa.

Beside each button (except the mentioned above) there is either a word or symbol or function written on a cover of the calculator. Those functionalities can be activated by starting with either shift or alfa depending on the colour of what is written. Alfa is written in purple colour, so to activate any button with a sub-function written in purple, start with alfa. Example, suppose we want to write letter x in a calculator

display, press the alfa button and then closing bracket button. Likewise, if we want to write pie (π) start with shift button and then **EXP** button.

Modes

Built – in functions of Casio – fx – 991MS. Mode button help in displaying several functions of the calculator. If mode button is pressed once the display shows “COMP” assigned number 1 and “CMPLX” assigned number 2. This means to select COMP press number 1 and to select CMPLX press number 2. CMPLX stands for complex numbers. If mode button is pressed twice, the display shows “SD, REG, BASE” which are assigned number 1, 2 and 3 respectively. To get the function you want press the number of the function.

Using non – programmable calculator

System of simultaneous equations

(a) Using the Casio calculator *fx – 991MS* can be used to solve system of equations of two unknowns and three unknowns.

To solve equations of the form
$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

Press mode 3 times, and select EQN (equation), select 2 unknowns, and then enter the coefficients of x and y and the constants.

Example 1

Use a non – programmable calculator to solve the system of equations.

$$\begin{cases} 3x + 5y = 30 \\ x - y = 2 \end{cases}$$

Solution

Press mode 3 times, select EQN and then 2 unknowns and then enter the coefficients and constants.

Given
$$\begin{cases} 3x + 5y = 30 \\ x - y = 2 \end{cases}$$

$a_1 = 3$, $b_1 = 5$ and $c_1 = 30$ then $a_2 = 1$, $b_2 = -1$ and $c_2 = 2$ after entering the coefficients the calculator will show the solution as $x = 5$ and $y = 3$

(b) Solving simultaneous equations with three unknowns, such as

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

press mode three times and select EQN and

then select three unknown and enter the coefficients and constants.

Example 2

Solve the following system using a non – programmable calculator

$$\begin{cases} 2x + 3y - 4z = -5 \\ x + 4y + 2z = 34 \\ 3x - 2y = -4 \end{cases}$$

Solution

Press mode three times and select EQN and then select 3 unknown and enter the coefficients and constants as follows, $a_1 = 2$, $b_1 = 3$, $c_1 = -4$ and $d_1 = -5$ then $a_2 = 1$, $b_2 = 4$, $c_2 = 2$ and $d_2 = 34$, then $a_3 = 3$, $b_3 = -2$, $c_3 = 0$ and $d_3 = -4$, at the end the results will be $x = 2$, $y = 5$, $z = 6$.

Note that, if the system of equations has many solutions or has no solution the calculator displays **Math ERROR**.

Suppose the following system is to be solved using a calculator

$$\begin{cases} 2x + y = 4 \\ 4x + 2y = 8 \end{cases}$$

this system of equations has infinity many solution, but the

calculator cannot solve because the determinant of the augmented

matrix is zero, like wise for the system of equations
$$\begin{cases} 2x + 3y + z = 5 \\ x - 3y + z = 4 \\ 4x + 6y + 2z = 7 \end{cases}$$
 the

system has no solution, therefore the calculator displays **Math ERROR** for both cases.

Solving degree 2 and 3 polynomial equations using a calculator

Example 3

Solve $x^2 - 3x + 2 = 0$ using a non – programmable calculator.

Solution

Press mode 3 times and then select EQN then scroll to the right to get degrees and choose degree 2,

Enter $a = 1$, $b = -3$ and $c = 2$, for the quadratic with two distinct roots the calculator display $x_1 = 2$ and $x_2 = 1$.

Example 4

Solve $16x^2 - 8x + 1 = 0$ using a non – programmable calculator.

Solution

Select degree 2 and enter the coefficients $a = 16$, $b = -8$ and $c = 1$ this equation has repeated root, therefore the calculator display $x = 0.25$.

Example 5

Solve $3x^2 + 2x + 5 = 0$, using a non – programmable calculator.

Solution

If the quadratic equation has complex roots, the calculator displays as repeated roots just because complex numbers occur in pairs, to display the imaginary part of the root press shift and equal sign (=), the roots

are $x_1 = -0.3 + 1.25i$ and $x_2 = -0.3 - 1.25i$

To know that the roots are complex the Casio calculator shows $R \Leftrightarrow i$ at the top right corner of the calculator display.

Example 6

Solve $3x^3 - 2x^2 - 3x + 2 = 0$ using a non – programmable calculator.

Solution

Choose EQN and select degree 3 and then enter the coefficients, the results are $x_1 = 1$, $x_2 = -1$ and $x_3 = 0.6$

Example 7

Solve $2x^3 + 3x^2 - 1 = 0$ using a non – programmable calculator

Solution

The results are $x_1 = 0.5$ and $x_2 = -1$, this cubic equation has two roots instead of three roots, this means that one of the root is a repeating root. Before concluding that the roots are repeating, check if there is $R \Leftrightarrow i$ at the right top corner of the display, if it is there the roots are complex roots.

Example 8

Solve $5x^3 - 4x^3 + 5x - 4 = 0$ using a non – programmable calculator

Solution

The roots are $x_1 = 0.8$, $x_2 = i$ and $x_3 = -i$.

Dealing with matrices using calculator

Matrices are rectangular arrangement of numbers in to rows and columns. The horizontal arrangement are called rows and the vertical arrangement are called column.

Using a calculator Casio *fx-991MS* it is possible to add, subtract, multiply, find determinant, transpose inverse of matrices.

If mode key is pressed three times, the built-in function written MAT is assigned number 2. MAT is for Matrices.

To enter the matrix in a calculator, activate MAT mode and then shift and press number 4, select dim (number 1) to enter the matrix or Edit (number 2) to edit the existing matrix or select Mat (number 3) to display the matrix which already in a calculator. If your calculator has no matrix, select dim (dimension), then select the name of your matrix (A, B or C), after naming your matrix, enter the dimension of your matrix (this is the size of the matrix). Here m is number of rows and n is number

of columns. Suppose the matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, name the matrix

A in your calculator, MatA(m × n), put 3 for “m” and press =, and for “n” put 3 and press =. The calculator display shows MatA11, this means the first element of first row and first element of first column. After you insert any element press = to enter the number to the matrix. Enter elements following the rows and **not** columns

Example 9

If $A = \begin{pmatrix} 9 & 1 \\ -3 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & 7 \\ 5 & -2 \end{pmatrix}$, use a calculator to evaluate the following:-

- | | |
|-----------|------------------|
| (a) $A+B$ | (d) $\det(A)$ |
| (b) $B-A$ | (e) $\det(A+2B)$ |
| (c) AB | |

Solution

Select Matrix mode, choose dim, and name it A, the size is 2 by 2 (m = 2 and n = 2).

Enter matrix $A = \begin{pmatrix} 9 & 1 \\ -3 & 8 \end{pmatrix}$, after you finish matrix A, Enter $B = \begin{pmatrix} 8 & 7 \\ 5 & -2 \end{pmatrix}$,

shift number 4, choose dim, and name it B, the size is 2 by 2 (m = 2, n = 2). Now your calculator has to matrices named A and B.

- (a) To add the matrices, press shift, then 4 then Mat then A, (The calculator shows (**MatA**), then press + (addition) and then press shift then 4, then mat and then B. To display the result press = and use the scroll key to get other elements and write the elements accordingly.

$$A+B = \begin{pmatrix} 17 & 8 \\ 2 & 6 \end{pmatrix}$$

- (b) Follow the procedures above and show that $B-A = \begin{pmatrix} -1 & 6 \\ 8 & -10 \end{pmatrix}$

- (c) Replace + in (a) above by × and the result is $AB = \begin{pmatrix} 77 & 61 \\ 16 & -37 \end{pmatrix}$

- (d) To a determinant of a matrix, press shift then 4 and scroll to the right and choose **Det**, press shift number 4 and choose **mat** and select **A**, so that the calculator shows **Det MatA**, press = sign to get the determinant, $\det(A) = 75$
- (e) Press shift then 4 then scroll find det, open brackets, repeat by pressing shift then 4, press 3 then A press +, press 2 then shift number 4, then mat then mat then B and close brackets. The result is $\det(A + 2B) = -5$.

Example 10

Use a non – programmable scientific calculator to evaluate the following

$$(a) \begin{vmatrix} 2 & 3 & 1 \\ 0 & 2 & 8 \\ 2 & 4 & 7 \end{vmatrix} \quad (b) \begin{pmatrix} 1 & 2 & 0 \\ -8 & 3 & -4 \\ 0 & 3 & 4 \end{pmatrix} \times \begin{pmatrix} 8 & 7 & 9 \\ -3 & -7 & 8 \\ 8 & 5 & 2 \end{pmatrix}$$

Solution

$$(a) \text{ Determinant } \begin{vmatrix} 2 & 3 & 1 \\ 0 & 2 & 8 \\ 2 & 4 & 7 \end{vmatrix} = 8$$

$$(b) \begin{pmatrix} 1 & 2 & 0 \\ -8 & 3 & -4 \\ 0 & 3 & 4 \end{pmatrix} \times \begin{pmatrix} 8 & 7 & 9 \\ -3 & -7 & 8 \\ 8 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -7 & 25 \\ -105 & -97 & -56 \\ 23 & -1 & 32 \end{pmatrix}$$

Example 11

If $B = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$ find (a) B^{-1} (b) BB^T

Solution

To find the inverse enter the matrix to the calculator and select it and press $[x^{-1}]$ to find its inverse.

$$(a) \text{ Inverse of } B = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix} \text{ is } B^{-1} = \begin{pmatrix} 1 & -0.5 \\ -2 & 1.5 \end{pmatrix}$$

$$(b) B^T \text{ is the transpose of matrix } B \text{ therefore } BB^T = \begin{pmatrix} 10 & 14 \\ 14 & 20 \end{pmatrix}$$

PRACTICE

Use a non-programmable calculator to solve the following system of equations

- | | |
|--|--|
| 1. $\begin{cases} 4x - 3y = 10 \\ 3y + x = 5 \end{cases}$ | 2. $\begin{cases} x + 2y = 5 \\ 1.5x + 3y = 4 \end{cases}$ |
| 3. $\begin{cases} 10x_1 + 2x_2 = 11 \\ x_1 + 4x_2 = 3 \end{cases}$ | 4. $\begin{cases} 2x + 3y = 10 \\ x - y = -2 \end{cases}$ |

Use a non-programmable calculator to solve the following equations, where necessary put your answer in 4 decimal places.

- | | |
|----------------------------|-------------------------------|
| 5. $3x^2 + 2x - 4 = 0$ | 6. $2x^2 + 7x + 6 = 0$ |
| 7. $4x^2 + 4x + 5 = 0$ | 8. $3x^2 - 2x + 4 = 0$ |
| 9. $3x^3 + x^2 - 3x = 1$ | 10. $x^3 + 3x^2 - 6x - 8 = 0$ |
| 11. $8y^3 + 6y^2 - 3y = 1$ | 12. $x^3 - x^2 - 4x + 4 = 0$ |

Use a non-programmable calculator to evaluate the following matrices operation

13. If $A = \begin{pmatrix} 2 & -4 & 4 \\ 0 & 1 & 9 \\ 10 & 11 & 0 \end{pmatrix}$ evaluate

- (a) $\det(A)$ (b) A^{-1} (c) $A^{-1}A^T$

14. If $B = \begin{pmatrix} 7 & 8 & 1 \\ 10 & 7 & -8 \\ 2 & 3 & 6 \end{pmatrix}$ and $C = \begin{pmatrix} 8 & 0 & 9 \\ 2 & 4 & 7 \\ 2 & 9 & 5 \end{pmatrix}$ evaluate

- (a) $B + 2C$ (b) $2(B + C^{-1})$
 (c) $BC + CB$ (d) $BC + C^2$

Vectors using a calculator

Likewise, matrices, vectors can be entered into a calculator by pressing mode 3 times and selecting VCT then after, press shift then number 5 and choose 1 to enter a new vector or number 2 to edit a vector or number 3 to display the vector.

Using a calculator, it is possible to find the addition, subtraction, cross product of vectors, to find dot product of vectors and to find the modulus/absolute/length of the vector.

Example 12

Given $\underline{v} = 3\underline{i} + 5\underline{j} - 5\underline{k}$ and $\underline{u} = 4\underline{i} + 6\underline{j} + 2\underline{k}$ find

- | | |
|--|---|
| (a) $\underline{v} + 3\underline{u}$ | (b) $4\underline{v} - 3\underline{u}$ |
| (c) $\underline{v} \times \underline{u}$ | (d) $\underline{v} \cdot \underline{u}$ |
| (e) $ \underline{v} + 2\underline{u} $ | (f) $3 \underline{v} $ |

Solution

- (a) Enter both vectors in a calculator, ($m = 3$) dimension of the calculator.

$$\text{Then } \underline{v} + \underline{u} = 15\underline{i} + 23\underline{j} + \underline{k}$$

- | | |
|--|---|
| (b) $4\underline{v} - 3\underline{u} = 2\underline{j} - 26\underline{k}$ | (c) $\underline{v} \times \underline{u} = 40\underline{i} - 26\underline{j} - 2\underline{k}$ |
| (d) $\underline{v} \cdot \underline{u} = 32$ | (e) $ \underline{v} + 2\underline{u} \approx 20.273$ |
| (f) $3 \underline{v} \approx 23.043$ | |

Statistics calculations using a non-programmable scientific calculator

Casio *fx-991MS* can be used to evaluate some of the statistics measures, these are mean, standard deviation, sum of the squares of the data, sum of the data and so on.

To be able to do the statistics calculation using a calculator, select SD (Standard Deviation) mode, enter the data, after every entry press M+ or DT (Data) button. At the end of your data, the calculator will show the total number of data, (n).

Example 13

Given the following numbers

100 201 150 200 112 123 145 140 120 130.
Find

- (a) Mean
- (b) Standard deviation
- (c) Variance
- (d) Sum of the square of the data
- (e) Sum of the data

Solution

Select SD mode, enter one number after the other while pressing DT for each entry.

- (a) $\bar{x} = 142.1$
- (b) $x\delta n = 32.51907133$
- (c) Variance = 1057.49
- (d) Sum of the square of the data = 212499
- (e) Sum of the data = 1421

Example 14

Given 55 54 51 55 53 53 54 52 51
 57 57 57 58 59 52. Find

- (a) Mean
- (b) Standard deviation
- (c) Variance
- (d) Sum of the square of data
- (e) Sum of the data

Solution

Enter data in your calculator.

Note that: to enter with 2 frequencies, press DT, DT. To enter a number with 4 frequencies, press DT four times.

- (a) $\bar{x} \approx 54.53$
- (b) Standard deviation, $x\delta n \approx 2.4998$
- (c) Variance ≈ 6.24889
- (d) Sum of the square of data, $\sum x^2 = 44702$

(e) Sum of the data, $\sum x = 818$.

Example 15

Given the frequency distribution table below

Data	10.3	10.5	10.7	10.9	11.1	11.3	11.5	11.7
Frequency	7	11	16	20	18	14	10	4

Find from the table

- (a) Mean
- (b) Standard deviation
- (c) Variance
- (d) Sum of the square of the data
- (e) Sum of the data

Solution

To enter numbers with frequencies given as in the table above, use the format $x;f$ where x is the data and f is its frequency.

Write 10.3 shift then ; then 7 (it frequency) and then press DT. Doing that, the answers are:-

- (a) Mean, $\bar{x} = 10.966$
- (b) Standard deviation, $\sigma_n = 0.3688414293$
- (c) Variance = 0.136044
- (d) Sum of square of numbers, $\sum x^2 = 12038.92$
- (e) Sum of the data, $\sum x = 1096.6$

Exponential display formats, fix and sci uses in a casio calculator

Casio *fx-991MS* calculator can display up to 10 numerals. If the answer exceeds 10 digits the answer is displayed in exponential forms.

FIX: Fix is used to set number of decimal places to be contained in the final answer.

SCI: Sci is used to set number of significant figures to be contained in the final answer.

Both FIX and SCI have 0 to 9 possible decimal places.

Example 16

Evaluate the following

(a) $\frac{\sqrt{20.17+4.34}}{\sqrt[3]{50.91}}$ correct to 4 decimal places

(b) $\frac{\sqrt{20.17+4.34}}{\sqrt[3]{50.91}}$ correct to 4 significant figures

Solution

(a) It is possible to write $\frac{\sqrt{20.17+4.34}}{\sqrt[3]{50.91}}$ in a calculator but it is advised

to evaluate one terms after the other following rule of BODMAS.

Evaluate $\sqrt{20.17} = 4.491102315$

Press Ans + 4.34 = 8.831102315

Press Ans $\div \sqrt[3]{50.91} = 2.382761371$

Therefore, the answer is 2.3828 (correct to 4 decimal places)

(b) $\frac{\sqrt{20.17+4.34}}{\sqrt[3]{50.91}} = 2.383$ (Correct to 4 significant figures)

Note the differences between the two answers, this first answer (a) has 4 decimal places while the second (b) has 3 decimal places but 4 significant figures.

Degrees and Radians

When dealing with trigonometrical functions, great care should be taken to change the calculator from Degree mode to radian mode or grad mode, it depends on your requirements.

Suppose you want to write $23^{\circ}45'57''$ is your calculator. Press 24 then the degree key, 45 press the degree key and 57 press the degree key, your screen shall look like $24^{\circ}45'57''$, no worry, the calculator knows to differentiate between them.

Example 17

Evaluate $238.98 + \sin^{-1}(0.3453)$ correct to 7 decimal places

Solution

Because of $\sin^{-1}(0.3453)$ we need to change the calculator mode to radians.

$$\text{So } 238.98 + \sin^{-1}(0.3453) = 239.3325584$$

Example 18

Evaluate $\frac{3.56 \times \tan^{-1}(0.1289)}{\cos^{-1}(0.2957) \times 45}$ give your answer correct to 5 significant figures

Solution

Calculator in radian mode: $\frac{3.56 \times \tan^{-1}(0.1289)}{\cos^{-1}(0.2957) \times 45} = 0.0079816$

Example 19

Evaluate $\frac{\sin(48^\circ 56') + 4.59}{0.1329}$ correct to 6 decimal places.

Solution

For this expression we need the calculator to be in degree mode.

$$\frac{\sin(48^\circ 56') + 4.59}{0.1329} = 40.210276$$

Example 20

Evaluate $2.798\text{rad} + 36^\circ 47'$ give your answer in

- (a) Degrees, minutes and seconds
- (b) Radian correct to 5 significant figures.

Solution

- (a) Let convert 2.798 rad to degree, Change the mode of your calculator to degree mode. Enter 2.798 and press shift then Ans button to get D R G, then select number 2 (R) for radian (You are telling the calculator that the number you've entered is radian) then after, press = to get 160.3135911 or $160^\circ 18' 49''$ then now add. Therefore $2.798\text{rad} + 36^\circ 47' = 197^\circ 05' 49''$
- (b) Change $36^\circ 47'$ to radians. Change the calculator mode to radians and enter $36^\circ 47'$, press shift then Ans to get D R G, and select

number 1 (D) and press = to display the answer which is 0.641990276. Now add.

$$2.798\text{rad} + 36^\circ 47' = 3.43999 \text{ rad}$$

Microsoft excel 2010 and earlier

Microsoft excel is a product in Microsoft office. Excel is a great tool to use for data collection and entry, and even to use for some derivation of other columns. However, Excel IS NOT the best tool to use to conduct advanced analyses, especially statistical analyses.

In Excel, the calculation can be specified using either a formula or a function.

(a) Formulas are self-defined instructions for performing calculations.

(b) Functions are pre-defined formulas that come with Excel.

In either case, all formulas and functions are entered in a cell and must begin with an equal sign '='. After the equal sign, a formula includes the addresses of the cells whose values will be manipulated with appropriate operands placed in between. The operands are the standard arithmetic operators.

Table below summarize the operations in excel

Operator	Meaning	Example
+	Addition	=(A3+B3)
-	Subtraction	=(A3 – B3)
*	Multiplication	=(A3*B3)
/	Division	=(A3/B3)
^	Exponents	=(A3^B3)

Excel spreadsheet has cells and each cell has a unique name which is the combination of letter and number of top row and the left most column.

	A	B	C	D	E	F	G
1						x	
2							
3			<i>b</i>				
4		<i>a</i>					
5				<i>c</i>			
6							<i>y</i>
7		<i>e</i>	<i>d</i>		<i>f</i>		

The cells above are:-

The cell with letter “a” is B4

The cell with letter “f” is E7

The cell with letter “d” is C7

Question

Name the cells with letter *b*, *c*, *e*, *x* and *y*

To be able to evaluate any formula or function, the cells address is very important for the calculations to be automatic.

Example 21

Suppose the following data are contained in excel spreadsheet as follows, A1 = 23, B1 = 34, C1 = 12, D1 = 44, E1 = 8, F1 = 24, G1 = 20. Write down the formula in H1 to find:-

- (a) The total
- (b) The average (mean)
- (c) The standard deviation.

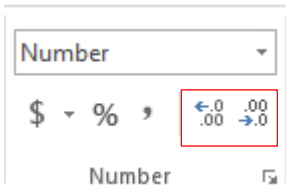
Solution

	A	B	C	D	E	F	G
1	23	34	12	44	8	24	20
2							

- (a) =SUM(A1:G1) enter
- (b) =AVERAGE(A1:G1) enter
- (c) =STDEV.S(A1:G1) enter

Approximation or rounding of values

To reduce decimal places on a number click on the decrease decimal button. Each click of this button reduces the number by one decimal place. Although the result shown might be rounded off to certain number of decimal places, but excel maintains an internal numeric precision of 15 numerals. When the excel formula is applied, excel looks at the actual value held in the cell, not the displayed number.

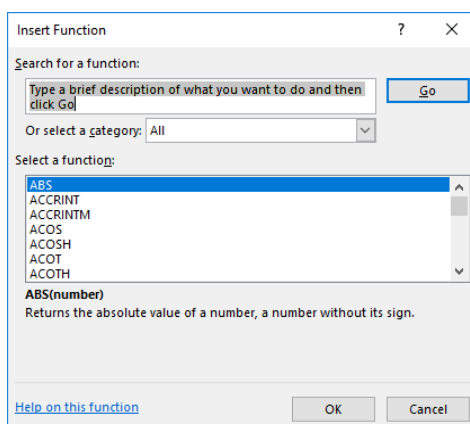


Suppose cells B1 has 22 and C1 has 7, if D1 has $= (B1/C1)$ then the result is 3.1429, approximated correct to 4 decimal places as shown. If another cells B2 has 33 and C2 has 7 and D2 has $= (B2/C2)$ then the result is 4.7143 approximated to 4 decimal places in excel as shown. If the sum of the results $= \text{SUM} (D1+D2)$ is computed by excel, excel refers the results of D1 as 3.14285714285714 and D2 as 4.71428571428571 so the sum will be 7.85714285714286.

Functions in excel

If you click on *fx insert Function* or of *fx* close to the formula bar a pop-up in form of dialog box will appear with several functions, in which you may search for a formula or select a category. If you are not sure with categories, select all.

You may choose any function according to what you want to do.



Relative cell reference

In relative reference the row and column references can change when you copy the formula to another cell because the references are actually offsets from the current row and column. Relative cell reference is a cell address without dollar sign (\$) in the row and column coordinates. The formula changes based on a relative position of rows and columns. By default, all references in excel are relative.

[Study more in your ICT class or find a book in this series called “EXCEL in EXCEL, written by Baraka Loibanguti, 2020 edition].

Maple 18 software

Maple 18 software is a powerful system that you can use to solve mathematical problems. *You can also create professional quality documents, presentations, and custom interactive computational tools in the Maple environment.*

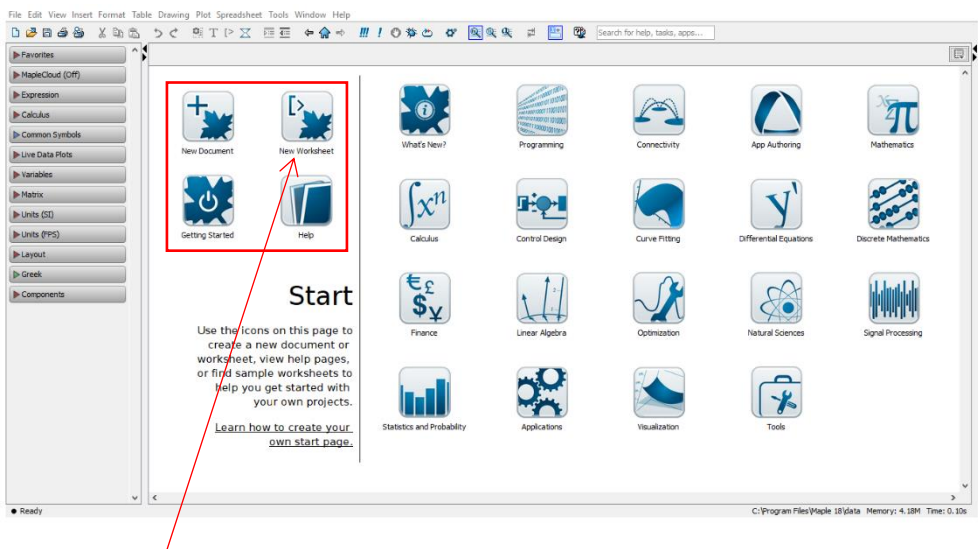
Introduction

To start Maple in Standard mode click on the red

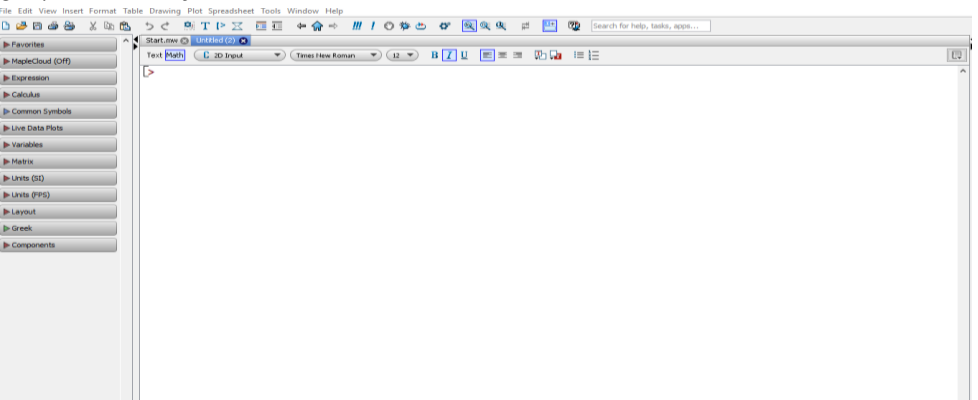


Maple icon

this opens the maple window below.



Click “New worksheet” to start. This action will open the *standard graphical interface*.

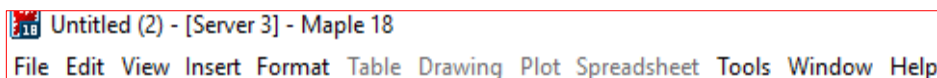


In this interface we have:-

Menu bar

Use the mouse to point and click to open the different menus. Take some time to investigate what each one does.

File menu



From the File Menu you can create **a new Maple file, open an existing file, save and print your work.** To save your work you will click on Save from the File Menu above. The Save Menu will pop up. Save your work with file extension **.mw** so as to identify your file as a Maple worksheet. You should save your work often so that you will not lose too much of it in case the computer crashes.

Help menu;

Maple has an excellent help system. The full text search allows you to type one or more words and you can use it to check command syntax or even to check if a particular function, feature or command even exists. Be curious, explore!

Tool bar



This changes depending on which input mode you are.

The font and style bar



By default Maple display its command **[>]** prompt: And you will see the blinking cursor just to the right of it. Maple expects you to enter a command.

Anything you type will be displayed in red and it will be interpreted as a Maple command or mathematics.

To switch between text and command mode

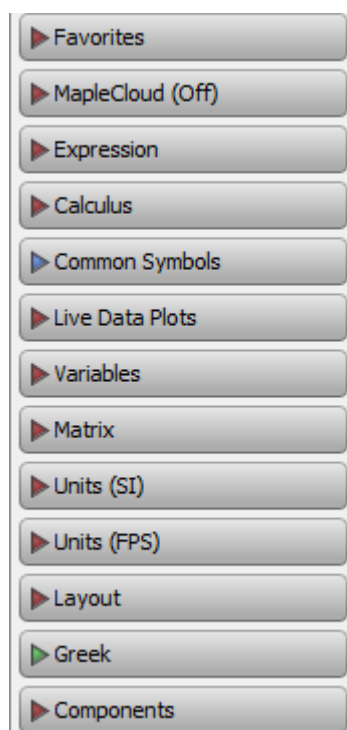
If you type normal text at the command prompt you will get an error message. You must go into the “text mode” by either clicking the Text Icon “T” from the tool bar or by pressing and holding the control key and then pressing the letter T from the keyboard [Ctrl + T]. You can always go back to the “command mode” by clicking the Command Icon [> from the toolbar, or by pressing and holding the control key and then pressing the letter M [Ctrl + M].

Palette Menu

This menu contains the most common mathematical operators, symbols, and expressions. When working in a project using Maple standard worksheet mode, you should always have the following palettes active and ready to use: “Expression”, “Calculus”, and “Common Symbols” etc. To active these palettes just click on their respective names.

Important notes

1. Every command in Maple must end with a semicolon [;]. The semicolon assures that the computer will execute your instruction. To execute your instructions you must press the Enter key.
2. Maple is case sensitive. For example, x is not the same as X because Maple distinguishes between upper case and lower case letters, and it will take them as being two different variables.



3. Always type Maple commands in lower case, unless your instructor tells you otherwise. In such cases be very careful to type everything exactly as your instructor tells you.
4. Errors can be corrected by moving a cursor to the point of correction and edit it, since maple work as a text editor, you can always make correction of the previous commands.

Mathematical computation in maple 18

In this section you will learn how to use Maple to do some standard numerical calculations. When solving problems Maple will give you an exact answer or a numerical approximation. You can use the standard characters found in a typical computer keyboard, and/or use the “Expression” in palette.

Addition (+) and subtraction (-)

Suppose we want to evaluate $12 + 5$ using maple. In a command line type the expression $12 + 5$; and then press enter key.

```
> 12 + 7;
```

19

As seen in the above screenshot, maple gave the answer as 19

Suppose the we wish to evaluate $102 - 50$, in a command line type, $102 - 50$; and press enter key, see the extract below

```
> 102 - 50;
```

52

Multiplication (*) and Division (/)

If we want to evaluate $12 * 12$, type $12 * 12$; and press enter key (see the extract below)

```
> 12·12;
```

144

Note that, the $*$ is multiplication sign, but maple changes the sign to dot ($\cdot$) as seen in $12 \cdot 12$.

To $30 \div 6$ in maple, type $30 / 6$; then press enter key. See below

When working with division, the sign $/$ expressed to fraction, see below

$$> \frac{30}{6};$$

5

Fractions in maple

To evaluate fractions in maple, use the sign $/$ (forward slash or division)

Suppose we wish to evaluate $\frac{7}{8} + \frac{5}{8}$, type it $\frac{7}{8} + \frac{5}{8}$; in maple and press enter key as shown below.

$$> \frac{7}{8} + \frac{5}{8};$$

$\frac{3}{2}$

Maple gave out the answer as $\frac{7}{8} + \frac{5}{8} = \frac{3}{2}$

By default Maple leaves the answer in fractional form. To change the answer to decimal use the command ***evalf()***. See below

$$> \text{evalf}\left(\frac{3}{2}\right);$$

1.500000000

By default Maple displays the answer using up to 10 digits, but you can specify how many digits to be displayed as follows

$$> \text{evalf}\left(\frac{3}{2}, 2\right);$$

1.5

Notice the difference between the first ***evalf()*** and the second ***evalf()*** commands.

Likewise, the expression $\frac{2}{7} \div \frac{1}{11}$, can be $\frac{2}{7} \div \frac{1}{11}$; the answer is as shown below

$$> \frac{\frac{2}{7}}{\frac{1}{11}};$$

$$\frac{22}{7}$$

To change the answer to decimal, with 30 digits use $\text{evalf}\left(\frac{22}{7}, 30\right)$. See below

$$> \text{evalf}\left(\frac{22}{7}, 30\right);$$

3.14285714285714285714285714286

This gives out the answer with 30 digits
(3.14285714285714285714285714286)

It is possible to use the previous result for the next computation processes. To do this we use **ditto** operator [%] see below

$$> \frac{4}{7} + \frac{5}{9};$$

$$\frac{71}{63}$$

$$> \text{evalf}(\%, 6);$$

1.12698

Maple evaluated $\frac{4}{7} + \frac{5}{9}$ and gave out $\frac{71}{63}$, the second command line uses the **ditto** operator to instruct maple to change the previous results to decimals with 6 digits.

Note that, any line which starts with # is called a **comment** in maple, maple ignore anything starting with the sign #. See below

$$> 4.8 + \frac{7}{11} - 6; \# \text{ This is a comment}$$

-0.563636364

Comments are used to help the reader to understand the command.

Exponential evaluation

The sign “^” in a computer keyboard helps in writing exponential numbers. Suppose we want to evaluate 45^7 , see below

```
> 457;  
373669453125
```

Square roots and absolute

This expressions can be evaluated using the command or be selected from palette menu.

```
> sqrt(9.1);  
3.016620626
```

Absolute of a number -7 is 7. See below

```
> abs(-7);  
7
```

Note that, all the evaluation done above are all available in palette menus. Expressions like **factorials**, **logarithms**, **nth roots** and many more are available in palette menus.

ALGEBRAIC COMPUTATION IN MAPLE

All of the arithmetic operators discussed before can be used with variables and algebraic expressions, just the same way as used in numbers.

```
> 3x + 7y - 8x + 2y + 10; # Maple collects like terms  
-5x + 9y + 10
```

Multiply polynomials

Use the command “*expand()*”. Using maple we can multiply polynomials as follows. Suppose we want to expand $(x+3)(2x-4)$, in maple we should write as $(x+3)*(2x-4)$;

```
> expand((x + 3) · (2x - 4)); # Maple expand brackets  
2x2 + 2x - 12
```

Suppose we want to expand $(x+2)^7$, see below

- (a) $\text{> expand}((x+2)^7);$
 $x^7 + 14x^6 + 84x^5 + 280x^4 + 560x^3 + 672x^2 + 448x + 128$
- (b) $\text{> expand}((a-b)^6);$
 $a^6 - 6a^5b + 15a^4b^2 - 20a^3b^3 + 15a^2b^4 - 6ab^5 + b^6$

Factorizing polynomials

Use the command “*factor()*”

Suppose we want to factorize the expression $x^3 + x^2 - 4x - 4$

- (a) $\text{> factor}(x^3 + x^2 - 4x - 4);$
 $(x-2)(x+2)(x+1)$
- (b) $\text{> factor}(x^3 - 3x^2 + x - 3);$
 $(x-3)(x^2+1)$

Simplifying polynomials

Use the command “*simplify()*”

- (a) $\text{> simplify}\left(\frac{x^3 + x^2 - 4x - 4}{x+1}\right);$
 $x^2 - 4$
- (b) $\text{> simplify}\left(\frac{a^3b^2 - 2a^2b^3}{3a^3b^2 + a^2b^3}\right);$
 $\frac{a-2b}{3a+b}$

We can assign a name to the result of a calculation or to an algebraic expression that we intend to use in subsequent calculations. To assign a name we can use a colon followed by an equal sign $[:=]$. When making your assignment, do not forget that Maple is case sensitive so try to use lower case letters. Keep the names short.

See below

$\text{> } f := x^2 + 5x + 6; \# \text{ the expression } x^2 + 5x + 6 \text{ is assigned letter } f$
 $f := x^2 + 5x + 6$

Factorizing $f := x^2 + 5x + 6;$

> factor(f); # referring to f above, maple factorize f

$$(x + 3) (x + 2)$$

Substitution, solving and unassigned letters

Suppose you had a function $p := 2x^2 + 7x + 3$, and you want to substitute $x = 10$ in p . This uses the command `subs()`; this command has two parameters, such that `subs(x, p)`; see below

(a) > $p := 2x^2 + 7x + 3$; # Assign "p" to the polynomial $2x^2 + 7x + 3$

$$p := 2x^2 + 7x + 3$$

> `subs(x = 10, p)`; # substitute $x = 10$ in "p"

$$273$$

(b) > $f := x^{10} - 2x^9 + 100$; # Assign $x^{10} - 2x^9 + 100 \rightarrow f$

$$f := x^{10} - 2x^9 + 100$$

> `subs(x = 3, f)` # Evaluate f when $x = 3$

$$19783$$

Solving polynomial equations in maple

To solve equation in maple use the command `solve()`;

Suppose we want to solve p above;

(a) > $p := 2x^2 + 7x + 3$;

$$p := 2x^2 + 7x + 3$$

> `solve(p = 0)`;

$$-\frac{1}{2}, -3$$

This give the values of x which are the solutions of the above polynomial. To be specific in our solution, use the command and parameters `solve (p = 0, {x})`; this specify values of x .

> $p := 2x^2 + 7x + 3$;

$$p := 2x^2 + 7x + 3$$

> `solve(p = 0, {x})`;

$$\left\{ x = -\frac{1}{2} \right\}, \{ x = -3 \}$$

To unassign p

> $p := 2x^2 + 7x + 3;$

$$p := 2x^2 + 7x + 3$$

> $p;$ #Check the value of p

$$2x^2 + 7x + 3$$

> $p := 'p';$

$$p := p$$

> $p;$ #Check the current value of p

$$p$$

(b) Suppose we want to solve for x in $4x + 10y = 13$, see below

> $f := 4x + 10y = 13;$

$$f := 4x + 10y = 13$$

> $\text{solve}(f, \{x\});$

$$\left\{ x = \frac{13}{4} - \frac{5}{2}y \right\}$$

Solving simultaneous equations

Suppose we want to solve $\begin{cases} 3x + 4y = 15 \\ 4x - y = 1 \end{cases}$ see below

> $\text{solve}(\{3x + 4y = 15, 4x - y = 1\}, \{x, y\});$

$$\{x = 1, y = 3\}$$

Solving absolute value equation

Solve $|x - 5| = 7$

> $\text{solve}(\text{abs}(x - 5) = 7, \{x\});$

$$\{x = 12\}, \{x = -2\}$$

Solving absolute inequalities $|x - 5| \geq 7$

> $\text{solve}(\text{abs}(x - 5) \geq 7, \{x\});$

$$\{x \leq -2\}, \{12 \leq x\}$$

Note that; To enter $\geq$ use $>=$ from the keyboard or select it from palette menu.

Packages in maple

Maple offers some very useful commands that are found in packages. A package is a group of routines or built-in functions related to a particular area of mathematics or a set of commands designed to help a particular audience. For example, the student package was designed with students like you in mind. In order to access commands contained in a package, you must use the *with* command first. Every time you see the command *with*, you should know that we are using a package. For example, to use the commands contained in the student package you must type *with(student);* as shown below. Example of the package is student package

```
> with(student);
```

```
[D, Diff, Doubleint, Int, Limit, Lineint, Product, Sum, Tripleint, changevar, completesquare,  
distance, equate, integrand, intercept, intparts, leftbox, leftsum, makeproc, middlebox,  
middlesum, midpoint, powsubs, rightbox, rightsum, showtangent, simpson, slope,  
summand, trapezoid]
```

You can choose whatever you want to use any command in this package. The other packages are *with (geometry); with (statistics), with (plots)*, see below

```
>with(geometry);
```

[Apollonius, AreCollinear, AreConcurrent, AreConcyclic, AreConjugate, AreHarmonic, AreOrthogonal, AreParallel, ArePerpendicular, AreSimilar, AreTangent, CircleOfSimilitude, CrossProduct, CrossRatio, DefinedAs, Equation, EulerCircle, EulerLine, ExteriorAngle, ExternalBisector, FindAngle, GergonnePoint, GlideReflection, HorizontalCoord, HorizontalName, InteriorAngle, IsEquilateral, IsOnCircle, IsOnLine, IsRightTriangle, MajorAxis, MakeSquare, MinorAxis, NagelPoint, OnSegment, ParallelLine, PedalTriangle, PerpenBisector, PerpendicularLine, Polar, Pole, RadicalAxis, RadicalCenter, RegularPolygon, RegularStarPolygon, SensedMagnitude, SimsonLine, SpiralRotation, StretchReflection, StretchRotation, TangentLine, VerticalCoord, VerticalName, altitude, apothem, area, asymptotes, bisector, center, centroid, circle, circumcircle, conic, convexhull, coordinates, detail, diagonal, diameter, dilatation, directrix, distance, draw, dsegment, ellipse, excircle, expansion, foci, focus, form, homology, homothety, hyperbola, incircle, inradius, intersection, inversion, line, medial, median, method, midpoint, orthocenter, parabola, perimeter, point, powerpc, projection, radius, randpoint, reciprocation, reflection, rotation, segment, sides, similitude, slope, square, stretch, tangentpc, translation, triangle, vertex, vertices]

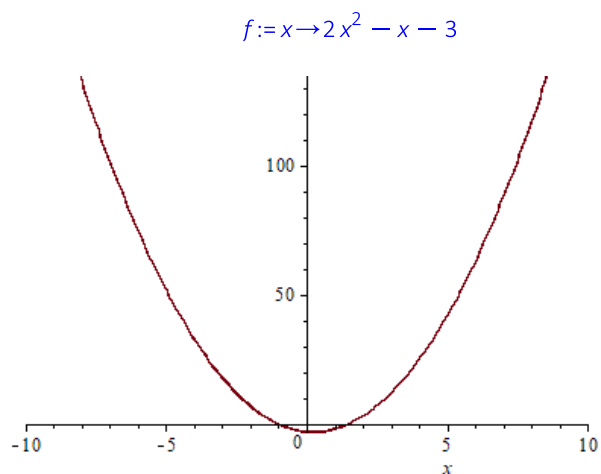
Graphing functions

To assign f to be a function of x such that $f(x) = 2x^2 - x - 3$, use $f := x \rightarrow$ (write the expression here).

> $f := x \rightarrow 2x^2 - x - 3$;

To plot the function, use the `plot ()`; command.

Notice the syntax above: $f(x)$. This means in math language: “ f of x ”. When you are graphing an equation you have to tell the software which variable is the independent variable. Here is x .



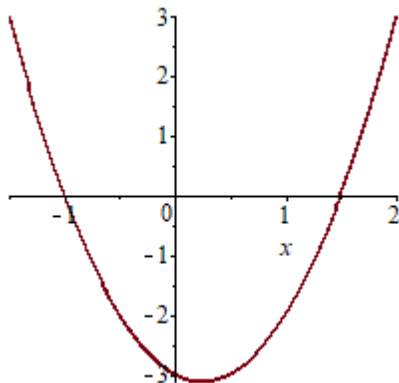
Notice also that by default Maple graphs the equations using x-values between -10 and 10. This range of values for x would be like the “domain” of the function just for graphing purposes. We can also change the scale to our convenience. Consider the equation below and its graph.

> $f := x \rightarrow 2x^2 - x - 3;$

$$f := x \rightarrow 2x^2 - x - 3$$

> $\text{plot}(f(x), x = -1.5..2);$

The syntax $\text{plot}(f(x), x = -1.5..2)$ is for plotting the graph for the given range, thus from $x = -1.5$ to $x = 2$. Note the number of dots between -1.5 and 2.



Many commands are available in maple, go ahead and explore the commands and learn how they work. Calculus commands are also available in palette menu.

Calculus in maple

(a) To differentiate $f(x) = \frac{1}{x^2} - \frac{2}{\sqrt[3]{x}} + 10x - 5$

In palette menu under calculus, select $\frac{d}{dx} f$ and edit f

$$> \frac{d}{dx} \left(\frac{1}{x^2} - \frac{2}{\sqrt[3]{x}} + 10x - 5 \right);$$

$$-\frac{2}{x^3} + \frac{2}{3x^{4/3}} + 10$$

(b) To integrate $\int \frac{x^2 + 3x - 4}{x + 3} dx$

$$> \int \left(\frac{x^2 + 3x - 4}{x + 3} \right) dx;$$

$$\frac{1}{2}x^2 - 4\ln(x + 3)$$

(c) To evaluate $\int_2^5 \frac{(x+2)(x-3)}{x+4} dx$

$$> \int_2^5 \frac{(x+2) \cdot (x-3)}{x+4} dx;$$

$$-\frac{9}{2} - 14 \ln(2) + 14 \ln(3)$$

> evalf(%);

1.17651152

MISCELLANEOUS PRACTICE

1. Write the numerical value of the following commands from your scientific calculator.

(a) [shift] $\rightarrow$ [V] $\rightarrow$ [8] $\rightarrow$ [9] $\rightarrow$ [alfa] $\rightarrow$ [$\int dx$] $\rightarrow$ [Ans] $\rightarrow$ [x] $\rightarrow$ [9]

$\rightarrow$ [=] $\rightarrow$ [=]

(b) [mode] $\rightarrow$ [mode] $\rightarrow$ [1] $\rightarrow$ [4] $\rightarrow$ [M+] $\rightarrow$ [6] $\rightarrow$ [M+] $\rightarrow$ [9] $\rightarrow$ [M+] $\rightarrow$ [5] $\rightarrow$ [M+] $\rightarrow$ [shift] $\rightarrow$ [2] $\rightarrow$ [2] $\rightarrow$ [=]

(c) [shift] $\rightarrow$ [V] $\rightarrow$ [1] $\rightarrow$ [shift] $\rightarrow$ [6] $\rightarrow$ [=]

(d) [hyp] $\rightarrow$ [tan] $\rightarrow$ [1] $\rightarrow$ [5] $\rightarrow$ [=]

2. Use your non-programmable scientific calculator to evaluate

(a) If
$$\begin{cases} 2x + 3y - z = 7 \\ x + y + 2z = -4 \\ 3x - 4y + 5z = 11 \end{cases}$$
 find the value of $\frac{z\sqrt{xy} + y\sqrt{xz} + x\sqrt{yz}}{x^2 + 3y^2 - 3z^2}$ correct to

4 decimal places.

(b) Find the value of $(\sqrt{x} + \sqrt{y})^5 + (\sqrt{x} - \sqrt{y})^5$ to 2 decimal places if

$$\begin{cases} 200x + 500y = 2433 \\ 300x - 100y = 989 \end{cases}$$

3. Evaluate each of the following using a non-programmable scientific calculator correct to 6 significant figures.

(a) $\tan^{-1}(4.145) - \tan^{-1}(2.341) + 32.431 \text{ rad}$

(b) $\sqrt{\frac{\sqrt{5} + 3\sin 50^\circ}{e^{\sqrt{2}} + e^3}}$

(c) $\left(\frac{(0.0048)^{2/3} + (6.45)^{-5}}{5.834 \times 0.321} \right)^{-3}$

(d) $\int_2^3 x \sin(x^3 + 1) dx$

4. Use a non-programmable scientific calculator to evaluate the following

(a) $\frac{d}{dx} \left(\frac{3}{7} x^4 + 3x^3 - 5x + 10 \right)$ at $x = 10$ correct to 2 decimal places.

(b) Find the modulus and argument of $z = (2 + 3i)^7$

(c) If $A = \begin{pmatrix} 1 & -3 & 4 \\ 2 & -3 & 0 \\ -8 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -8 & 1 & 4 \\ 7 & -1 & -9 \\ 0 & 2 & 7 \end{pmatrix}$ find

(i) $3A + 2B$ (ii) $5A - B$ (iii) $|A + 2B|$ (iv) $|2A - 4B|$

5. Evaluate using a non-programmable scientific calculator to evaluate

(a) $\tan^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{2}{3}\right)$ correct to 5 significant figures.

(b) $\log_6 10 - \ln\left(\frac{4}{7}\right)$ correct to 5 decimal places.

(c) From the numbers:

23 24 12 24 20 32 12 43

22 29 32 and 44. Find

(i) Average (ii) Standard deviation (ii) Variance.

6. (a) Given $\tan \theta = \frac{14.32 \tan(16^\circ 31')}{67.8}$, find θ correct to 4 decimal places

(b) Find the value of $\frac{42^\circ 57' + 2.43 \text{ rad}}{89.2}$ give your answer in radian correct to 4 significant figures.

7. Given the table below

Data	10101	20202	30303	40404	50505
Frequency	30	41	64	45	20

Find

(a) Average write correct to hundreds

(b) Variance write the full displayed answer

(c) $\left(\frac{\sum x^2}{N} - \left(\frac{\sum x}{N} \right)^2 \right)^{0.5}$ write the full displayed answer

8. (a) Solve $x^5 - 32x = 0$

(b) Evaluate $\int_1^5 \sin x dx$ correct to 5 decimal places

(c) Find the modulus and argument of $(1 + 2i)^{10}$

9. (a) What happen when the following is typed in maple text mode 4 * 13 and press enter?

10. If $A = \begin{pmatrix} 10 & 12 & 30 \\ 21 & 8 & 15 \\ 9 & 11 & 20 \end{pmatrix}$ evaluate

(a) A^3 (b) $(AA^{-1})^2$

(b) Using the calculator write each procedure, find the determinant of matrix $C = \begin{pmatrix} 1 & 0 & 1 \\ 9 & 1 & 0 \\ 1 & 3 & 4 \end{pmatrix}$ hence find the square of

it is determinant

(c) Evaluate $1087 + 25 \left(1 - \frac{1}{\frac{(1087-1320)}{10} - \frac{1}{600} + 1} \right)$ correct to three decimal places.

11. (a) By using a non-programmable scientific calculator, evaluate

(i) $k = \sum_{x=2}^6 \log \sqrt[3]{1 + \tan^{-1}(x)}$ correct to 6 decimal places

(ii) ${}^7C_4 + {}^6P_4 - \left(\begin{pmatrix} 7 & 4 & 11 \\ 9 & 3 & 6 \\ 2 & 1 & 2 \end{pmatrix} \right)^2$

(iii) $\sqrt{\frac{\sqrt{3} \cos 88^\circ + 4\sqrt{\arcsin(0.542)}}{\left(\sin^3\left(\frac{\pi}{3}\right)\right)^{2/5} \left(e^{\log 3}\right)^4}}$ correct to 7 significant figures.

(b) By using the statistical mode of a non-programmable calculator, find (i) Mean (ii) Variance correct to three decimal places (iii) Population standard deviation correct to three decimal places of the values shown in the table below

Values	250	230	210	190	170	150	130	110	90	70	50
Freq.	7	11	9	10	22	40	26	8	4	35	28

12. Write down all steps to follow in a non-programmable calculator to find

- (a) A dot product of a vector
- (b) A cross product of a vector
- (c) An magnitude of complex number
- (d) An argument of a complex number

13. If $\underline{a} = 3\underline{i} + 4\underline{j} - 7\underline{k}$ and $\underline{b} = 4\underline{i} - \underline{j} - 5\underline{k}$ find

- (a) $\underline{a} \times \underline{b}$
- (b) $2(\underline{a} \cdot \underline{b})$
- (c) $(\underline{a} \cdot \underline{b})(\underline{b} \times \underline{a})$

14. Evaluate $\tan\left(\ln\left(\sinh^{-1}\frac{\pi}{2}\right)\right) + \left(\sqrt[3]{\frac{4e^3\pi^4}{\cot\frac{4}{3\pi}}}\right)$ correct to 3 significant figures.

15. Find the positive difference between $A = \frac{(3.652 \times 10^e) + (e^{10} \times 0.00047)}{\sinh(2) + \tanh(3)}$

and $B = \frac{\sinh(2) + \tanh(3)}{(3.652 \times 10^e) + (e^{10} \times 0.00047)}$ and hence find

$C = e^{B/A} + e^B + 2e^{A/(A-B)}$ correct to 4 significant figures.

16. Evaluate $\int_0^{\pi/4} \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx$ correct to 9 significant figures

17. Use the extract of the spreadsheet below, answer the following questions.

	A	B	C	D	E	F	G	H	I
1	60	65	34	65	64	75	13		
2	23	54	32	55	65	76	83		
3	85	56	84	46	47	95	54		
4	28	66	48	06	45	36	55		

(a) Find standard deviation (STDEV.P) of the data in row 1, put your answer in H1 correct to 5 decimal places.

(b) Find average of the data in row 1, put your answer in I1 correct to 5 decimal places.

(c) Find standard deviation (STDEV.S) of the data in row 2, put your answer in H2 correct to 5 decimal places.

(d) Find standard deviation (STDEV.P) of the data in row 3 and row 4, put your answer in I4 correct to 5 decimal places.

(e) Find minimum and maximum of the whole data, put your answer in H3 and I3 respectively.

18. Solve for x: (a) $64 \times 2^x - 30 = 2^{x+2}$ (b) $5^{3x} = 34$ correct to 4 decimal places.

19. Given statistical table below

Intervals	07 – 10	17 – 20	27 – 30	37 – 40	47 – 50	57 – 60
Frequency	10	40	80	100	70	50

Calculate;

(a) Mean $\times$ 3 standard deviation to 4 decimal places.

(b) Variance to 4 decimal places.

- (c) $\sum x^2 \pm \sum x$
20. If $\sqrt{\frac{3.783+20k^3}{9.839}} = 35.7627$, Find the value of k correct to 3 decimal places.
21. Compute each of the following:-
- (a) $\sin\left(\frac{\pi}{13}\right) + \cos\left(\frac{\pi}{17}\right) - \tan\left(\frac{\pi}{7}\right)$ to 4 decimal places
- (b) $\frac{\pi}{23} - \sqrt{2.43 + \pi} + \cot\left(\frac{\pi}{31}\right)$ to 4 decimal places
- (c) $65^\circ 67' + 3.3\pi - 89^\circ$ give your answer in degree to 4 decimal places
22. (a) Evaluate $\left(\ln\left(\frac{\ln 9.26}{\log 9.26}\right) + \log\left(\frac{\log 7.88}{\ln 7.88}\right)\right)^2$ correct to 3 decimal places
- (b) Given $\log(5^{2x} - 30) = 2$ solve for x for 4 significant figures.
23. Evaluate each of the following correct to 5 significant figures
- (a) $\sqrt[7]{\frac{(2.78)^9 \times \sin^3(36.89^\circ)}{\cot^2(68.5^\circ) \times \sqrt{6.41}}}$
- (b) $\ln\left(\sqrt{\frac{e^2 \log(42.38)}{2^{5/5} \sqrt{6.768}}}\right)^{5/3}$
- (c) $\frac{\sec^{-1}(1.7313) + \csc^{-1}(2.3892)}{\tan^{-1}(2.1789) - \cot^{-1}(2.1789)}$
24. Evaluate $\left(\begin{vmatrix} 1 & 2 & 4 \\ 3 & 1 & 3 \\ 2 & 1 & -2 \end{vmatrix}\right)^{-2} + e^{-13} - \ln\left(\frac{e^{13}}{13}\right)$ correct to 8 significant figures
25. (a) Evaluate $\frac{(8.24)^4 - (2.75)^3}{9.7 + 3!}$ correct to 2 significant figures
- (b) Evaluate $\sec^{-1}(2) + (1.842)^{-1}$ correct to 2 decimal places
26. (a) Evaluate $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$ writing full display of your calculator.

(b) Evaluate $\frac{d}{dx} \left(\frac{xe^{\tan x}}{1 + (\ln x)^2} \right)^x$ when $x = \pi/4$ writing the full display of your calculator.

(c) Evaluate $\csc^{-1}(2) + \sec^{-1}(3) + \cot^{-1}(0.2)$ give your answer to 4 significant figures.

27. (a) Evaluate using a non-programmable calculator

$\left(\frac{{}^7C_3 \times \ln 2.7}{{}^3\sqrt{34}} \times \frac{{}^7P_3 \times \log 2.7}{{}^3\sqrt{34}} \right)$ give answer to 3 significant figures.

(b) Use a scientific non-programmable calculator to evaluate

$\left(\frac{\sqrt{9!}}{0!} \times \frac{0!}{\sqrt{8!}} \right)^{3.3}$ correct to 4 significant figures.

(c) Evaluate $\frac{5\sqrt{11.3} + e^{\log 5}}{(1.01)^2 \sin(46^\circ 34')}$ correct to 4 decimal places.

(d) Evaluate $\sum_{1}^5 \left(\frac{x}{2x+1} \right)^x$ correct to 5 significant figures.

28. (a) Evaluate $\sum_{2}^9 (2x+10)$

(b) Evaluate $\sum_{4}^{10} (n^2 - 2n)$

(c) Evaluate $\prod_{5}^9 (2n-3)$

(d) Evaluate $\prod_{8}^{10} (n^2 + 3n - 2)$

(e) Evaluate

$\sum_{2}^8 \left(e^{\frac{1}{10}x} \right)$ write

the full displayed answer.

POSSIBLE ANSWERS TO THE PRACTICE QUESTIONS

Double check the answers

PRACTICE

-
- | | |
|---|---|
| <p>1. $x = 3$ and $y = 2/3$</p> <p>3. $x_1 = 1$ and $x_2 = 1/2$</p> <p>5. $x = 0.8685$ or $x = -1.5352$</p> <p>7. $x = -0.5 \pm i$</p> <p>9. $x = \pm 1$ or $x = -1/3$</p> <p>11. $y = 1/2$, $y = -1$ or $y = -1/4$</p> | <p>2. No solution</p> <p>4. $x = 0.8$ and $y = 2.8$</p> <p>6. $x = 1.5$ or $x = -2$</p> <p>8. $x = 0.3333 \pm 1.1055i$</p> <p>10. $x = 2$, $x = -4$ or $x = -1$</p> <p>12. $x = \pm 2$ or $x = 1$</p> |
|---|---|
13. (a) -598 (b) $\begin{pmatrix} 99/598 & -22/299 & 20/299 \\ -45/299 & 20/299 & 9/299 \\ 5/299 & 31/299 & -1/299 \end{pmatrix}$
- (c) $\begin{pmatrix} 267/299 & 158/299 & 11/13 \\ -134/299 & 101/299 & -10/13 \\ -118/299 & 22/299 & 17/13 \end{pmatrix}$
14. (a) $\begin{pmatrix} 23 & 8 & 19 \\ 14 & 15 & 6 \\ 6 & 21 & 16 \end{pmatrix}$ (b) $\begin{pmatrix} 1821/127 & 1951/127 & 290/127 \\ 2536/127 & 1756/127 & -1994/127 \\ 498/127 & 834/127 & 1492/127 \end{pmatrix}$
- (c) $\begin{pmatrix} 148 & 132 & 124 \\ 146 & 21 & 111 \\ 148 & 160 & 29 \end{pmatrix}$ (d) $\begin{pmatrix} 156 & 122 & 241 \\ 116 & 35 & 180 \\ 78 & 147 & 175 \end{pmatrix}$

MISCELLANEOUS PRACTICE

1. (a) 40.183 (b) 1.8708 (c) 10 (d) 1
2. (a) -3.2487 (b) 848.56
3. (a) 32.5980 (b) 0.432866 (c) 282382 (d) -0.0416330
4. (a) 2609.29 (b) Mod = 3.61 and arg = 56.31
- (c) (i) $\begin{pmatrix} -13 & -7 & 20 \\ 20 & -11 & -18 \\ -24 & 7 & 17 \end{pmatrix}$ (ii) $\begin{pmatrix} 13 & -16 & 16 \\ 3 & -14 & 9 \\ -40 & 3 & -2 \end{pmatrix}$ (iii) 351 (iv) 20216
5. (a) 1.4045 (b) 1.84471 (c) (i) 26.417 (ii) 9.8 (iii) 96.08
6. (a) 0.2178 (b) 0.03565
7. (a) 29500 (b) 145250194.2 (c) 12051.97885

8. (a) $x = 0$, $x = \pm 2.38$ and $x = \pm 2.38i$ (b) 0.25664 (c) $\text{mod} = 3125$, $\text{arg} = 63.43$
9. 52
10. (a) (i) $\begin{pmatrix} 27406 & 23712 & 48450 \\ 24681 & 21554 & 43605 \\ 21261 & 18525 & 37970 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (b) 900 (c) 1104.744
11. (a) (i) 0.599624 (ii) 314 (iii) 7.006475 (b) (i) 131.4 (ii) 3350.04 (iii) 58.024
13. (a) $-27\underline{i} - 13\underline{j} - 19\underline{k}$ (b) 86 (c) $1161\underline{i} + 559\underline{j} + 817\underline{k}$
14. 3.87
15. 415.2760396, $C = 7.439$
16. 0.110468027
17. (a) 20.35200 (b) 53.71429 (c) 21.90021 (d) 22.71440 (e) Min = 6 and Max = 95
18. (a) $x = -1$ (b) 0.7304
19. (a) 1488.3852 (b) 171.1020 (c) $\sum x^2 + \sum x = 576662.5$ and $\sum x^2 - \sum x = 550112.5$
20. $k = 8.568$
21. (a) 0.7407 (b) -0.1473 (c) 658.6333°
22. (a) 0.091 (b) 1.512
23. (a) 2.9543 (b) -1.2565 (c) 1.9526
24. -10.433158
25. (a) 290 (b) 1.59
26. (a) 2.4674011 (b) 6.47066935 (c) 3.128
27. (a) 38.7 (b) 37.54 (c) 6.7397 (d) 113640
28. (a) 168 (b) 273 (c) 135135 (d) 1166848 (e) 11.77321997

ADVANCED MATHEMATICS

FOR PAPER 1



This book is intended to cover the first topic of advanced mathematics-The calculating devices. Enjoy!

All topics are available